

The ideal versus the reality: topology and turbulence in the current density in tokamak

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Abstract. Models for the current density profile in magnetically confined plasma have been proposed starting from first principles, basically in analogy with the self-organization of the vorticity in the two-dimensional Euler fluid. The principle of maximum entropy has led to the derivation of the Liouville equation, under some criticism regarding the possibility of a statistical approach. We identify a class of asymptotic stationary states for the coupled fields of current-density and vorticity, using the Backlund transform. For this class of asymptotic states the maxima of the vorticity and current density must coincide. We also discuss a possible relevance of the field-theoretical solutions *sphaleron* for magnetic reconnection in low-collision regimes.

1. Introduction

The current and the magnetic configuration of the experimental thermonuclear fusion device “tokamak” offer an interesting field of application of methods derived from topological concepts. The high temperature plasma has low collisionality and the invariants are quasi-ideally preserved. On the other hand the large gradients of the plasma parameters (density, temperatures, electric current, etc.) are sources of free energy for instabilities that evolve to turbulence (Wesson, 2004). Therefore in the tokamak plasma both extremes are present: exact topological structures and turbulence, the same as, for instance, in astrophysical or solar plasmas.

Although the basic expectation is axisymmetry (invariance along the toroidal direction) the experiments reveal exceptions: a persistent, robust, helical structure of the magnetic field (island); generation of a stable filament of particle density and current density sitting on a magnetic surface close to the axis (“snake”); a stable distribution of the current showing a hole centered on the magnetic axis, etc. These are nontrivial states not fully understood.

As many other systems, the physics of tokamak plasma is investigated using *conservation* equations (density, momentum, energy, etc.) which are hardly able to identify privileged states, if any. The signature that some states are special is the so-called “profile resiliency”, proved experimentally: even with high input of energy and high transport rate, the profiles of the main plasma parameters are almost unchanged. It can be explained by: (1) Self - Organized Criticality (SOC). This is an universal type of behavior of systems composed of a large number of quasi-independent sub-systems, each having a threshold-type dynamics (Bak, 1996). Under a slow drive the sub-systems evolve approaching the threshold for instability. When the limit is exceeded for one sub-system, its reaction of return to equilibrium (*i.e.* under the threshold) is



very fast and is propagated to the neighbors that are, themselves, at marginal stability. They become active and the effect is a perturbation (an avalanche) which can extend on a wide range of scales, given by the space extensions of the clusters of marginally-stable sub-systems. On large time scale the SOC is a stationary state of statistical stability with minimum rate of entropy production. The Tokamak plasma there are instabilities with fast growth rate beyond a threshold given by the gradient of the temperature (Dimits *et al.*, 2000). Transport events of “avalanche” - type have been observed in experiments (Garbet *et al.*, 2004), suggesting that the SOC may be the adequate physical picture. However the large eddies and structures do not allow the simultaneous presence of all spatial scales of correlations, specific to criticality. (2) Natural or “privileged” states. They are stable and stiff since they are the extrema of an action functional, defined on a space of functions representing the plasma states. The shape of the functional around the extremum may explain the resiliency. We are then led to examine the problem of natural profiles of the current density in tokamak. This problem is actually not solved at this time.

2. Natural current profiles in Tokamak

2.1. The relaxed state of the current in tokamak

For slow motion (relative to the Alfvén speed), taking the rotational of the momentum equation at equilibrium, we have $\nabla \times (\mathbf{J} \times \mathbf{B}) = 0$ or

$$B_0 \frac{dJ_z}{dz} + (\mathbf{B}_\perp \cdot \nabla_\perp) J_z = 0 \quad (1)$$

where J_z is the current density along the toroidal direction, \mathbf{B}_\perp is the magnetic field component laying in the meridional section of the torus, B_0 the magnitude of the main, toroidal, magnetic field. Introducing the scalar function ψ , the z component of the magnetic potential, $\mathbf{A}_z = \psi \hat{\mathbf{e}}_z$, it results $\mathbf{B}_\perp = -\nabla\psi \times \hat{\mathbf{e}}_z$ and $\Delta_\perp \psi = J_z$.

The magnetic surfaces are surfaces of constant ψ : $\mathbf{B} \cdot \nabla\psi = 0$. The solution for the scalar function ψ is

$$\Delta_\perp \psi = J(\psi) \quad (2)$$

where J is an *arbitrary function* of ψ . Taylor (1993) assumed that the current density is in a slightly chaotic state. The current exists as *filaments* with unique magnitude j_0 , and their intersection with the meridional plane of the torus is a set of points. Now consider that the z coordinate is like the *time* variable, $z \rightarrow t$. The interaction between these filaments is

$$j_0 \frac{dx_i}{dt} = \frac{1}{B_0} \frac{\partial H}{\partial y_i}, \quad j_0 \frac{dy_i}{dt} = -\frac{1}{B_0} \frac{\partial H}{\partial x_i} \quad (3)$$

The Hamiltonian is $H = \sum_{i>j} j_0^2 U(\mathbf{r}_i, \mathbf{r}_j)$. The potential $U(\mathbf{r}_i, \mathbf{r}_j)$ is generated in the point \mathbf{r}_i of the plane by the filament (“rod”) located at \mathbf{r}_j . For a large plane box of linear dimension L , the potential is $U(\mathbf{r}_i, \mathbf{r}_j) \approx (2\pi)^{-1} \ln(|\mathbf{r}_i - \mathbf{r}_j|/L)$, the Green function of the Laplace operator.

The idea of Taylor was to use statistical methods to analyze the ensemble of discrete, point-like objects, the puncture in plane of the filaments of J_z . First it is discretized the plane box and it is “counted” the number of filaments in each elementary cell, n_k . The microcanonical ensemble is the statistical ensemble of realizations of the distributions of N current filaments, with two constants: energy $E = \sum_k \sum_i j_0^2 n_k U_{ki} n_i$ and total number (*i.e.* total current) $N = \sum_k n_k$. In analogy with the set of discrete point-like vortices of the Euler equation for the ideal fluid, it is calculated the distribution function of the microcanonical ensemble, $\rho(\{\mathbf{r}_i\}) = \delta(E - H(\{\mathbf{r}_i\}))$.

The average density of filaments is

$$\langle \rho(\mathbf{r}) \rangle = \int \delta[E - H(\{\mathbf{r}, \mathbf{r}_i\})] \prod_{i=2}^N d\mathbf{r}_i \quad (4)$$

This average can be calculated on the basis of the maximum entropy principle (for this system the statistical temperature is *negative*), under preservation of the total energy, E , and the total number of filaments, N .

$$\langle \rho(\mathbf{r}) \rangle = \sum_{\{n_i\}} \exp \left(- \sum_i n_i \ln n_i \right) \delta \left[E - \sum_k \sum_i j_0^2 n_k U_{ki} n_i \right] \delta \left(N - \sum_k n_k \right) \quad (5)$$

At the continuum limit one defines the scalar function $\psi(\mathbf{r}) = \int d\mathbf{r}' j_0 U(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}')$ and the distribution of the current density over the plane is (K is a constant)

$$J = j_0 \langle \rho(\mathbf{r}) \rangle = K \exp(-\mu j_0 \psi(\mathbf{r})) \quad (6)$$

The equation for the current density is obtained from the equation for the magnetic potential

$$\Delta \psi = A \exp(-\lambda \psi) \quad (7)$$

which is the Liouville equation. Although criticized, this approach has a subtle difference relative to others: it formulates the problem in terms of matter (density of point-like objects in plane), field (long range, Coulombian potential) and interaction. This suggests a formalism of classical field theory.

2.2. A field theoretical formalism for the current in tokamak

Inspired by the field theoretical description of the coherent flows at relaxation of the 2D ideal and incompressible (Euler) fluid, we would try to build a similar formalism for the current density. It will be sufficient to do this for the two-dimensional plasma, *i.e.* the current is transversal on the plane representing the meridional cross-section of the tokamak. The current is non-zero within a circular region of radius a .

From the beginning we note that we do not attempt to tackle here the 2D MHD problem, where two fields (\mathbf{v}, \mathbf{B}) [and their rotationals (ω, j)] are evolving in interaction. The separation of the models for j and for ω requires the factorization of a unique model and may be possible at, and in close proximity of, the relaxation states.

The essential step in developing a field-theoretical framework for the Euler fluid in 2D was the equivalence of the physical model (the Euler equation) with the dynamics of the discrete set of point-like vortices interacting in plane by a self-determined, long-range, potential. For the latter model we have been able to write a Lagrangian density, constructed such as to reflect the Lorentz-type motion (relative gyration) of the elements of vorticity (Spineanu & Vlad, 2003, 2005). The nature of the elementary vortices is reflected in the fact that the scalar (matter) field ϕ must be a mixed spinor, an element of $sl(2, \mathbf{C})$ algebra. At self-duality this becomes the $su(2)$ algebra. As discussed before, there exists a discrete model for the current density: it neglects the resistivity, the transport processes, the change in the electric field due to induction, the convection of the current by the plasma flow but retains the essential aspect: the Biot-Savart interaction of parallel current filaments leading to Lorentz-type relative motion. The model is then similar to the one for the Euler fluid and we expect to be able to write a Lagrangian density.

We immediately see that there is an essential difference between the elements of vorticity and the elements of current. The elementary vortex is a spin-like entity while the elementary current

is not. Then the model for the latter should be Abelian. Assuming a certain universality which places the $2D$ point-like objects in the same class as interacting “charges” in plane, we take as reference Jackiew and Pi (1990) and the expression of the Lagrangian density is written:

$$\mathcal{L} = \frac{\kappa}{4} \varepsilon^{\alpha\beta\gamma} A_\alpha F_{\beta\gamma} + i\hbar\psi^* \left(\frac{\partial}{\partial t} + \frac{ie}{\hbar} A^0 \right) \psi - \frac{\hbar^2}{2m} |\mathbf{D}\psi|^2 + \frac{g}{2} (\psi^*\psi)^2 \quad (8)$$

Here ψ is the matter field, A_μ is the gauge field that carries the interaction and the last term is the scalar - field nonlinearity. This model has actually been constructed to describe the quantum Hall model but its classical counterpart is adequate for the dynamics of point-like objects moving in plane according to the Lorentz-type interaction. It is found that, when the relationship $\frac{g}{2} \pm \frac{e^2\hbar}{2m\kappa} = 0$ between the parameters exists one obtains the Self-Duality state, whose signature is the possibility of the Bogomolnyi procedure of writing the action as a sum of squares. The self-dual states of this model are solutions of $D_1\phi = \pm iD_2\phi$ or

$$D_-\phi = 0 \quad (9)$$

and this is formally solved by

$$\mathbf{A} = \nabla\chi \pm \frac{\hbar c}{2e} \nabla \times \ln \rho \quad (10)$$

where $\phi = \rho^{1/2} \exp(i\chi)$. For the scalar field one obtains

$$\nabla^2 \ln \rho = \pm 2 \frac{e^2}{\hbar c \kappa} \rho \quad (11)$$

This is exactly the equation derived by Taylor (1993) for the natural profiles of the current density in $2D$ plasma: $\Delta \ln \rho = -\gamma\rho$, after the z component of the magnetic potential, ψ , is introduced by $\rho = \exp(\psi)$. It then results that the asymptotic stationary distribution of current density obeys the Liouville equation, confirming the Taylor’s result derived from statistical physics. We recall that this result is for pure current, plasma is not taken into account.

Naturally, the simultaneous consideration of vorticity and current density makes the problem more difficult and returns to MHD relaxation.

3. MHD relaxation in $2D$: the simplest asymptotic states via Backlund transformations

At relaxation in the in $2D$ plasma there is a strong correlation between the spatial patterns of the vorticity and of the current density. The numerical simulations (Kinney *et al.*, 1994) show that in the stationary $2D$ MHD states exhibit coincidence of the local maxima of the current and of the vorticity. More generally, when ω and j are overlapped, they move together. When they are initially distinct they try to overlap (Yatsuyanagi *et al.*, 2002). In a plasma with a helical configuration where the current density \mathbf{J} and the vorticity ω are parallel (*i.e.* aligned) the total rate of energy dissipation (ohmic and viscous) is minimal (Montgomery *et al.*, 1989). Then a cylindrical, axially periodic plasma which is initially axisymmetric will spontaneously bifurcate to a state where a first order helical configuration with \mathbf{J} and ω aligned is present. This state is favorable since it has minimum rate of entropy production. Looking for states of minimum of the energy content under the constraint of constant cross helicity it is found the condition (Montgomery, 1992)

$$\bar{\mathbf{j}} = \left(\frac{\nu}{\eta} \right)^{1/2} \bar{\omega} \quad (12)$$

This suggests the existence of a certain similitude in the stationary asymptotic states of the fields ψ (fluid streamfunction) and a_z (z - component of the magnetic potential), with the vorticity $\omega \hat{\mathbf{e}}_z \rightarrow \omega = \Delta\psi$ and the current density $J_z \hat{\mathbf{e}}_z \rightarrow J_z = \Delta a_z$.

Consider the 2D MHD equations

$$\begin{aligned} \frac{d\omega}{dt} &= \nabla_{\parallel} J_{\parallel} = \left[\frac{\partial}{\partial z} + (-\nabla_{\perp} a_z \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] J_{\parallel} \\ \frac{\partial a_z}{\partial t} + (\mathbf{v} \cdot \nabla_{\perp}) a_z &= 0 \end{aligned} \quad (13)$$

The first equation can be written

$$\left[\frac{\partial}{\partial t} + (-\nabla_{\perp} \psi \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] \Delta\psi = \left[\frac{\partial}{\partial z} + (-\nabla_{\perp} a_z \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] \Delta a_z \quad (14)$$

The structure of the two sides is identical, if we adopt the following approximation $J_{\parallel} \approx J_z$. Then the following limits of these two scalar fields are seen to solve the equations

$$\begin{aligned} \left[\frac{\partial}{\partial t} + (-\nabla_{\perp} \psi \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] \Delta\psi &= 0 \\ \left[\frac{\partial}{\partial z} + (-\nabla_{\perp} a_z \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] \Delta a_z &= 0 \end{aligned} \quad (15)$$

in the following particular states

$$\begin{aligned} \psi &\equiv \text{stationary } \frac{\partial \psi}{\partial t} = 0 \\ a_z &\equiv \text{invariant on the } z\text{-direction, } \frac{\partial a_z}{\partial z} = 0 \end{aligned} \quad (16)$$

In this case the first equation in Eq.(15) can be seen as the asymptotic form of the Euler equation and this is known to lead to states where the streamfunction $\psi(x, y)$ verifies the *sinh*-Poisson equation

$$\Delta\psi + \sinh \psi = 0 \quad (17)$$

The second equation is formally identical and the solution a_z should be one of the solutions of the same equation

$$\Delta a_z + \sinh a_z = 0 \quad (18)$$

Then from the first MHD equation it results that a possible class of stationary asymptotic MHD states consists of two functions $\psi(x, y)$ and $a_z(x, y)$ that are both solutions of the *sinh*-Poisson equation.

However there is the second MHD equation

$$\frac{\partial a_z}{\partial t} + [(-\nabla_{\perp} \psi \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp}] a_z = 0 \quad (19)$$

where we see that the scalar magnetic potential a_z is advected by the velocity of the *other* field, ψ .

If now we choose at random two different solutions of the *sinh*-Poisson equation, for ψ and respectively for a_z then introducing them in the equation above it will result in general a *time-variation* of the function a_z . We would like the system to be asymptotically stationary, $\partial/\partial t \equiv 0$.

Then one needs that the two solutions of the *sinh*-Poisson equation that have been chosen to be such that

$$[(-\nabla_{\perp}\psi \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp}] a_z = 0 \quad (20)$$

One possibility consists of simply taking a solution of *sinh*-Poisson for a_z with the gradient ∇a_z perpendicular in plane on the streamlines of the other field ψ

$$-\nabla_{\perp}\psi \times \hat{\mathbf{e}}_z \perp \nabla a_z \quad (21)$$

This can be made by either choosing for ψ and for a_z the *same* solution of the *sinh*-Poisson Eq., or by choosing a solution for ψ and then choosing another solution for a_z but with equilines that are everywhere *parallel* to the streamlines of ψ . Then, whatever is the amplitude of a_z in the point (x, y) (different of the amplitude of $\psi(x, y)$) the geometrical form ensures the fulfillment of the condition.

The question is: is it possible to find two different solutions of the *sinh*-Poisson equation that have the same geometrical form of the equilines? We now recall that a fundamental property of the *sinh*-Poisson equation is the exact integrability. For integrable equations in general there exists the property that solutions are connected between them by Backlund transforms. Since both a_z and ψ are solutions of the *sinh*-Poisson equation we must first find a solution (say ψ) then find a_z by a Backlund transform and look for the degenerate case Eq.(20).

3.1. Backlund transform for the *sinh* - Poisson equation

Indeed it is possible to find solutions of Eq.(17) and Eq.(18) that also verifies Eq.(20). The first suggestion for a positive answer comes from the properties of an equation which is similar to *sinh*-Poisson, the *sine* - Gordon equation. One starts from a pseudospherical surface Σ which is a surface with a negative total curvature (Rogers & Schief, 2002)

$$\mathcal{K} = -\frac{1}{\rho^2} \quad (22)$$

Consider a point P on Σ and a segment of fixed length PP' tangent to Σ in P . If the way this segment is moved on Σ is given by the equations of a Backlund transformation between solutions of *sine* - Gordon equation, then the end point P' will trace another surface Σ' which is also pseudospherical and has the *same curvature* as the initial Σ . We take this as a suggestion to look for geometrical structures where the *sinh* - Poisson equation may be of certain relevance and try to extract from the properties of the geometrical structures results for the this equation. Fortunately, such connection exists.

The *sinh* - Poisson equation (also known as “elliptic *sinh* - Gordon”) is not only the equation describing the asymptotic states of the 2D Euler fluid but also the equation of the conformal metric of Constant Mean Curvature (CMC) surfaces in the 3D Euclidean space (Bobenko, 1991): the first differential form can be written $ds^2 = 4 \exp(\psi) (dx^2 + dy^2)$, with ψ verifying the *sinh* - Poisson equation. For these surfaces there is a property of periodicity: a surface that is parallel to a CMC surface and is at a distance $1/H$ has the same property, *i.e.* has constant mean curvature. This chain of periodicity defines a family of CMC surfaces and implicitly a family of solutions of the *sinh* - Poisson equation. The two surfaces are *dual* to each other and the associated streamfunctions are in the relation

$$\psi_2 = -\psi_1 + \ln(4/H^2) \quad (23)$$

Our conjecture is that the stationary asymptotic states of the 2D MHD can be realized by streamfunction ψ and magnetic potential a_z belonging to this family of solutions. If this is unique or if there are other Backlund transforms, remains to be checked.

An alternative way is to start directly from the exact analytical solution of the *sinh*-Poisson equation (Ting *et al.*, 1987)

$$u(x, y) = 2 \ln \frac{\Theta(\mathbf{l} + \frac{1}{2}\mathbf{l})}{\Theta(\mathbf{l})} + K \quad (24)$$

where K is a constant, Θ is the Riemann's hyperelliptic *theta* function and $\mathbf{l} = \mathbf{k}x + \omega y + \mathbf{l}_0$ is the vector of linear combinations arising from the linearization on the Jacobi torus. Finding (ψ, a_z) that verify the constraint Eq.(20) now consists of looking for adequate constants in Eq.(24).

4. The possible ideal route to magnetic reconnection: the *Sphaleron* solution

The fundamental content of a magnetic reconnection event is the change of the topology. If this is to be realized in the absence of any dissipation the field must traverse a singularity. An elementary representation of the singularity can be obtained as follows. Consider two closed, double periodic lines winding around a torus, given by $\xi_1 = m_1\theta - n_1\varphi$ and $\xi_2 = m_2\theta - n_2\varphi$, with different pairs of integers $(m_1, n_1) \neq (m_2, n_2)$. The lines belong to distinct topological classes and no smooth (homotopic) deformation can connect one to the other. They can only be connected if one of them traverses the singular line which is the axis of the torus. There θ is not defined so that state is singular. The process consists of the continuous reduction of the radius on which we imagine that the line (m_1, n_1) is sitting, with preservation of the periodicity, until the radius becomes zero and the line (m_1, n_1) is superposed with the axis of the torus. We actually may call this process straightening of the line (m_1, n_1) . From the axis of the torus, which is a singular state, the line emerges with a different periodicity (m_2, n_2) . This is the topological transition.

The basic idea is that the process that takes place at the reconnection is the *alignment* of the magnetic field lines that arrive from both sides to the X point. Only if the two lines are *aligned* they are indistinguishable and so can be considered reconnected.

Plasma at high temperature (like in tokamak reactor) or at very low density (like in astrophysics) has weak collisionality which only produces a slow rate of magnetic reconnection. This is frequently in contradiction with the experiments where fast rates of reconnection are observed. We are led to look for mechanisms of reconnection, which are fast even at very low collisionality. The *sphaleron* transition may be one of them.

The *sphaleron* (from the greek "ready to fall") has been found in field theory and represents an unstable solution which connects in the space of functions solutions that are separated by a barrier. The initial and final states can have different topologies. In magnetic reconnection, we expect the sphaleron transition to consist of: (1) starting from a topology (m, n) ; (2) first straightening the line, *i.e.* the line goes through the singularity where (m) is not defined; (3) emerge from the straight (singular) state with a different (m, n) . This time the "line" is moving in 3D during this process *i.e.* the point of intersection of the line with the transversal plane comes from the up region, goes through the X point and moves away along the neutral plane, expelled from the X point.

In order to study such possibility we need a formulation in which the magnetic field lines are described in terms of field theory. For only an illustration we consider the Abelian-Higgs with the spatial coordinate compactified to a circle (Park *et al.*, 2001). The space is (x, t) with $x \in S^1$ and the Euclidean action is

$$S = \int dx dt \left[\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \phi)^* (D_\mu \phi) + \frac{\lambda}{4} \left(|\phi|^2 - \frac{v^2}{2} \right)^2 \right] \quad (25)$$

where $D_\mu = \frac{\partial}{\partial x^\mu} - igA_\mu$. The gauge field has two components in this two-dimensional space

$A_\mu \equiv (A_0, A_1)$ and the gauge is fixed by $A_0 = 0$. It is found the sphaleron

$$\begin{aligned} A_{1sph} &= A \equiv \text{const} \\ \phi_{sph} &= \frac{kb(k)}{\sqrt{\lambda}} \exp(igAx) \text{sn}[b(k)x] \end{aligned} \quad (26)$$

where sn is the Jacobi function and $b(k) = \sqrt{\frac{\lambda}{2}} v \left(\frac{2}{1+k^2} \right)^{1/2}$.

5. Conclusions

The problems raised by the tokamak plasma are common to other plasma systems, in particular astrophysical plasma. We have shown that stationary asymptotic states of the current density, which have been suggested to be derived from extremum entropy principle can also be obtained from a field theoretical formulation. For 2D MHD, (which is a good approximation of the tokamak plasma), we found that the asymptotic states of vorticity and current density can be connected by Backlund transforms that correspond to a geometrical properties of Constant Mean Curvature surfaces. For magnetic reconnection at very low collisionality we have proposed a description based on the *sphaleron* solutions of the field theoretical formulation.

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