

# Supersymmetric closed string tachyon cosmology: a first approach

V Vázquez-Báez<sup>1,2</sup> and C Ramírez<sup>2</sup>

<sup>1</sup>Benemérita Universidad Autónoma de Puebla, Facultad de Ingeniería, Blvd. Valsequillo esq. Av. San Claudio s/n, Ciudad Universitaria, Edificio 108-C, 72570 Puebla, México

<sup>2</sup>Benemérita Universidad Autónoma de Puebla, Facultad de Ciencias Físico Matemáticas, P.O. Box 165, 72000 Puebla, México

E-mail: `manuel.vazquez@correo.buap.mx`

**Abstract.** We give a worldline supersymmetric formulation for the effective action of closed string tachyon in a FRW background. This is done considering that, as shown by Vafa, the effective theory of closed string tachyons can have worldsheet supersymmetry. The Hamiltonian is constructed by means of the Dirac procedure and written in a quantum version. By using the supersymmetry algebra we are able to find solutions to the Wheeler-DeWitt equation via a more simple set of first order differential equations.

## 1. Introduction

String theory has in its lowest mode tachyons which for the lack of knowledge on how to handle unstable configurations were ignored for many years. With the inclusion of supersymmetry they can be consistently eliminated by the GSO truncation. Some years ago it has been seen that the evolution of these tachyonic instabilities can be described by the condensation of the tachyonic modes. This was first performed in the somewhat simpler case of open strings, resumed by the well known Sen conjectures [1]. For closed strings the situation is more complicated, because it involves the structure of space-time. An interesting fact in this case, is that closed string tachyons, which are nonsupersymmetric in target space, can have worldsheet supersymmetry [2]. In [3] a Lagrangian of supersymmetric tachyons in the framework of a FRW background has been worked out, in a worldline superspace where the time variable is extended to the superspace of supersymmetry [6, 7]. This work is performed along such approach considering the covariant formulation of one-dimensional supergravity of the so called ‘new’  $\Theta$  variables [8, 9], which allows in a straightforward way to write supergravity invariant actions. Supersymmetric cosmology was studied in a variety of different schemes, we refer the interested reader to the well known books by D’Eath and Moniz for a review [4, 5],

## 2. Closed String Tachyon Effective Action

The bosonic closed string tachyon effective action is given according to [10] as

$$S = \frac{1}{2\kappa_D^2} \int \sqrt{-g} e^{-2\phi} \left[ R + 4(\partial\phi)^2 - (\partial T)^2 - 2V(T) \right] d^D x, \quad (1)$$



where  $T$  is the closed string tachyon field,  $V(T)$  is the tachyon potential and  $\phi$  is the dilaton field. This action can be written in the Einstein frame by means of  $g_{\mu\nu}^{string} = e^{\phi} g_{\mu\nu}^{Einstein}$ , which is more suitable for our cosmological approach. For a four dimensional FRW metric and in the Einstein frame, the action takes the form

$$S = \int \left[ -\frac{3\dot{a}^2 a}{\kappa^2 N} + \frac{3Nka}{\kappa^2} + \frac{a^3}{\kappa^2 N} \dot{\phi}^2 + \frac{a^3}{2\kappa^2 N} \dot{T}^2 - \frac{a^3 N e^{2\phi} V(T)}{\kappa^2} \right] dt, \quad (2)$$

where  $N$  is the lapse function and  $a$  is the scale factor. This Lagrangian is invariant under time reparametrizations and this invariance is extended to supersymmetry introducing a Grassmann superspace associated to the bosonic time coordinate  $t$  (see Tkach et al. in [6, 7]).

### 3. Supersymmetric Closed String Tachyon Model

As stated in [11] superspace is the natural framework for a geometrical formulation of supersymmetry and supergravity, it extends spacetime by anticommuting Grassmann variables,  $x^m \rightarrow (x^m, \theta^\mu)$ , and the field content of the superfields is given by the Grassmann power expansion in the anticommuting variables  $\phi(z) = \sum_n 1/n! \theta^{\mu_1} \dots \theta^{\mu_n} \phi_{\mu_1 \dots \mu_n}(x)$ .

The supersymmetric cosmological model is obtained upon an extension of the time coordinate into a supermultiplet  $t \rightarrow (t, \Theta, \bar{\Theta})$ . Due to this generalization the fields of the theory are generalized as superfields, in this the expansion mentioned above is given by

$$\begin{aligned} \mathcal{A}(t, \Theta, \bar{\Theta}) &= a(t) + i\Theta\bar{\lambda}(t) + i\bar{\Theta}\lambda(t) + B(t)\Theta\bar{\Theta}, \\ \mathcal{T}(t, \Theta, \bar{\Theta}) &= T(t) + i\Theta\bar{\eta}(t) + i\bar{\Theta}\eta(t) + G(t)\Theta\bar{\Theta}, \\ \Phi(t, \Theta, \bar{\Theta}) &= \phi(t) + i\Theta\bar{\chi}(t) + i\bar{\Theta}\chi(t) + F(t)\Theta\bar{\Theta}, \end{aligned} \quad (3)$$

where,  $\mathcal{A}$ ,  $\mathcal{T}$  and  $\Phi$  are the superfields of  $a$ ,  $T$  and  $\phi$ .

The supersymmetric generalization of the action is given by

$$S = S_{Rsusy} + S_{Msusy}, \quad (4)$$

where,  $S_{Rsusy}$  is the cosmological supersymmetric generalization of the free FRW model

$$S_{Rsusy} = \int \left( \frac{3\mathcal{E}}{\kappa^2} \mathcal{A} \nabla_{\bar{\Theta}} \mathcal{A} \nabla_{\Theta} \mathcal{A} - \frac{3\sqrt{k}}{\kappa^2} \mathcal{E} \mathcal{A}^2 \right) d\Theta d\bar{\Theta} dt, \quad (5)$$

and the supersymmetric matter term is

$$S_{Msusy} = \frac{1}{\kappa^2} \int \left[ -\mathcal{E} \mathcal{A}^3 \nabla_{\bar{\Theta}} \Phi \nabla_{\Theta} \Phi - \frac{1}{2} \mathcal{E} \mathcal{A}^3 \nabla_{\bar{\Theta}} \mathcal{T} \nabla_{\Theta} \mathcal{T} + \mathcal{E} \mathcal{A}^3 W(\Phi, \mathcal{T}) \right] d\Theta d\bar{\Theta} dt, \quad (6)$$

where  $W(\Phi, \mathcal{T})$  is the superpotential, and  $\mathcal{E}$  is an invariant density playing the role of a supersymmetric  $\sqrt{-g}$  in the action and given by

$$\mathcal{E} = -e - \frac{i}{2} (\Theta \bar{\Psi} + \bar{\Theta} \psi), \quad (7)$$

see [3, 8, 9] for details of its calculation.

The superpotential expansion is  $W(\Phi, \mathcal{T}) = W(\phi, T) + \frac{\partial W}{\partial \phi} (\Phi - \phi) + \frac{\partial W}{\partial T} (\mathcal{T} - T) + \frac{1}{2} \frac{\partial^2 W}{\partial T^2} (\mathcal{T} - T)^2 + \frac{1}{2} \frac{\partial^2 W}{\partial \phi^2} (\Phi - \phi)^2 + \frac{\partial^2 W}{\partial T \partial \phi} (\mathcal{T} - T)(\Phi - \phi)$ . This expansion is finite because the terms  $(\mathcal{T} - T)$  and  $(\Phi - \phi)$  are purely grassmannian and nilpotency holds for these kind of variables.

Integrating over the Grassmann parameters, the total Lagrangian is

$$\begin{aligned}
 L = & -\frac{3a\dot{a}^2}{e\kappa^2} + \frac{3a\dot{a}}{e\kappa^2} (\psi\lambda - \bar{\psi}\bar{\lambda}) + \frac{a^3\dot{T}^2}{2e\kappa^2} - \frac{a^3\dot{T}}{2e\kappa^2} (\psi\eta - \bar{\psi}\bar{\eta}) + \frac{3ia^2\dot{T}}{2\kappa^2} (\lambda\bar{\eta} + \bar{\lambda}\eta) + \frac{a^3\dot{\phi}^2}{e\kappa^2} \\
 & - \frac{a^3\dot{\phi}}{e\kappa^2} (\psi\chi - \bar{\psi}\bar{\chi}) + \frac{3ia^2\dot{\phi}}{\kappa^2} (\lambda\bar{\chi} + \bar{\lambda}\chi) + \frac{3ia}{\kappa^2} (\lambda\dot{\lambda} + \bar{\lambda}\dot{\lambda}) - \frac{ia^3}{2\kappa^2} (\eta\dot{\eta} + \bar{\eta}\dot{\eta}) - \frac{ia^3}{\kappa^2} (\chi\dot{\chi} + \bar{\chi}\dot{\chi}) \\
 & + \frac{6e\sqrt{k}\lambda\bar{\lambda}}{\kappa^2} + \frac{3ia\sqrt{k}}{\kappa^2} (\psi\lambda + \bar{\psi}\bar{\lambda}) - \frac{6aeW\lambda\bar{\lambda}}{\kappa^2} - \frac{3ia^2W}{2\kappa^2} (\psi\lambda + \bar{\psi}\bar{\lambda}) - \frac{ia^3W_T}{2\kappa^2} (\psi\eta + \bar{\psi}\bar{\eta}) \\
 & + \frac{3a^2eW_T}{\kappa^2} (\bar{\lambda}\eta - \lambda\bar{\eta}) - \frac{ia^3W_\phi}{2\kappa^2} (\psi\chi + \bar{\psi}\bar{\chi}) + \frac{3a^2eW_\phi}{\kappa^2} (\bar{\lambda}\chi - \lambda\bar{\chi}) - \frac{a^3eW_{TT}}{\kappa^2}\eta\bar{\eta} \\
 & + \frac{a^3eW_{T\phi}}{\kappa^2} (\bar{\chi}\eta - \chi\bar{\eta}) - \frac{a^3eW_{\phi\phi}}{\kappa^2}\chi\bar{\chi} - \frac{3a\psi\bar{\psi}\lambda\bar{\lambda}}{2e\kappa^2} + \frac{a^3\psi\bar{\psi}\eta\bar{\eta}}{4e\kappa^2} + \frac{a^3\chi\bar{\chi}\psi\bar{\psi}}{2e\kappa^2} - \frac{3ae\lambda\bar{\lambda}\eta\bar{\eta}}{\kappa^2} \\
 & - \frac{6ae\lambda\bar{\lambda}\chi\bar{\chi}}{\kappa^2} - \frac{3aB^2e}{\kappa^2} + \frac{6aBe\sqrt{k}}{\kappa^2} - \frac{3Be\lambda\bar{\lambda}}{\kappa^2} - \frac{3a^2Be\eta\bar{\eta}}{2\kappa^2} - \frac{3a^2Be\chi\bar{\chi}}{\kappa^2} - \frac{3a^2BeW}{\kappa^2} \\
 & + \frac{a^3eG^2}{2\kappa^2} + \frac{3a^2eG}{2\kappa^2} (\lambda\bar{\eta} - \bar{\lambda}\eta) - \frac{a^3eGW_T}{\kappa^2} + \frac{a^3eF^2}{\kappa^2} + \frac{3a^2eF}{\kappa^2} (\lambda\bar{\chi} - \bar{\lambda}\chi) - \frac{a^3eFW_\phi}{\kappa^2},
 \end{aligned}$$

where the subscripts in  $W$  denote partial differentiation with respect to  $\phi$  and  $T$  respectively.

When we perform the variation of the Lagrangian with respect to the fields  $B$ ,  $F$  and  $G$ , as usual algebraic constraints are obtained, that is  $B$ ,  $F$  and  $G$  play the role of auxiliary fields, and they can be eliminated from the Lagrangian. Upon solving for the auxiliary fields and making the further rescalings for convenience  $\lambda \rightarrow \kappa a^{-1/2}\lambda$ ,  $\bar{\lambda} \rightarrow \kappa a^{-1/2}\bar{\lambda}$ ,  $\eta \rightarrow \kappa a^{-3/2}\eta$ ,  $\bar{\eta} \rightarrow \kappa a^{-3/2}\bar{\eta}$ ,  $\chi \rightarrow \kappa a^{-3/2}\chi$ ,  $\bar{\chi} \rightarrow \kappa a^{-3/2}\bar{\chi}$ , we find the Lagrangian

$$\begin{aligned}
 L = & -\frac{3a\dot{a}^2}{e\kappa^2} + \frac{3\sqrt{a}\dot{a}}{e\kappa} (\psi\lambda - \bar{\psi}\bar{\lambda}) + \frac{3eka}{\kappa^2} + \frac{\dot{T}^2a^3}{2e\kappa^2} - \frac{\sqrt{a^3}\dot{T}}{2e\kappa} (\psi\eta - \bar{\psi}\bar{\eta}) + \frac{3i\dot{T}}{2} (\lambda\bar{\eta} + \bar{\lambda}\eta) \\
 & + \frac{\dot{\phi}^2a^3}{e\kappa^2} - \frac{\sqrt{a^3}\dot{\phi}}{e\kappa} (\psi\chi - \bar{\psi}\bar{\chi}) + 3i\dot{\phi} (\lambda\bar{\chi} + \bar{\lambda}\chi) + 3i (\lambda\dot{\lambda} + \bar{\lambda}\dot{\lambda}) - \frac{i}{2} (\eta\dot{\eta} + \bar{\eta}\dot{\eta}) \\
 & - i (\chi\dot{\chi} + \bar{\chi}\dot{\chi}) + \frac{3e\sqrt{k}\lambda\bar{\lambda}}{a} - \frac{3e\sqrt{k}\eta\bar{\eta}}{2a} - \frac{3e\sqrt{k}\chi\bar{\chi}}{a} + \frac{3i\sqrt{a}k}{\kappa} (\psi\lambda + \bar{\psi}\bar{\lambda}) \\
 & + \frac{3eW^2a^3}{4\kappa^2} - \frac{3e\sqrt{k}a^2W}{\kappa^2} - \frac{eW_T^2a^3}{2\kappa^2} - \frac{eW_\phi^2a^3}{4\kappa^2} - \frac{9}{2}eW\lambda\bar{\lambda} + \frac{3}{4}eW\eta\bar{\eta} + \frac{3}{2}eW\chi\bar{\chi} \\
 & - \frac{3ia^{3/2}W}{2\kappa} (\psi\lambda + \bar{\psi}\bar{\lambda}) - \frac{i\sqrt{a^3}W_T}{2\kappa} (\psi\eta + \bar{\psi}\bar{\eta}) + \frac{3eW_T}{2} (\bar{\lambda}\eta - \lambda\bar{\eta}) - \frac{i\sqrt{a^3}W_\phi}{2\kappa} (\psi\chi + \bar{\psi}\bar{\chi}) \\
 & + \frac{3eW_\phi}{2} (\bar{\lambda}\chi - \lambda\bar{\chi}) - eW_{TT}\eta\bar{\eta} + eW_{T\phi} (\bar{\chi}\eta - \chi\bar{\eta}) - eW_{\phi\phi}\chi\bar{\chi} + \frac{3e\kappa^2}{4a^3}\eta\bar{\eta}\chi\bar{\chi} - \frac{3}{2e}\psi\bar{\psi}\lambda\bar{\lambda} \\
 & + \frac{1}{2e}\psi\bar{\psi}\chi\bar{\chi} + \frac{1}{4e}\psi\bar{\psi}\eta\bar{\eta}.
 \end{aligned}$$

#### 4. Hamiltonian analysis

We have the following canonical momenta

$$\begin{aligned}\pi_a &= -\frac{6a\dot{a}}{e\kappa^2} - \frac{3\sqrt{a}\bar{\psi}\bar{\lambda}}{e\kappa} + \frac{3\sqrt{a}\psi\lambda}{e\kappa}, \\ \pi_T &= \frac{a^3\dot{T}}{e\kappa^2} + \frac{\sqrt{a^3}\bar{\psi}\bar{\eta}}{2e\kappa} - \frac{\sqrt{a^3}\psi\eta}{2e\kappa} + \frac{3a^{3/2}i\lambda\bar{\eta}}{2\sqrt{a^3}} + \frac{3a^{3/2}i\bar{\lambda}\eta}{2\sqrt{a^3}}, \\ \pi_\phi &= \frac{2a^3\dot{\phi}}{e\kappa^2} + \frac{\sqrt{a^3}\bar{\psi}\bar{\chi}}{e\kappa} - \frac{\sqrt{a^3}\psi\chi}{e\kappa} + \frac{3a^{3/2}i\lambda\bar{\chi}}{\sqrt{a^3}} + \frac{3a^{3/2}i\bar{\lambda}\chi}{\sqrt{a^3}}, \\ \pi_\lambda &= -3i\bar{\lambda}, \quad \pi_{\bar{\lambda}} = -3i\lambda, \\ \pi_\eta &= \frac{i}{2}\bar{\eta}, \quad \pi_{\bar{\eta}} = \frac{i}{2}\eta, \\ \pi_\chi &= i\bar{\chi}, \quad \pi_{\bar{\chi}} = i\chi.\end{aligned}$$

It can be seen, as usual, the appearance of the fermionic constraints

$$\begin{aligned}\Omega_\lambda &= \pi_\lambda + 3i\bar{\lambda}, \quad \Omega_{\bar{\lambda}} = \pi_{\bar{\lambda}} + 3i\lambda, \\ \Omega_\eta &= \pi_\eta - \frac{i}{2}\bar{\eta}, \quad \Omega_{\bar{\eta}} = \pi_{\bar{\eta}} - \frac{i}{2}\eta, \\ \Omega_\chi &= \pi_\chi - i\bar{\chi}, \quad \Omega_{\bar{\chi}} = \pi_{\bar{\chi}} - i\chi.\end{aligned}\tag{8}$$

According to the Dirac formalism, the previous constraints are second class and the dynamics of the system is obtained when we impose the set of constraints (8) and introduce the Dirac brackets, we have in this case

$$\begin{aligned}\{a, \pi_a\}_D &= 1, \quad \{\phi, \pi_\phi\}_D = 1, \quad \{T, \pi_T\}_D = 1, \\ \{\lambda, \bar{\lambda}\}_D &= -\frac{1}{6i}, \quad \{\chi, \bar{\chi}\}_D = -\frac{i}{2}, \quad \{\eta, \bar{\eta}\}_D = -i,\end{aligned}\tag{9}$$

as the only non zero brackets of the theory.

Following Dirac's procedure, using the standard definition for the Hamiltonian and imposing the constraints (8), the Hamiltonian of the system can be written as

$$H = NH_0 + \frac{1}{2}\psi S - \frac{1}{2}\bar{\psi}\bar{S},\tag{10}$$

where

$$\begin{aligned}H_0 &= -\frac{\kappa^2\pi_a^2}{12a} + \frac{\kappa^2\pi_T^2}{2a^3} - \frac{3i\kappa^2\pi_T}{2a^3}(\lambda\bar{\eta} + \bar{\lambda}\eta) + \frac{\kappa^2\pi_\phi^2}{4a^3} - \frac{3i\kappa^2\pi_\phi}{2a^3}(\lambda\bar{\chi} + \bar{\lambda}\chi) - \frac{3a^3}{4\kappa^2}W^2 \\ &\quad + \frac{3\sqrt{ka^2}}{\kappa^2}W - \frac{3ka}{\kappa^2} + \frac{a^3}{2\kappa^2}W_T^2 + \frac{a^3}{4\kappa^2}W_\phi^2 + \frac{9}{2}W\lambda\bar{\lambda} - \frac{3}{4}W\eta\bar{\eta} - \frac{3}{2}W\chi\bar{\chi} \\ &\quad + \frac{3}{2}W_T(\lambda\bar{\eta} - \bar{\lambda}\eta) + \frac{3}{2}W_\phi(\lambda\bar{\chi} - \bar{\lambda}\chi) + W_{TT}\eta\bar{\eta} + W_{T\phi}(\chi\bar{\eta} - \bar{\chi}\eta) \\ &\quad + W_{\phi\phi}\chi\bar{\chi} - \frac{3\sqrt{k}}{a}\lambda\bar{\lambda} + \frac{3\sqrt{k}}{2a}\eta\bar{\eta} + \frac{3\sqrt{k}}{a}\chi\bar{\chi} - \frac{9\kappa^2}{2a^3}\lambda\bar{\lambda}\chi\bar{\chi} - \frac{9\kappa^2}{4a^3}\lambda\bar{\lambda}\eta\bar{\eta} - \frac{3\kappa^2}{4a^3}\eta\bar{\eta}\chi\bar{\chi},\end{aligned}\tag{11}$$

$$\begin{aligned}S &= \frac{\kappa\pi_a}{\sqrt{a}}\lambda + \frac{\kappa\pi_T}{\sqrt{a^3}}\eta + \frac{\kappa\pi_\phi}{\sqrt{a^3}}\chi - \frac{6i\sqrt{ak}}{\kappa}\lambda + \frac{3i\sqrt{a^3}}{\kappa}W\lambda \\ &\quad + \frac{i\sqrt{a^3}}{\kappa}W_T\eta + \frac{i\sqrt{a^3}}{\kappa}W_\phi\chi + \frac{3i\kappa}{2a^{3/2}}\lambda\eta\bar{\eta} + \frac{3i\kappa}{a^{3/2}}\lambda\chi\bar{\chi},\end{aligned}\tag{12}$$

$$\begin{aligned}\bar{S} &= \frac{\kappa\pi_a}{\sqrt{a}}\bar{\lambda} + \frac{\kappa\pi_T}{\sqrt{a^3}}\bar{\eta} + \frac{\kappa\pi_\phi}{\sqrt{a^3}}\bar{\chi} + \frac{6i\sqrt{ak}}{\kappa}\bar{\lambda} - \frac{3i\sqrt{a^3}}{\kappa}W\bar{\lambda} \\ &\quad - \frac{i\sqrt{a^3}}{\kappa}W_T\bar{\eta} - \frac{i\sqrt{a^3}}{\kappa}W_\phi\bar{\chi} - \frac{3i\kappa}{2a^{3/2}}\bar{\lambda}\eta\bar{\eta} - \frac{3i\kappa}{a^{3/2}}\bar{\lambda}\chi\bar{\chi},\end{aligned}\tag{13}$$

are the set of first class constraints satisfying the Dirac algebra  $\{S, \bar{S}\}_D = 2H_0$ ,  $\{H_0, S\}_D = \{H_0, \bar{S}\}_D = 0$ .

### 5. Wave function of the universe

In a previous work, see for instance [12], we computed a wave function of the universe for a supersymmetric tachyon in a FRW background, providing a normalizable solution in which the unstable nature of the tachyon can be appreciated; in such study the action (2) was used with the dilaton turned off. Now we address the solution of the Wheeler-DeWitt equation for the complete theory.

The constraints (11)-(13) become conditions on the wave function of the universe, i.e,  $H\Psi = \bar{S}\Psi = S\Psi = F\Psi = 0$ , which due to the operator superalgebra, specifically  $\{S, \bar{S}\} = 2H$ , imply that any state wave function,  $\Psi$ , satisfying  $\bar{S}\Psi = S\Psi = 0$  is also solution of the Wheeler-DeWitt equation,  $H\Psi = 0$ .

There are bilinear terms in the constraints  $S$  and  $\bar{S}$ , eqs. (14) and (13), that need to be ordered to produce a suitable quantum theory, we make use of the anticommutation properties of Grassmann variables at the classical level to write such terms as commutators e.g.  $\lambda\chi\bar{\chi} \rightarrow \lambda[\chi, \bar{\chi}]$ . Thus we have for the quantum constraints:

$$S = \frac{\kappa\pi_a}{\sqrt{a}}\lambda + \frac{\kappa\pi_\phi}{\sqrt{a^3}}\chi + \frac{\kappa\pi_T}{\sqrt{a^3}}\eta + \frac{3i\sqrt{a^3}W}{\kappa}\lambda + \frac{i\sqrt{a^3}W_T}{\kappa}\eta + \frac{i\sqrt{a^3}W_\phi}{\kappa}\chi \quad (14)$$

$$+ \frac{3i\kappa}{4a^{3/2}}\lambda[\eta, \bar{\eta}] + \frac{3i\kappa}{2a^{3/2}}\lambda[\chi, \bar{\chi}] - \frac{6i\sqrt{ak}}{\kappa}\lambda,$$

$$\bar{S} = \frac{\kappa\pi_a}{\sqrt{a}}\bar{\lambda} + \frac{\kappa\pi_\phi}{\sqrt{a^3}}\bar{\chi} + \frac{\kappa\pi_T}{\sqrt{a^3}}\bar{\eta} - \frac{3i\sqrt{a^3}W}{\kappa}\bar{\lambda} - \frac{i\sqrt{a^3}W_T}{\kappa}\bar{\eta} - \frac{i\sqrt{a^3}W_\phi}{\kappa}\bar{\chi} \quad (15)$$

$$- \frac{3i\kappa}{4a^{3/2}}\bar{\lambda}[\eta, \bar{\eta}] - \frac{3i\kappa}{2a^{3/2}}\bar{\lambda}[\chi, \bar{\chi}] + \frac{6i\sqrt{ak}}{\kappa}\bar{\lambda}.$$

We introduce a representation for the Grassmann variables in terms of suitable linear combinations of 8-dimension Dirac matrices, thus the wave function will be an eight-component spinor  $\Psi = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8)$ . Upon application of the operators  $S$  and  $\bar{S}$  on this state, we get two sets of eight equations, from which we see that the only nonzero components are  $\psi_1$  and  $\psi_6$ , which satisfy the following equations

$$\left(\hbar\kappa\frac{\partial}{\partial\phi} - \frac{a^3}{\kappa}\frac{\partial W}{\partial\phi}\right)\psi_1 = 0,$$

$$\left(\hbar\kappa\frac{\partial}{\partial T} - \frac{a^3}{\kappa}\frac{\partial W}{\partial T}\right)\psi_1 = 0, \quad (16)$$

$$\left(\hbar a\kappa^2\frac{\partial}{\partial a} - 3a^3W + 6a^2k + \frac{9}{8}\right)\psi_1 = 0,$$

and

$$\left(\hbar\kappa^2\frac{\partial}{\partial\phi} + a^3\frac{\partial W}{\partial\phi}\right)\psi_6 = 0,$$

$$\left(\hbar\kappa\frac{\partial}{\partial T} + \frac{a^3}{\kappa}\frac{\partial W}{\partial T}\right)\psi_6 = 0, \quad (17)$$

$$\left(\hbar a\kappa^2\frac{\partial}{\partial a} + 3a^3W - 6a^2k + \frac{9}{8}\right)\psi_6 = 0.$$

Solving these equations we find that

$$\psi_1(\phi, T, a) = a^{-\frac{9}{8\hbar}} \exp\left[\frac{a^{3/2}\left(\sqrt{a^3}W(\phi, T) - 3\sqrt{ak}\right)}{\kappa^2\hbar}\right],$$

$$\psi_6(\phi, T, a) = a^{-\frac{9}{8\hbar}} \exp \left[ -\frac{a^{3/2} \left( \sqrt{a^3} W(\phi, T) - 3\sqrt{ak} \right)}{\kappa^2 \hbar} \right].$$

## 6. Conclusions

We have constructed a supersymmetric Lagrangian for the complete supersymmetric cosmological model of the closed string tachyon in a FRW background, in the ‘new’- $\Theta$  variables formalism. In order to quantize this theory and extract information of the wave function of the universe, we computed the Hamiltonian of this theory, upon a well suited representation of the Grassmann variables as linear combinations of Dirac matrices. We have found a set of first order partial differential equations for the components of the wave function by means of the supersymmetry algebra. This solution is dependent on the superpotential of the theory, as well as on the scale factor. It would be interesting to study the effects of tachyon in other cosmological models, following the lines of these work.

## Acknowledgments

We thank VIEP-BUAP and PIFI-SEP for the support, V. Vázquez-Báez also thanks CONACyT for the studies grant during this work and the organizers of the conference for supporting his attendance.

## References

- [1] Sen A 2005 *Int. J. Mod. Phys. A* **20** 5513  
Sen A 2005 *Phys. Scripta* **T117** 70-75 (*Preprint* hep-th/0312153)  
Sen A 2003 *Int. J. Mod. Phys. A* **18**, 4869  
Sen A 2002 *Mod. Phys. Lett. A* **17**, 1797 (2002);  
Sen A 2002 *J. High Energy Phys.* JHEP0207065;  
Sen A 2002 *J. High Energy Phys.* JHEP0204048  
Sen A 2000 *J. High Energy Phys.* JHEP003002
- [2] Vafa C 2001 Mirror symmetry and closed string tachyon condensation *Preprint* hep-th/0111051
- [3] García-Jiménez G, Ramírez C and Vázquez-Báez V 2014 *Phys. Rev. D* **89** 043501
- [4] D’Eath P D 1996 *Supersymmetric quantum cosmology* (Cambridge, UK : Cambridge University Press)
- [5] Moniz P V 2010 *Quantum Cosmology - The Supersymmetric Perspective* vol 2 Advanced Topics Lect. Notes Phys. 804 (Berlin: Springer)
- [6] Obregón O, Rosales J J, and Tkach V I 1996 *Phys. Rev. D* **53** R1750
- [7] Tkach V I, Obregón O, Rosales J J 1997 *Class. Quant. Grav.* **14** 339
- [8] Wess J and Bagger J 1992 *Supersymmetry and Supergravity* (Princeton, New Jersey: Princeton University Press)
- [9] C. Ramírez 1985 *Ann. Phys.* **186** 43
- [10] Yang H and Zwiebach B 2005 *J. High Energy Phys.* JHEP0508046
- [11] Salam A and Strathdee J 1974 *Nucl. Phys.* **B76**
- [12] Vázquez-Báez V and Ramírez C 2013 *AIP Conf. Proc.* **1548** 227 (2013) 9th Mexican School on Gravitational Physics: Cosmology for the 21st Century 3-7 December 2012 (Puerto Vallarta: Jalisco México)