

Stellar Stability in the Braneworld

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Abstract. We study the properties of astrophysical systems in a *Brane World* scenario. In particular, the consequences of local terms that emerge from extra-dimensional gravity are explored through the modified Tolman-Oppenheimer-Volkoff equation. We compute the induced changes into mass, density and compactness of objects like white dwarfs and neutron stars. The main corrections in comparison with the known results of General Relativity are given in terms of the ratio ρ_0/λ , where ρ_0 is the central energy density of the stellar distribution and λ is the brane tension. As a result of these modifications, We obtain that the compactness of both, neutron stars and white dwarf decreases with respect to the value of General Relativity.

1. Introduction.

It is well known that stellar systems are excellent laboratories to study *General Relativity* (GR), being this theory confirmed over and over again since its almost one hundred years (see for example [1]). However there are astrophysical systems that possess high energies due to their extremely high densities and pressures, so GR corrections are expected.¹

For this reason, it is important to have modifications to GR. An extension to the Einstein theory of gravity is offered by the *Brane World* models (BW), which states that our Universe has a space-time of five dimensions. These models assume that interactions of the standard model are confined to the (3+1)-dimensional hypersurface (*brane*), while gravity can leak to the extra dimension; this is the reason why gravity is perceived weaker compared with the other fundamental interactions, as proposed by Randall-Sundrum (RS) in the models RSI and RSII [3, 4]. The effects of extra dimension can be observed in the Einstein equations, which are modified by the appearance of new terms: the local or high energy term, which depend on quadratic terms of the energy-momentum tensor, the nonlocal or geometric term provided by the Weyl tensor and finally the possible matter content in the extra dimension. The presence of these new terms, provide us with new dynamic in the study of astrophysical systems.

Based in these ideas, this work is dedicated to extend the results provided by several authors [5, 6] to the case of a polytropic equation of state (EoS). For simplicity, we exclude the nonlocal terms, preserving the quadratic part of the energy-momentum tensor. We will be interested in three regimes: high, comparable and low energy regime, whose main distinction is given by the ratio ρ_0/λ .

We will proceed as follows: In Section 2 a RSII-like scenario is established. The effective Einstein equations on the brane will arise following the methodology of [7, 8] and then in

¹ A good system to test GR was discovered few month ago as reported at [2].



Section 3, we will obtain the modified Tolman-Oppenheimer-Volkoff (TOV) equation by the local term of the extra-dimensional gravity. By using this TOV equation will be possible to analyze a particular astrophysical system to study the consequences on the stellar object due to extra dimension, as shown in Section 4 where two astrophysical distributions are studied: 1.- a neutron star with constant density, which analytical expressions for pressure, the bound for the value of brane tension and the limit for gravitational collapse are obtained, and 2.- a white dwarf, which matter content is well described by a polytropic EoS, this is, a relationship between pressure P and density ρ described by $P \propto \rho^\gamma$, where γ is a fractional exponent. Finally, in Section 5 we discuss the results and perspectives of this work.

2. Einstein Field Equations on the brane (a mathematical review).

Let us start by writing the equations describing a (3+1) brane embedded in a five dimensional space-time using the Randall-Sundrum II (RSII) model [4, 7, 8]. We first assume that the gravitational equation of the 5D Universe has the same mathematical structure of the Einstein field equations in four dimensions,

$$G_{AB} + \Lambda_{(5)}g_{AB} = \kappa_{(5)}^2 {}^{(5)}T_{AB}, \quad (1)$$

where G_{AB} denotes the Einstein tensor, ${}^{(5)}T_{AB}$ refers to the energy-momentum tensor, $\Lambda_{(5)}$ represents the cosmological constant, and $\kappa_{(5)}^2$ is the gravitational coupling, all of them in 5D. In order to write the gravitational equations of motion in the 4D brane, we use the Gauss and Codacci equations respectively,

$${}^{(4)}R_{\beta\gamma\delta}^\alpha = {}^{(5)}R_{\nu\rho\sigma}^\mu q_\mu^\alpha q_\beta^\nu q_\gamma^\rho q_\delta^\sigma + K_\gamma^\alpha K_{\beta\delta} - K_\delta^\alpha K_{\beta\gamma}, \quad (2)$$

$$D_\nu K_\mu^\nu - D_\mu K = {}^{(5)}R_{\rho\sigma} n^\sigma q_\mu^\rho, \quad (3)$$

where the extrinsic curvature over the 4D manifold \mathcal{M} is given by $K_{\mu\nu} = q_\mu^\alpha q_\nu^\beta \nabla_\alpha n_\beta$, $K = K_\mu^\mu$, and D_μ is the covariant differentiation of $q_{\mu\nu}$. It is important to remark that in the BW scenario, our 4D world is described by a 3-brane $(\mathcal{M}, q_{\mu\nu})$ within a 5D spacetime $(\mathcal{V}, g_{\mu\nu})$. We denote the unit vector normal to \mathcal{M} by n^α , and the induced metric onto it by $q_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ [8]. After an appropriate computation, it is possible to demonstrate that the modified 4D Einstein's equation can be written as [7, 8]

$$G_{\mu\nu} + E_{\mu\nu} + \Lambda_{(4)}g_{\mu\nu} = \kappa_{(4)}^2 T_{\mu\nu} + 6\frac{\kappa_{(4)}^2}{\lambda}\Pi_{\mu\nu} + 4\frac{\kappa_{(4)}^2}{\lambda}F_{\mu\nu}, \quad (4)$$

where

$$\Lambda_{(4)} = \frac{1}{2}\Lambda_{(5)} + \frac{\kappa_{(5)}^4}{12}\lambda^2, \quad (5)$$

$$\kappa_{(4)}^2 = 8\pi G_N = \frac{\kappa_{(5)}^4}{6}\lambda, \quad (6)$$

$$\Pi_{\mu\nu} = -\frac{1}{4}T_{\mu\alpha}T_\nu^\alpha + \frac{TT_{\mu\nu}}{12} + \frac{g_{\mu\nu}}{24}(3T_{\alpha\beta}T^{\alpha\beta} - T^2), \quad (7)$$

$$F_{\mu\nu} = \frac{2}{3}{}^{(5)}T_{AB}g_\mu^A g_\nu^B + \frac{2g_{\mu\nu}}{3}\left[{}^{(5)}T_{AB}n^A n^B - \frac{{}^{(5)}T}{4}\right], \quad (8)$$

$$E_{\mu\nu} \equiv {}^{(5)}C_{AFB}^E n^E n^F g_\mu^A g_\nu^B. \quad (9)$$

Here, $\Lambda_{(4)}$ is the four dimensional cosmological constant, $\kappa_{(4)}$ is the 4D gravitational constant related to the Newton gravitational constant G_N , λ is the brane tension, and ${}^{(5)}C_{AFB}^E$ is the 5D Weyl tensor [7, 8]. Note that we will use the most-plus signature ($\text{diag}(-, +, +, +)$) for the line element, and natural units in which $c = \hbar = 1$.

3. The Tolman-Oppenheimer-Volkoff equation with high energy corrections.

Now, we turn our attention to solve the modified Einstein equations given by (4) under the following considerations: At astrophysical scales the cosmological constant on the brane is zero, *i.e.*, $\Lambda_{(4)} = 0$. This agrees with the fact that the observed cosmological constant is practically zero, so the 5D spacetime is Anti de Sitter, because Eq.(5) yields

$$\Lambda_{(4)} \sim 0 = \frac{1}{2} \left(\Lambda_{(5)} + \frac{1}{6} \kappa_{(5)}^4 \lambda^2 \right), \quad \Lambda_{(5)} = -\frac{1}{6} \kappa_{(5)}^4 \lambda^2. \quad (10)$$

It will be also assumed for simplicity that the nonlocal terms are negligible at astrophysical scales, and then $E_{\mu\nu} = 0$. This is a very strong assumption because it is expected that an observer on the brane may be able to detect nonlocal effects due to the 5D gravity. In fact, notice that by taking the relativistic limit $\lambda \rightarrow \infty$ in Eq. (4), the term containing the Weyl tensor remains. However, in special cases the symmetries of the system allow us to simplify the Weyl tensor. For example, $E_{\mu\nu} = 0$ for a 5D conformally flat spacetime [7].²

Without matter content in the extra dimension, we will have ${}^{(5)}T_{AB} = 0$, and this implies that $F_{\mu\nu} = 0$ [5, 7]. Also, we find that $\nabla^\mu T_{\mu\nu} = 0$, because Eqs. (1) and (3), together with the Israel conditions[10], imply that

$$\nabla^\mu T_{\mu\nu} = -2 {}^{(5)}T_{AB} n^A g_\nu^B = 0, \quad (11)$$

which is consistent with the Bianchi identity in 4D gravity.³

With all the assumptions given previously, Eqs. (4) becomes

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu} + \frac{48\pi G_N}{\lambda} \Pi_{\mu\nu}. \quad (12)$$

The tensors $T_{\mu\nu}$ and $\Pi_{\mu\nu}$ will describe a perfect fluid, and their explicit forms are

$$T_{\mu\nu} = \rho u_\mu u_\nu + P h_{\mu\nu}, \quad \Pi_{\mu\nu} = \frac{1}{12} \rho [\rho u_\mu u_\nu + (\rho + 2P) h_{\mu\nu}], \quad (13)$$

where $\rho(r)$ and $P(r)$ are the energy density and pressure respectively. To solve Eqs. (12), let us consider the following static and spherically symmetric line element,

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2[d\theta^2 + \sin^2(\theta)d\varphi^2], \quad (14)$$

where $A(r)$ and $B(r)$ are the gravitational potentials. By writing in components the Eq.(12), and by eliminating the gravitational potentials $A(r)$ and $B(r)$, it is possible to obtain a first order differential equation in terms of the physical quantities of the star: the energy density $\rho(r)$ and pressure $P(r)$,

$$-r^2 \frac{dP(r)}{dr} = \frac{G_N M(r) \rho(r)}{1 - 2G_N M(r)/r} \left[1 + \frac{P(r)}{\rho(r)} \right] \left[1 + \frac{4\pi P(r) r^3}{M(r)} \left(1 + \frac{\rho(r)}{2\lambda} \left[2 + \frac{\rho(r)}{P(r)} \right] \right) \right], \quad (15)$$

² Solutions that contain $E_{\mu\nu} \neq 0$ can be seen at [5, 6]. For a case where $E_{\mu\nu} \neq 0$ but $\Pi_{\mu\nu} = 0$ see for instance [9].

³ See [11] for cases where $\mathcal{F}_{\mu\nu} \neq 0, \Lambda \neq 0$.

where the mass $M(r)$ is given by

$$M(r) = 4\pi \int_0^r dr' \rho(r') r'^2 \left[1 + \frac{\rho(r')}{2\lambda} \right] = M_0(r) + \frac{2\pi}{\lambda} \int_0^r dr' \rho^2(r') r'^2. \quad (16)$$

Notice that we have also defined the physical mass as $M_0(r)$, which coincides with the standard GR definition of mass for a stellar distribution. The complete mass function $M(r)$ includes a correction term that depends on the brane tension λ .

Therefore, Eq. (15) is the TOV equation with corrections coming from 5D gravity. Clearly, if we take the relativistic limit $\rho/\lambda \rightarrow 0$ we obtain the TOV equation of GR, and then it is natural to consider corrections to the standard results in terms of that ratio. For instance, according to this criterion, the mass of the star configuration $M \rightarrow M_0$ in the limit $\rho/\lambda \rightarrow 0$, as can be seen from Eq.(16).

4. Inner solutions from modified TOV equation.

We will describe solutions of the modified TOV equation (15) for two astrophysical systems:

- 1.- Neutron star with a constant energy density: $\rho = \text{const.}$
- 2.- White dwarf with a polytropic equation of state: $P(r) = K\rho^\gamma(r)$.

The first case is of particular interest because, even if it is not realistic, their solutions may provide a useful insight on the nature of the brane corrections that modify the well known GR results. The second case may have a more realistic interpretation in terms of a white dwarf since they are objects that can be observed in nature, and therefore represent a “natural laboratory” to test the theoretical models.

4.1. The neutron star case. (A review of uniform density)

This case was studied in detail in [5], and we give here a brief review of the main results. Because the energy density is a constant quantity, we can find an exact solutions of Eqs.(16) and (15). The mass $M(r)$ from Eq.(16) is given by

$$M(r) = \int_0^r 4\pi r'^2 \rho \left(1 + \frac{\rho}{2\lambda} \right) dr' = \frac{4\pi r^3}{3} \rho \left(1 + \frac{\rho}{2\lambda} \right), \quad (17)$$

and then Eq.(15) becomes

$$\frac{dP(r)}{dr} = -\frac{G_N M(r) \rho}{r^2} \left[1 + \frac{P(r)}{\rho} \right] \left[1 + \frac{4\pi P(r) r^3}{M(r)} \left(1 + \frac{\rho}{2\lambda} \left[2 + \frac{\rho}{P(r)} \right] \right) \right] \left[1 - \frac{2G_N M(r)}{r} \right]^{-1}. \quad (18)$$

We then obtain the following exact solution of Eq.(18),

$$\frac{P(r)}{\rho} = \frac{\sqrt{1 - 2G_N M/R} - \sqrt{1 - 2G_N M r^2/R^3}}{\sqrt{1 - 2G_N M r^2/R^3} - 3 \left(\frac{1+\rho/\lambda}{1+2\rho/\lambda} \right) \sqrt{1 - 2G_N M/R}}, \quad (19)$$

where $M \equiv M(R)$ and R represent the total mass and radius of the star respectively. From Eq.(19) is possible to see that in the limit $\rho/\lambda \rightarrow 0$ we recover the well known solution of GR (see for instance [12, 13, 14]). Several cases of the solution obtained in Eq.(19) are shown in Fig.2 for different values of the ratio ρ/λ .

From condition which establishes that pressure is regular at the origin, we find that

$$\frac{G_N M_0}{R} < \frac{4}{9} \left[\frac{(3 + 3\rho/\lambda)^2 - (1 + 2\rho/\lambda)^2}{8(1 + \rho/2\lambda)(1 + \rho/\lambda)^2} \right], \quad (20)$$

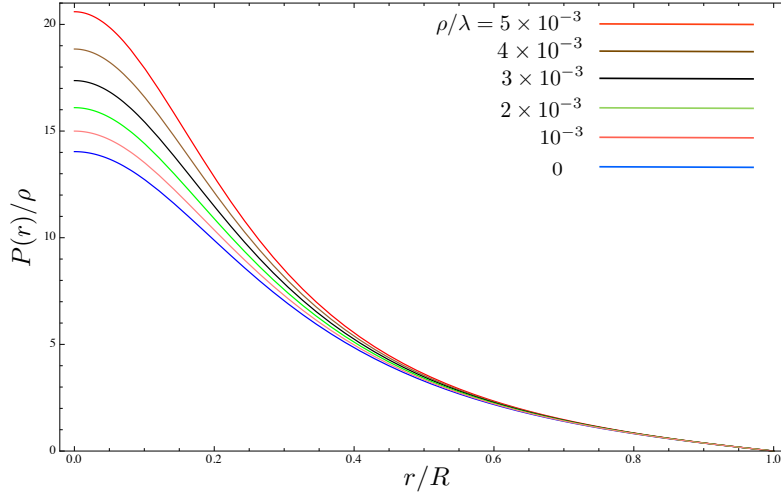


Figure 1. The radial profile of the ratio $P(r)/\rho$ for a star with a constant energy density, for different values of the ratio ρ/λ are shown. The most remarkable effect arises for the central value of the pressure, which increases for larger values of the ratio ρ/λ .

where $M_0 = 4\pi\rho R^3/3$ is the total physical mass, (see Eq.(17)). This result must be compared with the GR expression given by $G_N M/R < 4/9$, which is readily obtained in the limit $\rho/\lambda \rightarrow 0$. Following the procedure of [5], we expand Eq.(20) up to first order in ρ/λ to obtain

$$\frac{G_N M_0}{R} < \frac{4}{9} - \frac{1}{3} \left(\frac{\rho}{\lambda} \right). \quad (21)$$

By considering the typical values of the mass, radius and mean energy density of a neutron star, it is found that the brane tension must obey the bound $\lambda > 6.320 \times 10^{10} \text{MeV}^4$, which agrees with other reported values in [7, 5, 15, 16], and also with the bound imposed by nucleosynthesis, $\lambda \geq 1 \text{MeV}^4$.

4.2. The white dwarf case.

Now, we turn our attention to the case of a white dwarf. The matter content of the star will be described by a polytropic equation of state, which has the form: $P(r) = K\rho^\gamma(r)$, where K is a proportionality constant and γ is a fractional exponent; which value will be $\gamma = 4/3$. To solve the modified TOV equation, it is convenient to define the following dimensionless variables: $\bar{M} \equiv \sqrt{G_N} M(r)$, $\bar{\rho} \equiv G_N^2 \rho$, $\bar{r} \equiv r/\sqrt{G_N}$, $\bar{K} \equiv K/G_N^{2/3}$, $\bar{\lambda} \equiv G_N^2 \lambda$.

Thus, the system of Eqs.(15) and (16) are written as

$$\frac{d\bar{M}(\bar{r})}{d\bar{r}} = 4\pi\bar{\rho}(\bar{r})\bar{r}^2 \left[1 + \frac{\bar{\rho}(\bar{r})}{2\bar{\lambda}} \right], \quad (22)$$

$$\frac{d\bar{\rho}(\bar{r})}{d\bar{r}} = -\frac{3\bar{M}(\bar{r})\bar{\rho}^{2/3}(\bar{r})}{4\bar{K}\bar{r}^2} \left(\frac{1 + \bar{K}\bar{\rho}^{1/3}(\bar{r})}{1 - 2\bar{M}(\bar{r})/\bar{r}} \right) \left[1 + \frac{4\pi\bar{K}\bar{\rho}^{4/3}(\bar{r})\bar{r}^3}{\bar{M}(\bar{r})} + \frac{2\pi\bar{\rho}(\bar{r})\bar{r}^3}{\bar{M}(\bar{r})} (1 + 2\bar{K}\bar{\rho}^{1/3}(\bar{r})) \frac{\bar{\rho}(\bar{r})}{\bar{\lambda}} \right]. \quad (23)$$

It must be noticed that, like in the case of constant energy density discussed above, the equations of motion depend on ρ/λ , and the latter is invariant under the change of variables taken, that is, $\rho/\lambda = \bar{\rho}/\bar{\lambda}$. However, this time the energy density depends on

the radial coordinate, and then we will define three different regimes according to the ratio $\bar{\rho}_0/\bar{\lambda}$, which compares the central value of the energy density with the brane tension. The regimes are discussed in the following subsections, where we take the value of central density as $\bar{\rho}(r=0) = \bar{\rho}_0 = 0.01$.

4.2.1. High energy regime $\bar{\rho}_0/\bar{\lambda} \gg 1$. The numerical solutions of Eqs.(22) and (23) for this case are shown in Fig.2. The numerical value for the brane tension is given by $\bar{\lambda} = 0.001$. First of all, we notice that density profile is significantly modified, as it decreases more abruptly that in the standard GR case (without brane correction).

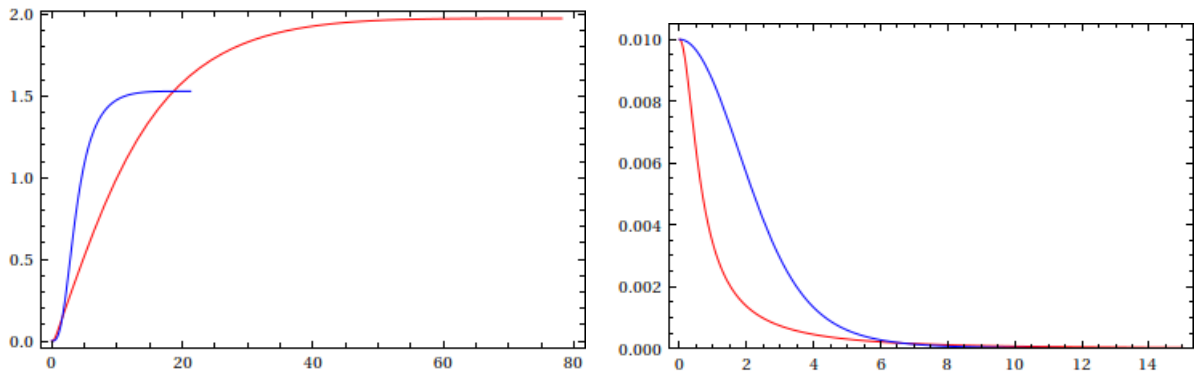


Figure 2. *Solution of the modified TOV equation in the case of a stellar configuration with a polytropic equation of state (red solid lines) and its comparison with the solution of the standard TOV equation in the pure GR case (blue solid lines). See the text for more details.*

In order to have a proper comparison between the brane case and the standard GR case, we also show the behavior of the physical mass M_0 for both cases; we only need to recall that physical mass does not contain brane corrections (see Eq.(17)). Thus, for GR we have that $M_0 = 1.53$. Is important to note that, the radius of the stellar distribution R , is given by the numerical value for which energy density and therefore, pressure vanish.

We follow the same criterion for the compactness of the star, and define the latter in terms of the physical mass M_0 . The results indicate that compactness in GR is $M_0/R = 0.072$. In this way, the modifications suffered by physical mass and compactness due to extra dimensional gravity are given by

$$M_0 = \int_0^R 4\pi\bar{r}^2\bar{\rho}(\bar{r})d\bar{r} = 1.74, \quad \frac{M_0}{R} = 0.022. \quad (24)$$

4.2.2. Comparable energies $\bar{\rho}_0/\bar{\lambda} \sim 1$. In this case we obtain the numerical solutions shown in Fig. 3, in which case, the numerical value for the brane tension is given by $\bar{\lambda} = 0.01$. We see that the total mass of the star with effects of the extra dimension (red line) is still larger than the relativistic total mass (blue line) and the stellar radius is larger than the radius of the star without BW corrections. However these increments in both mass and radius are smaller than the high energy regime.

Also in Fig. 3, we note that density with corrections of extra dimension (red line) is less than the relativistic case (blue line). On the other hand, the physical mass M_0 and compactness in this regime are still lesser that the GR solution.

$$M_0 = 1.50, \quad \frac{M_0}{R} = 0.042. \quad (25)$$

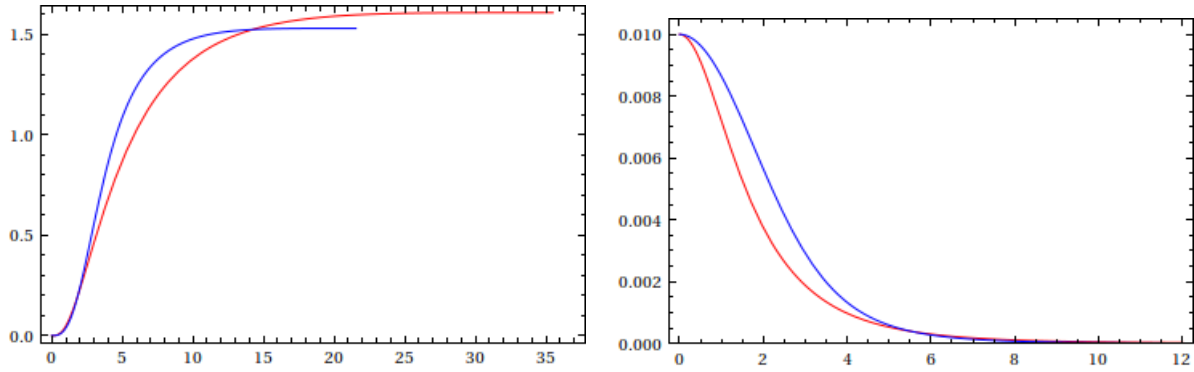


Figure 3. The effects due to extra dimensional gravity on the mass $\bar{M}(r)$ (left) and energy density $\bar{\rho}(r)$ (right) are shown with a red line, while the blue line represents the stellar distribution without branes. In this regime is still present the effects of extra dimension, but is less than the high energy regime.

4.2.3. Low energies $\bar{\rho}_0/\bar{\lambda} \ll 1$. Finally, we see in the following figures that is possible to obtain the relativistic solutions in this low energies regime. The numerical value for the brane tension is given by $\bar{\lambda} = 10$.

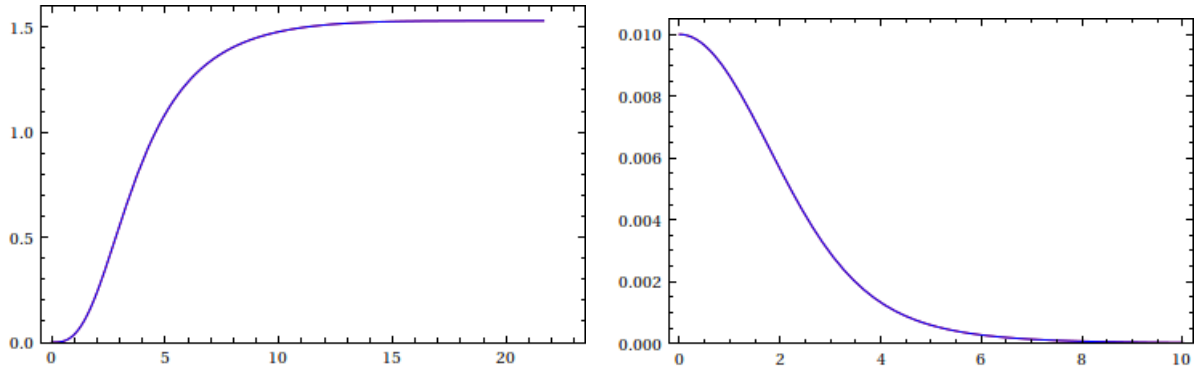


Figure 4. The effects due to extra dimensional gravity on the mass $\bar{M}(r)$ (left) and energy density $\bar{\rho}(r)$ (right) are shown, and is clearly seen that both curves coincide. Thus, solutions of GR case are recovered.

We can see in Fig. 4, that both curves describing the stellar mass are practically the same, *i.e.*, the total mass of the star with effects of the extra dimension (red line) has the same value that the relativistic mass (blue line) and the stellar radius is also the same. In fact, the physical mass and compactness in this regime are given by

$$M_0 = 1.53, \quad \frac{M_0}{R} = 0.071. \quad (26)$$

In Fig. 4, we also note that density recovers its behavior and coincides with the GR case. The following Table summarizes the results obtained above

Regime	Physical Mass M_0	Stellar Radio R	Compactness M_0/R
$\rho_0/\lambda \gg 1$	1.74	78.15	0.022
$\rho_0/\lambda \sim 1$	1.50	35.50	0.042
$\rho_0/\lambda \ll 1$	1.53	21.61	0.071

Table 1. The numerical values obtained for the physical mass, stellar radio and compactness of the star, all of them with corrections due to the presence of extra dimension are shown. For GR case we have that physical mass, stellar radio and compactness are given by $M_0 = 1.53$, $R = 21.45$ and $M_0/R = 0.072$ respectively.

Therefore, it is clear that is possible to recover the GR solutions, as can be seen in the low energies regime, while the effects of extra dimensions are more noticeable as the ratio $\bar{\rho}_0/\bar{\lambda}$ increases.

5. Discussion and Conclusions.

A modification to the TOV equation by considering the local term of the extra dimensional gravity was presented. This equation represents the simplest extension in these BW scenarios, because there were not considered nonlocal effects and possible matter content in the extra dimension.

Subsequently, the modified TOV equation was applied to the case of a star with constant energy density, which results are manifested in the increase of central pressure. In a more realistic case, a white dwarf was analyzed, and it was possible to see the effects that it suffers due to the extra dimension: the values for total mass and stellar radio are bigger than GR case, this effect is more noticeable in the high energies regime, while in the low energies regime these effects are practically negligible. In both neutron stars and white dwarf, the compactness decreases with respect to the GR value, but such a value is recovered when the extra dimensional term is not relevant in the equations, *i.e.*, when $\rho_0/\lambda \ll 1$.

An important fact is that all our results show that we successfully recover the relativistic solutions by taking the appropriate limit $\rho/\lambda \rightarrow 0$, which is a favorable feature of the model, if we consider it as a generalization of GR.

An extension of these results would be to consider non-local terms that arise from the projection of the Weyl tensor. This implies a new difficulty level: the system of equations given by the components of modified Einstein equation and the conservation equations is not determinated in a closed way due to the additional geometric variables. A couple of contributions that address that issue can be found in [6, 17].

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