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Vanishing condition for the heat flux of a relativistic fluid in a moving frame

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Abstract. It has been asked if is appropriate to introduce the heat flow in the energy-momentum tensor, due to the non-mechanical nature of heat [1]. Although this could be answered by both kinetic and symmetry arguments, we address the problem by checking the validity of the second law of thermodynamics in a fluid that is boosted by a Lorentz transformation to a non comoving frame. In this contribution we found that this only can happen under certain conditions. Indeed, we found that there are a family of reference frames that satisfies these conditions, where Landau-Lifshitz frame is one of those. Additionally we relate such conditions with the null energy condition and the entropy production.

1. Introduction

The description of relativistic fluids is need to describe astrophysical systems such as neutron stars or primordial gases in cosmological models and in experiments such as collisions between heavy nuclei at high speeds at RHIC or LHC at CERN and recently to model systems such as graphene [2].

Heat flow and its constitutive equations in relativistic systems have been the subject of various interpretations and discussions in Eckart's formalism, where the constitutive equation of heat flow contains an acceleration term that seems to be the major source of instability in the fluctuations, which implies for instance, that water at room temperature becomes unstable and the nonexistence of the Rayleigh-Brillouin spectrum [3, 4].

Recently, constitutive equations that attempt to solve this problem have been proposed [4, 5, 6]. One alternative, comes from the relativistic kinetic theory [7] where a pressure gradient appears naturally instead of the acceleration term, and also turns theory stable [6, 4]. In the mid 80's the relativistic extension of the Meixner Prigogine formalism was made [8]. In this theory the heat flux is not included in the energy-momentum tensor [9], it appears as an independent 4-vector and as a consequence such unstabilites does not appear. It was argued that the only reason to include the heat flux in the energy momentum tensor is to fulfill the energy conservation [1]. It was claimed because the heat is a non mechanical kind of energy and also has properties of momentum and inertia, can not be included into energy-momentum tensor.

Given such objections the question whether to introduce the heat flux in the energy momentum tensor is in order. For its transformation properties [10] and also from the kinetic theoretical point of view the heat flux must be part of the energy tensor [7]. However we address this question from a rather simple point view.

Consider two inertial frames connected by a Lorentz transformation such that in the comoving frame the fluid has heat flow and in the moving one it does not. This might be interpreted that through a transformation between reference systems one could convert non-mechanical energy into purely mechanical. Apparently, this fact contradicts the second law of thermodynamics [11]. On the other hand, it can be thought that the frame with no heat flux is just the Landau-Lifshitz frame, the so-called frame *that moves with the heat*, however, as we shall see this frame has very strong conditions that are not essential.

In this work we found that, in order to cancel the heat flux with a Lorentz boost, some conditions must be satisfied by the fluid. We obtain those conditions and relate them with the null energy condition for the positivity of the energy density [12]. We also compute the entropy production and notice that is in agreement with the second law. We found that for a fluid that fulfill the conditions there are a plenty of reference frames in which the heat flow vanishes, one such is used in the Landau-Lifshitz approach.

2. Convective and non-convective energy flows

From kinetic arguments there is no objection to introduce the heat flux in the energy-momentum tensor [4, 7]. However there is a subtlety to consider when comparing with the non-relativistic case.

In case the non-relativistic energy flow can be written as [13]:

$$\frac{1}{2}\int v_i' v'^2 f dv' = u_i\left(\frac{1}{2}nmu^2+e\right)+u_ip_{ij}+\frac{1}{2}\int c_i' c'^2 f dv', \tag{1}$$

where $f = f(v')$ is the distribution function, v' is the molecular velocity, u is the hydrodynamic velocity molecules, c' the chaotic velocity (which is the molecular velocity in the comoving frame), nm mass density, e the energy and p the pressure.

There are three terms in (1): the first is the energy flow due to macroscopic convection and the second should be interpreted macroscopically due to work done by the stresses per unit time. The third term represents another kind of energy flow, which is usually called the heat flux vector, and is denoted by q:

$$q_i = \frac{1}{2}\int c_i' c'^2 f dv', \tag{2}$$

where it plays the same role in the macroscopic equations [13]. However, the name heat flow is somewhat misleading, because there are situations when $q \neq 0$ and the temperature is practically constant everywhere, in this case one has to talk of heat flux at constant temperature. Hence, the name non-convective energy flow would be more appropriate for q but is not used.

In the special relativistic case, we notice that by taking the Lorentz transform of the energy-momentum tensor to a moving frame, we obtain both a convective and a non-convective flux.

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Thus, it seems that we are converting non-mechanical to mechanical kind of energy. If a Lorentz transformation become all non-mechanical contribution, purely mechanical, we could be facing a system that does not satisfy the contents of the second law of thermodynamics [11].

In next section we obtain the conditions to be met by a fluid for make this possible, and verify if second law is violated.

3. Vanishing heat flux in a moving frame

From the point of view of kinetic theory, the energy momentum tensor is identified as the second statistical moment of the distribution function [7],

$$T^{\mu\nu} = \int p^\mu p^\nu f \, \frac{d^3p}{p^0}, \tag{3}$$

where p^0 is the energy, p^μ the 4-momentum and f the relativistic distribution function. In terms of its components is written as follows

$$T^{\mu\nu} = \begin{pmatrix} ne & \vec{q}^T \\ \vec{q} & \mathbb{P} \end{pmatrix}, \tag{4}$$

with $\mathbb{P}$ the viscosity stress tensor, $\vec{q}$ the heat flux vector, and ne the energy density that transforms with two Lorentz factors, one for the energy per particle and another by the density [10]. For a perfect fluid in its matrix form is represented as,

$$T^{\epsilon\sigma} = \begin{pmatrix} ne & 0 & 0 & 0 \\ 0 & p_x & 0 & 0 \\ 0 & 0 & p_y & 0 \\ 0 & 0 & 0 & p_z \end{pmatrix},$$

where it can be considered p_x, p_y, p_z as anisotropic pressures.

The matrix form of the Lorentz transformation in the direction x is,

$$\Lambda^\alpha_\epsilon = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where the Lorentz factor is given by $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$, and $\beta = \dfrac{v}{c}$ and c is the velocity of light. The Lorentz transformed quantities of the energy momentum tensor can be calculated as follows

$$\overline{T}^{\alpha\beta} = \Lambda^\alpha_\epsilon \Lambda^\beta_\sigma T^{\epsilon\sigma}. \tag{5}$$

The heat flux is given by

$$\bar{q}_x = -\beta\gamma^2(en + p_x). \tag{6}$$

As the reader can see there is a convective flow induced by the Lorentz transformation.

Next we consider a fluid with a heat flux in x direction as seen in its comoving frame. Then we compute a Lorentz transform in x direction where the heat flux turns to be,

$$\bar{q}_x = [q_x\beta^2 - (ne + p_x)\beta + q_x]\gamma^2. \tag{7}$$

Comparing (6) and (7) we notice that the total heat flux in this frame can be split into two parts, a convective and non-convective flux, as follows,

$$\bar{q}_x = \bar{q}_C + \bar{q}_{NC} \tag{8}$$

where

$$\bar{q}_C = -\beta\gamma^2(ne + p_x)$$
$$\bar{q}_{NC} = q_x\gamma^2(1+\beta^2).$$

Now, we set to zero the heat flux in the transformed tensor, which would mean that through a transformation between the reference systems we could convert convective into non-convective flux. With this end, we seek for the conditions that the thermodynamical quantities must satisfy in order to guarantee the existence of β i.e. such that $0 < \beta < 1$, and therefore the corresponding Lorentz transformation.

It is therefore necessary solve the following equation,

$$\alpha\beta^2 - \beta + \alpha = 0, \tag{9}$$

that comes form Eq. (7), where $\alpha = q_x/(ne + p_x)$. The solution to this last equation is easy to find and can be seen in Figure 1, namely,

$$\beta(\alpha) = \frac{1 \pm \sqrt{1-4\alpha^2}}{2\alpha}. \tag{10}$$

For $0 < \beta \leq 1$, the function α should be $\alpha \leq 0.5$. In particular when $\alpha = 0.5 \Rightarrow \beta = 1$ and is well known that photons do not define a frame of reference. So, in order for the heat flux vanish in this frame, the fluid in the comoving frame, should satisfy the following condition

$$q_x < \frac{1}{2}(ne + p_x). \tag{11}$$

As it is well known to make contact with the Landau Lifshitz scheme form the Eckart's frame, is possible to define a Lorentz frame only if $(q_x/nh_E)^2 \approx 0$, this condition is contained in Eq. (11) when $(q_x/nh_E)^2 < 1/4$, where $h_E = e + p_x/n$ [7]. Is important to notice that this implies that there is more than one reference frame where $\bar{q} = 0$.

This condition is simple and seems that can be obtained from another restriction. The null energy condition states that the energy density of any matter distribution measured by any null observer must be non-negative [12], and this happens when,

$$T^{\mu\nu}k_\mu k_\nu \geq 0, \tag{12}$$

where k_μ, k_ν are null vectors. If we choose the vector $k^\nu = (c, c, 0, 0)$, and the energy-momentum tensor with heat flux, then with a little algebra we obtain that,

$$\frac{1}{2} \geq \frac{q_x}{ne + p_x}, \tag{13}$$

which is the same condition as (11).

In order to verify the validity of the second law on a fluid under condition (11), let us calculate the entropy production in the Eckart frame [7]. Because we are considering a non viscous heat conducting fluid, the entropy production is reduced to

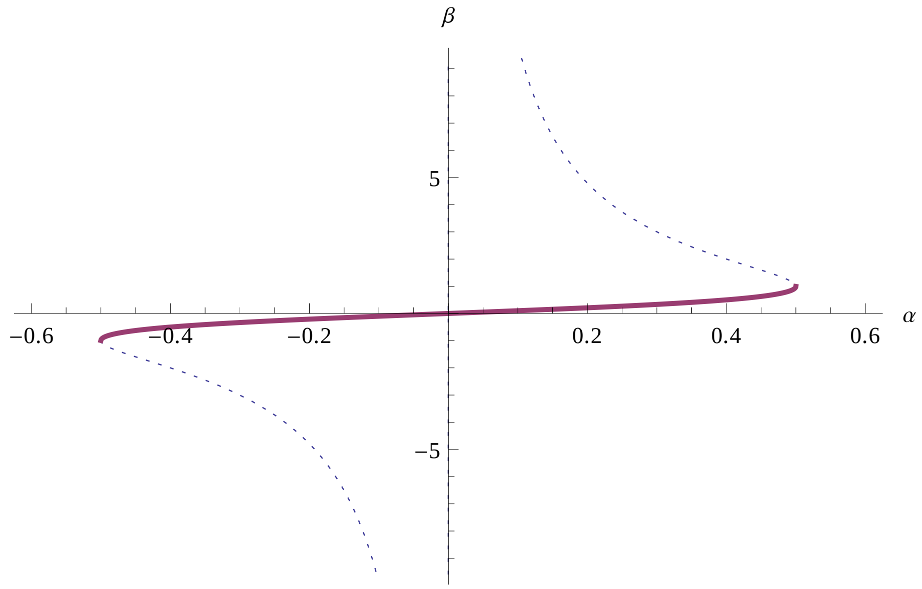

Figure 1. Figure shows that in order to has $0 < \beta < 1$, it is necessary that $0 < \alpha < \frac{1}{2}$, where $\alpha = \frac{q_x}{ne + p_x}$.

$$\varsigma = -\frac{1}{T^2}\left(q^\alpha \nabla_\alpha T - \frac{T}{c^2} q^\alpha D U_\alpha\right), \tag{14}$$

where T is the temperature, q^α the heat flux, c the speed of light, U_α the four velocity and D the convective time derivative. We also know that $q^\alpha = \Delta^\alpha_\gamma U_\beta T^{\beta\gamma}$, $\nabla_\alpha = \Delta^\beta_\alpha \partial_\beta$ and $D = U^\alpha \partial_\alpha$, where Δ^α_γ is the projector in the conmoving frame [7].

Assuming that the heat flux has only x component the entropy production takes the form,

$$\varsigma = -\frac{q_x c}{T^2}\left[\partial_x T - \frac{T}{c^2} D U_x\right]. \tag{15}$$

The term $-TDU_x/c^2$ in (15), has no correspondence in the non-relativistic case and represents the isothermal heat flux when the fluid is accelerated. This term acts in a direction opposite to the acceleration and is said to be due to the inertia of energy [15]. It is precisely this term which has caused controversy in the validity of Eckart's constitutive equation and seems to cause that generic instabilities [3].

We may express (15) in another way by eliminating the time derivative of the 4-velocity in the so-called Eulerian regime [7, 3],

$$\frac{n h_E}{c^2} D U^\alpha = \nabla^\alpha p, \tag{16}$$

where $h_E = e + p/n$. Then (15) takes the form,

$$\varsigma = -\frac{q_x c}{T^2}\left[\partial_x T - \frac{T}{n h_E} \partial_x p\right]. \tag{17}$$

Using condition (11) in the production of entropy (17), we have

$$\varsigma > -\frac{cnh_E}{2T^2}\partial_x T + \frac{1}{2T}\partial_x p, \tag{18}$$

which is positive and satisfies the second law, whenever,

$$\partial_x p > cnh_E \frac{\partial_x T}{T}. \tag{19}$$

4. Discussion

We study the heat flow behavior contained in the energy-momentum tensor under Lorentz transformations finding that there is a total heat flow that contains a convective and nonconvective part. When the heat flux is in the same direction as the Lorentz transformation was found that satisfying the condition (11), i.e. $q_x < \frac{1}{2}(ne + p_x)$ (this means that through a transformation between reference frames) is possible to convert some kind of convective energy in non-convective one. This condition can also be found through the null energy condition. When the Lorentz transformation is performed in a direction orthogonal to the heat flow it has a convective flow in the direction transformation and therefore the components do not mix each other. With the condition imposed in (11) is possible to find most of the reference systems, besides than the Landau-Lifshitz one for that the heat flux in a moving frame vanish. Moreover when we introduce the condition (11) in the entropy production it gives non-violation to the second law of thermodynamics. This happens only when the the pressure gradient is greater than the temperature gradient in the $x-$direction.

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