

# Hidden Color

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**Abstract.** With the acceptance of QCD as the fundamental theory of strong interactions, one of the basic problems in the analysis of nuclear phenomena became how to consistently account for the effects of the underlying quark/gluon structure of nucleons and nuclei. Besides providing more detailed understanding of conventional nuclear physics, QCD may also point to novel phenomena accessible by new or upgraded nuclear experimental facilities. We discuss a few interesting applications of QCD to nuclear physics with an emphasis on the hidden color degrees of freedom.

## 1. Introduction

The 12 GeV upgrade of the continuous electron beam accelerator facility (CEBAF) in Jefferson Laboratory (JLab) provides an opportunity to investigate novel nuclear phenomena predicted by QCD. In particular, a JLab collaboration proposed to measure the deuteron tensor structure function which may provide a probe of exotic QCD effects due to hidden color in a six-quark configuration[1]. It is also interesting to note the recent measurements of electron scattering from high-momentum nucleons in nuclei performed at Hall C(E02-019) in JLab[2], which may allow for an improved determination of the strength of two- and three-nucleon short-range correlations for several nuclei. This result may indicate the existence of hidden color in multi-quark configurations. These new developments in experimental facilities motivate us to discuss a number of applications of QCD to nuclear physics which is the main concern of nuclear chromodynamics (NCD)[3]. Its goal is to give a fundamental description of nuclear dynamics and nuclear properties in terms of quark and gluon fields at short distance, and to obtain a synthesis at long distances with the normal nucleon, isobar, and meson degrees of freedom. NCD provides an important testing ground for coherent effects in QCD and nuclear effects at the interface between perturbative and non-perturbative dynamics.

NCD may imply in some cases a breakdown of traditional nuclear-physics concepts. For example, one can identify where off-shell effects modify traditional nuclear physics-formulas, such as the impulse approximation for elastic nuclear form factors. At high momentum transfer, nuclear amplitudes are predicted to have a power-law fall off in QCD in contrast to the Gaussian or exponential fall off usually assumed in nuclear physics.

In QCD, the fundamental degrees of freedom of nuclei as well as hadrons are postulated to be the spin-1/2 quark and spin-1 gluon quanta. Nuclear systems are identified as color-singlet composites of quark and gluon fields, beginning with the six-quark Fock component of the deuteron. An immediate consequence is that nuclear states are a mixture of several color representations which cannot be described solely in terms of the conventional nucleon, meson, and isobar degrees of freedom: there must also exist hidden-color multi-quark wave-function



components, i.e., nuclear states which are not separable at large distances into the usual color-singlet nucleon clusters.

Because of color confinement, one may expect that virtually any color-singlet hadronic configuration of quarks and gluons can form either bound states or resonances as discussed in a recent paper by Bashkanov, Brodsky and Clement[4]. Color confinement can lead to  $q\bar{q}q\bar{q}$  tetraquark systems[5, 6] such as the charged  $Z_c(3900)$ [7, 8] and possibly  $qqqq\bar{q}$  pentaquark states<sup>1</sup> in addition to the familiar  $q\bar{q}$  mesons,  $qqq$  baryons, the  $gg$  and  $ggg$  glueball states[9], as well as nuclei. Recently, the LHCb collaboration has collected a strong enough signal to declare that  $Z(4430)$  is a bona fide particle[10], which joins other exotic particles, such as  $Z_c(3900)$ . The quark content of  $Z(4430)$  as well as  $Z_c(3900)$  posed a puzzle: its decay implied it contained a charm quark and anticharm, while its charge required two more quarks (a down and anti-up, for example)-giving a total of four<sup>2</sup>. The LHCb experiment at CERN in Geneva, which is primarily set up to study bottom-quark physics in the LHCs proton-proton collisions, has collected 25,000 relevant  $B_0$  decays at energies of 7 and 8 TeVs. This sample is a factor of 10 larger than the data sets of Belle and BaBar. The analysis by the LHCb collaboration shows a highly significant signal (about 14 standard deviations above background) that removes any doubt that  $Z(4430)$  is a real particle. The team also confirms that the particle has a spin of 1 and a positive parity, which rules out the interpretation of the particle signatures as merely arising from a pair of (two-quark)  $D$  mesons. The only remaining explanation appears that  $Z(4430)$  is a bound state of four quarks.

As described in Ref.[4], mesonic nuclei[11, 12, 13, 14, 15, 16, 17] and nuclear-bound quarkonium[18, 19, 20] are also possible. Resonances in the  $\bar{q}\bar{q}q\bar{q}qq$  channel just below the  $B\bar{B}$  threshold could explain the anomalously large rates[21] seen in  $e^+e^- \rightarrow p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}$  at threshold. The anomalously large transverse spin-spin correlation  $A_{NN}$  observed in large-angle proton-proton elastic scattering near the strangeness and charm thresholds[22] could be explained by the effects of  $|uud\bar{u}u\bar{d}Q\bar{Q}\rangle$  baryon number  $B = 2$  resonances in the  $J = L = 1$   $pp$  s-channel[23, 24].

The possible mechanisms underlying confinement multiply as the number of quarks and gluon constituents in a hadronic system increase. A key question is whether such states are bound by fundamental QCD interactions or do the constituents always cluster as color-singlet subsystems? In the case of nuclei, the quark constituents evidently cluster as color-singlet nucleons bound by virtual meson exchange, the analog of covalent binding in molecular physics due to quark interchange or exchange. When there are no covalence quarks in common, QCD also predicts attractive multigluonic van der Waals forces which are due to glueball exchange. The attractive QCD van der Waals potential leads to the prediction of bound states of heavy quarkonium to heavy nuclei[18, 19, 25]. However, there are also rare configurations in which other multi-quark color configurations (hidden color [26, 27, 28]) can enter.

As an extreme example, one may expect a huge enhancement of hidden color degrees of freedom in the neutron stars. They are extreme cases of rare isotopes naturally born after going through the stellar explosion process. Neutron stars can be made in the aftermath of type II supernovae explosions which result from the gravitational core collapse of massive stars. While the masses of type II supernovae are greater than  $8M_\odot$ , where the solar mass  $M_\odot = 2 \times 10^{33}$  g, the masses of neutron stars are between one and two solar masses[29]. The radius of the neutron star is only about 10 km, while the radius of our sun is  $6.96 \times 10^5$  km. Thus, the central density could reach as high as five to ten times of the normal nuclear density,  $\rho_0 = 2.65 \times 10^{14}$  g cm<sup>-3</sup>. For comparison, the solar density is only  $1.4$  g cm<sup>-3</sup>. In such dense nuclei, it appears natural to expect a large enhancement of the hidden color effect. The cooling mechanism of such neutron

<sup>1</sup> The nomenclature  $q\bar{q}$  etc. refers to the lowest particle number Fock state of the hadronic eigensolution of the QCD light-front Hamiltonian.

<sup>2</sup> This was presented as one of the Synopses in the website <http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.112.222002>.

stars has been the subject of numerous studies[30, 31, 32, 33, 35, 36, 37, 38, 39].

On the other hand, it was also claimed in the past that the concept of hidden color is just an artifact of a certain technique of constructing group representations and therefore has no true physical meaning[40]. To understand the main point of such claim, one may consider an example of four-fermion spin system using the  $SU(2)$  group instead of jumping into the discussion of the multi-quark color system with the  $SU(3)$  group. Consider a system of four spin-1/2 particles and try to find the totally spin-neutral (or spin-singlet) system. If the four-particle system is represented by the two clusters of the two-particle system, then there are two independent representations which provide the totally spin-neutral system: i.e.

$$\begin{aligned}
 |\text{Singlet I}\rangle_{1234} &= \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} = |\text{Singlet}\rangle_{12}|\text{Singlet}\rangle_{34}, \\
 |\text{Singlet II}\rangle_{1234} &= \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} = |\text{Triplet}\rangle_{12}|\text{Triplet}\rangle_{34} \\
 &= (2|\text{Singlet}\rangle_{13}|\text{Singlet}\rangle_{24} - |\text{Singlet}\rangle_{12}|\text{Singlet}\rangle_{34})/\sqrt{3}.
 \end{aligned} \tag{1}$$

The key point of the claim in Ref.[40] is that the representation of  $|\text{Singlet II}\rangle_{1234}$  originally expressed as the product of the two triplet-states can be rewritten as a combination of the entirely singlet products. One should note, however, that the required singlet clusters are not just  $|\text{Singlet}\rangle_{12}$  and  $|\text{Singlet}\rangle_{34}$  but also  $|\text{Singlet}\rangle_{13}$  and  $|\text{Singlet}\rangle_{24}$  in order to express  $|\text{Singlet II}\rangle_{1234}$  in terms of only the singlet-states. In other words, the clustering should be modified in this expression compared to the original clustering of the two triplet states and thus another degree of freedom beyond the  $|\text{Singlet}\rangle_{12}|\text{Singlet}\rangle_{34}$  is clearly necessary. The new degree of freedom beyond the  $|\text{Singlet}\rangle_{12}|\text{Singlet}\rangle_{34}$  is necessary no matter how one arranges the clustering. In  $SU(3)$  color system, such new degrees of freedom beyond the original clustering of singlet-states correspond to the hidden-color states and they are clearly necessary to express all the totally color-singlet states of any multi-quark system beyond the ordinary meson and baryon systems.

In the next section, Section 2, we discuss the hidden-color degrees of freedom along with the first-principle QCD evolution of multi-quark systems. In particular, we pay attention to the six- and nine-quark hidden-color configurations and show how fast the number of hidden-color degrees of freedom increases as the number of quarks increase. In Section 3, we review the work by Bashkanov, Brodsky and Clement[4] for the recent observation of a hadronic resonance  $d^*$  in the proton-neutron system with isospin  $I = 0$  and spin-parity  $J^P = 3^+$  which raises the possibility of producing other novel six-quark dibaryon configurations allowed by QCD. The width and decay properties of such six-quark resonances could be regarded as manifestations of hidden-color six-quark configurations. Finally, in Section 4 we discuss one of the basic problems in the analysis of nuclear scattering amplitudes, namely how to consistently account for the effects of the underlying quark/gluon component structure of nucleons. We review the idea of the reduced nuclear amplitudes and discuss its application to the coherent pion photoproduction on the deuteron. Summary and conclusions follow in Section 6.

## 2. Multiquark Evolution in QCD and Hidden Color Degrees of Freedom

The short-distance behavior of multi-quark wave functions can be systematically computed in pQCD. A simple illustration may be found in Ref. [27], where the wave function of a four-quark color-singlet bound state in  $SU(2)_C$  has been analyzed as an analogue to the six-quark problem in  $SU(3)_C$ . The QCD evolution equation was solved for the multi-quark distribution amplitude at short distances in the basis of completely anti-symmetrized quark representations. The eigensolutions of the evolution kernel correspond to a spectrum of candidate states of the relativistic multi-quark system. The four-quark antisymmetric representations are then connected to the eigensolutions to the physical two-cluster basis of  $SU(2)_C$  dibaryon ( $NN$ ,

$N\Delta$ ,  $\Delta\Delta$ ) and hidden-color ( $CC$ ) components. It provides constraints on the effective nuclear potential between two clusters. Anomalous states are also found in the spectrum which cannot exist without substantial hidden-color degrees of freedom.

As technical details are presented in Ref. [27], a given four-quark antisymmetric representation ( $A$ ) can be decomposed onto two clusters ( $A_1 \otimes A_2$ ) using the following steps:


- (i) Represent the four-quark antisymmetric representation as an inner product form  $A = C \times T \times S \times O$ .
- (ii) Decompose each four-quark representation  $C, T, S$ , and  $O$  as an outer product of 2 two-quark representations using fractional parentage coefficients, e.g.,  $C = C_1 \otimes C_2$ .
- (iii) Recombine the representations as an inner product:  $A = (C_1 \otimes C_2) \times (T_1 \otimes T_2) \times (S_1 \otimes S_2) \times (O_1 \otimes O_2)$ .
- (iv) Commute the order of inner product and outer product, gathering together representations of the same cluster:  $A = (C_1 \times T_1 \times S_1 \times O_1) \otimes (C_2 \times T_2 \times S_2 \times O_2) \equiv A_1 \otimes A_2$ .
- (v) It is sufficient to consider only the coefficient of the symmetric orbitals  $O_1$  and  $O_2$  to classify the clusters such as  $NN$ ,  $N\Delta$  and  $\Delta\Delta$ .

With this method, the four-quark eigensolutions can be expanded on the physical basis of effective clusters which are the analogs of the  $NN$ ,  $\Delta\Delta$ ,  $N\Delta$ , and  $CC$  states in QCD. By analyzing the behavior of  $\phi(x_i, Q)$  at large  $Q$ , one can predict the effective potential between two clusters. For example, it was found that one of the hidden-color states has a large projection on the eigensolution with leading anomalous dimension (dominant at short distances), whereas the states analogous to  $NN$  and  $\Delta\Delta$  in QCD have an almost negligible leading component. This implies that the effective potential tends to be repulsive between color-singlet clusters and attractive between colored clusters at short distance. Two other types of four-quark states were also found in  $SU(2)_C$ , which cannot be identified with dibaryon degrees of freedom. One of these states has equal  $NN$ ,  $\Delta\Delta$  and  $CC$  components. The other state is an anomalous hidden-color two-cluster system orthogonal to the usual hidden-color state which has the unusual feature that it has very small projection on the eigensolutions which dominate at short distance, i.e., the effective potential between the colorful clusters of the anomalous hidden-color state tends to be repulsive. One may speculate that the analogous anomalous states in QCD could be quasistable non-nucleonic nuclear systems, possibly related to the anomalous phenomena apparently observed in nuclear collisions [41, 42, 43, 44]. These results also give some support to the conjecture that multi-quark hidden-color components exist in ordinary nuclei [43].

The results in Ref. [27] represented a first attempt to extract exact results for the composition and interactions of multi-quark nuclear systems at short distances. Although just the four-quark bound state in  $SU(2)_C$  was analyzed in Ref. [27] for simplicity, many of the derived properties were indeed extended to six-quark states in QCD as discussed in Ref. [28]. In particular, since the leading eigensolution at high-momentum transfer has 80% hidden-color probability, we expect a transition of the ordinary nuclear state to non-nucleonic degrees of freedom as one evolves from long to short distances. The set of eigensolutions of the evolution equation represent all the possible degrees of freedom of the multi-quark bound-state system since its kernel has the same invariances and symmetries of the full QCD Hamiltonian. We thus expect that the eigensolutions of the evolution kernel, which are dominantly hidden-color, to correspond to actual states and excitations of ordinary nuclei. A careful experimental search for these exotic resonances should be made. Possible channels where signals for such states may be observed include Compton scattering and pion photo-production on a deuteron target at large angles, as well as the electron scattering from high-momentum nucleons in nuclei.

### 2.1. Six-Quark Evolution

Six-quark states can be classified by their symmetries under  $SU(3)_C$  (color),  $SU(2)_T$  (isospin),  $SU(2)_S$  (spin), and spatial symmetry. Since the physical states are color singlets, the Young

symmetry of the color-singlet states of the six-quark system is  $[222]$  or . Since three colors are shared by six quarks, there are five independent color-singlet states corresponding to five different Yamanouchi labels of  $[222]$  symmetry. The explicit representations of the five independent color-singlet states and their correspondence to Yamanouchi labels are given in the Appendix of Ref. [28].

Harvey[45, 46, 47] has classified the color-singlet six-quark states in terms of a physical cluster decomposition. In this classification, the physical deuteron state (i.e., a bound state of two color-singlet clusters) is represented as a linear combination of several different kinds of totally antisymmetric color-singlet six-quark states. For example, the two well-separated nucleons  $|NN\rangle$  are given by

$$|NN\rangle = \frac{1}{3}|[6]\{33\rangle + \frac{2}{3}|[42]\{33\rangle - \frac{2}{3}|[42]\{51\rangle, \quad (2)$$

where  $[]$  and  $\{\}$  represent the orbital and spin-isospin symmetry and color symmetry  $[222]$  is abbreviated. Similarly, the two well-separated deltas  $|\Delta\Delta\rangle$  and the hidden-color states  $|CC\rangle$  are given by

$$|\Delta\Delta\rangle = \sqrt{\frac{4}{45}}|[6]\{33\rangle + \sqrt{\frac{16}{45}}|[42]\{33\rangle + \sqrt{\frac{25}{45}}|[42]\{51\rangle, \quad (3)$$

$$|CC\rangle = \sqrt{\frac{4}{5}}|[6]\{33\rangle - \sqrt{\frac{1}{5}}|[42]\{33\rangle, \quad (4)$$

respectively.

Since this classification itself does not include the dynamics of strong interactions between the constituents, the dynamical evolution equation of six-quark systems was formulated [28] to include the dynamics between the quarks and solved to give the general form of the quark distribution amplitude  $\phi_d(x_i, Q)$ :

$$\phi_d(x_i, Q) = (CTS)\phi(x_i, Q), \quad (5)$$

where  $(CTS)$  is a tensor representation obtained from the Young symmetry of  $SU(3)_C$ ,  $SU(2)_T$ , and  $SU(2)_S$ , and the orbital distribution amplitude is given by

$$\phi(x_i, Q) = x_1 x_2 x_3 x_4 x_5 x_6 \sum_{n=0}^{\infty} a_n \tilde{\phi}_n(x_i) \left( \ln \left( \frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n}. \quad (6)$$

One may then project Eq. (2) to momentum space:

$$\phi_{NN}(x_i, Q) = \frac{1}{3}\phi_{[6]\{33}(x_i, Q) + \frac{2}{3}\phi_{[42]\{33}(x_i, Q) - \frac{2}{3}\phi_{[42]\{51}(x_i, Q). \quad (7)$$

In the limit  $Q \rightarrow \infty$ , the dependence on  $Q$  is determined by the leading anomalous dimension; all other terms which have nonleading anomalous dimensions are suppressed by logarithmic damping factors. The orbital symmetry of the eigensolution which has the leading anomalous dimension cannot be  $[42]$  but is  $[6]$ . This means only the first term of Eq. (7) survives at the large- $Q$  limit. The  $NN$  amplitude itself is not sufficient. One can show that an 80% hidden-color state is necessary to saturate the normalization of the six-quark amplitude when six quarks approach

the same position in impact space. One may call this new degree of freedom an anomalous state since it does not correspond to the usual nucleonic degrees of freedom of the nucleus. The physical implication of the anomalous state was discussed in the previous toy model analysis [27].

The QCD predictions for high- $Q$  behavior of the deuteron form factor and the form of the deuteron distribution amplitude at short distances were presented in Ref. [26]. The fact that the six-quark state is 80% hidden color at small transverse separation implies that the deuteron form factors cannot be described at large  $Q$  by meson-nucleon degrees of freedom alone, and that the nucleon-nucleon potential is repulsive at short distances. Since the basic scale of QCD,  $\Lambda_{\text{QCD}}$ , is phenomenologically of the order of a few hundred MeV or less, QCD predicts a transition from the traditional meson and nucleon degrees of freedom of nuclear physics to quark and gluon degrees of freedom at inter-nucleon separations of a fm or less. In this respect, it may be important to realize the 12 GeV upgrade of JLab as a viable opportunity to investigate novel nuclear phenomena predicted by QCD. As an example, a JLab collaboration proposed to measure the deuteron tensor structure function  $b_1$  [1]. This leading twist tensor structure function of spin-1 hadrons provides a tool to study partonic effects, while also being sensitive to coherent nuclear properties in the simplest nuclear system. Although shadowing effects are expected to dominate this structure function at low values of the Bjorken scaling variable  $x$ , it may provide a probe of exotic QCD effects due to hidden color in 6-quark configuration at larger values of  $x$ . Since the deuteron wave function is relatively well known, any novel effects are expected to be readily observable. An analysis including the hidden-color degrees of freedom appeared recently [48]. Most available models predict a small or vanishing value of  $b_1$  at moderate  $x$ . However, the first measurement of  $b_1$  at HERMES revealed a crossover to an anomalously large negative value in the region  $0.2 < x < 0.5$ , albeit with relatively large experimental uncertainty. The proposal [1] describes an inclusive measurement of the deuteron tensor asymmetry in the region  $0.15 < x < 0.45$ , for  $0.8 < Q^2 < 5.0 \text{ GeV}^2$ . It might be possible to determine  $b_1$  with sufficient precision to discriminate between conventional nuclear models, and the more exotic behaviour which is hinted at by the HERMES data. This measurement will provide access to the tensor quark polarization, and allow a test of the Close-Kumano sum rule [49], which vanishes in the absence of tensor polarization of the quark sea.

## 2.2. Nine-Quark Color Singlets

In this subsection, we provide the counting of the number of hidden-color states in nine-quark systems. It is well known that the baryon multiplets can be constructed from the three triplets (quarks) in  $SU(3)$ : i.e.,

$$\square \otimes \square \otimes \square = \square\square\square \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \quad (8)$$

or

$$\{3\} \otimes \{3\} \otimes \{3\} = \{10\} \oplus \{8\}_S \oplus \{8\}_A \oplus \{1\}. \quad (9)$$

Applying it in the color degrees of freedom, i.e.  $SU(3)_C$ , one can see that only one color singlet appears in the three-quark system, or baryon. However, more color singlets independent of each other can be formed as the number of quarks gets increased.

In the six-quark system such as deuteron, five color singlets are formed owing to the fact that the three color degrees of freedom are shared by the six quarks in the system. In other words, not only the singlet and singlet but also the octet and octet can form six-quark color-singlet

bound-states. Similar to Eq. (8), one may explicitly get

$$\begin{aligned} & \square \otimes \square \otimes \square \otimes \square \otimes \square \otimes \square = \square \square \square \square \square \square \oplus 5 \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & & & \\ \hline \end{array} \\ & \oplus 9 \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & & & \\ \hline \end{array} \oplus 15 \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & & & \\ \hline \square & & & \\ \hline \end{array} \oplus 16 \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & & \\ \hline \square & & \\ \hline \end{array} \oplus 5 \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & & \\ \hline \square & & \\ \hline \end{array} \oplus 5 \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \square & \\ \hline \end{array} \end{aligned} \quad (10)$$

or

$$\begin{aligned} & \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} = 729 \\ & = \{28\} \oplus 5\{35\} \oplus 9\{27\} \oplus 15\{10\} \oplus 16\{8\} \oplus 5\{10^*\} \oplus 5\{1\}. \end{aligned} \quad (11)$$

Among the five color-singlet states in Eq. (11), just one of them stems from the two color-singlet clusters of three-quark system such as  $N$  and  $\Delta$  while the remaining four states correspond to the two color-octet clusters denoted as  $CC$  or hidden-color states.

Now, let's count here how many hidden-color states are available in the nine-quark system such as the  ${}^3\text{He}$  nucleus. Extending the same color algebra, we get

$$\begin{aligned} & \square \otimes \square \otimes \square \otimes \square \otimes \square \otimes \square \otimes \square \otimes \square \otimes \square = \square \square \square \square \square \square \square \square \square \oplus 27 \begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & \\ \hline \end{array} \\ & \oplus 8 \begin{array}{|c|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & & \\ \hline \end{array} \oplus 48 \begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & \\ \hline \square & & & & & \\ \hline \end{array} \oplus 42 \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \end{array} \oplus 105 \begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & \\ \hline \square & & & & & \\ \hline \square & & & & & \\ \hline \end{array} \\ & \oplus 162 \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \end{array} \oplus 28 \begin{array}{|c|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & & \\ \hline \square & & & & & & \\ \hline \square & & & & & & \\ \hline \end{array} \oplus 84 \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \end{array} \oplus 120 \begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & \\ \hline \square & & & & & \\ \hline \square & & & & & \\ \hline \square & & & & & \\ \hline \end{array} \\ & \oplus 168 \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \square & & & & \\ \hline \end{array} \oplus 42 \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & & & \\ \hline \square & & & \\ \hline \square & & & \\ \hline \end{array} \end{aligned} \quad (12)$$

or

$$\begin{aligned} & \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} = 19683 \\ & = \{55\} \oplus 27\{81\} \oplus 8\{80\} \oplus 48\{64\} \oplus 42\{35^*\} \oplus 105\{35\} \oplus 162\{27\} \oplus 28\{28\} \oplus 84\{10^*\} \\ & \oplus 120\{10\} \oplus 168\{8\} \oplus 42\{1\}. \end{aligned} \quad (13)$$

This shows a remarkable proliferation of the hidden color degrees of freedom in the nine-quark system compare to the six-quark system[50], i.e.  $(42 - 1)/(5 - 1) = 41/4 > 10$ . Here<sup>3</sup> we find forty-one hidden-color states in the nine-quark system since only one of the forty-two color-singlet states stem from the three color-singlet clusters of three-quark systems. Compared to just four hidden-color states in the six-quark system, we now find more than an order of magnitude increase of hidden-color states in the nine-quark system. It amounts to almost an order of magnitude increase in the hidden-color sectors.

It is interesting to note the recent measurements of electron scattering from high-momentum nucleons in nuclei performed at JLab Hall C(E02-019)[2]. These data allowed for an improved determination of the strength of two- and three-nucleon correlations for several nuclei, including

<sup>3</sup> The number of color singlets can also be found from a specific combinatoric counting technique using Young tableaux. The number of hidden-color degrees of freedom presented in Ref.[50] is corrected in this work.

light nuclei where clustering effects can be examined. At  $x > 2$ , where three-nucleon short-range correlations (3N-SRCs) are expected to dominate, their  $A/{}^3\text{He}$  ratios were significantly higher than the previous CLAS data and suggested that contributions from 3N-SRCs in heavy nuclei are larger than previously believed. Interestingly, there were large differences between the  ${}^4\text{He}/{}^3\text{He}$  ratios from E02-019 ( $Q^2 \approx 2.9 \text{ GeV}^2$ ) and those from CLAS ( $\langle Q^2 \rangle \approx 1.6 \text{ GeV}^2$ ). If these results indeed indicate a large dependence on  $Q^2$  for the  $x > 2$  plateau region, then it might be associated with the dramatic increase of the hidden color degrees of freedom that we found in the three-nucleon systems. It will be interesting to see if this ratio difference between Hall C and CLAS is truly due to the  $Q^2$  difference between these two experiments. This may be tested in the upcoming experiments to be performed after the 12 GeV upgrade in JLab.

### 3. Double Pionic Fusion and “ABC” Effect

Bashkanov, Brodsky and Clement[4] discussed the recent observation of a hadronic resonance  $d^*$  in the proton-neutron system with isospin  $I = 0$  and spin-parity  $J^P = 3^+$  and the possibility of producing novel six-quark dibaryon configurations allowed by QCD. As Clement reviewed the process of two-pion production in nucleon-nucleon collisions[51], it provides a suitable means to study single as well as mutual nucleon excitations by the well-understood  $t$ -channel meson exchange and gives access to study the mutual excitation of the colliding nucleons into their first excited state, the  $\Delta(1232)$ , in great detail. The question of whether there are more eigenstates in the system of two baryons than just the deuteron ground state has been a strong motivation for this process from the perspective of quantum chromodynamics (QCD) since the possible existence of so-called dibaryons was first envisaged by Jaffe[52]. Although the worldwide dibaryon searches have met with no success so far, the two-pion production is of special interest due to the possible detailed study of baryon-baryon excitations with particular emphasis on the  $\Delta\Delta$  system.

In particular, the purely isoscalar reaction channel  $pn \rightarrow d\pi^0\pi^0$  exhibits a narrow resonance-like structure with  $m = 2.37 \text{ GeV}$  and  $\Gamma = 70 \text{ MeV}$  in the total cross section, which is correlated with an intriguing low-mass enhancement in the  $\pi\pi$  invariant mass spectrum known as the ABC effect[53] as well as with the formation of a  $\Delta\Delta$  system in the intermediate state. From the angular distributions, the spin and parity of this structure have been derived resulting in  $I(J^P) = 0(3^+)$ . For this resonance-like structure to be the ground-state  $S$ -wave of the totally antisymmetric six-quark system, it is inevitably represented by[28]

$$|(1^6) \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \rangle_{CTSO} = \frac{1}{\sqrt{5}} \sum_{Y_T} \sum_{Y_{CS}} \eta_T \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} Y_T >_T \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} Y_{CS} >_{CS} \begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \end{array} [111111]_O >_O, \quad (14)$$

where  $Y_T$  is the allowed Yamanouchi label of isospin symmetry and the phase  $\eta_T = \pm 1 = (-1)^{Y_T}$  depends on whether  $Y_T$  is obtained from the Yamanouchi label with the indices in natural order by an even or odd number of transpositions. Similarly,  $Y_{CS}$  is the allowed Yamanouchi label of color-spin symmetry and  $[111111]_O$  is the only allowed Yamanouchi label for the  $S$ -wave orbital symmetry. As phrased long ago in terms of inevitable dibaryon[54], it is interesting to note that an inevitable symmetry is consequential from a particular quantum number such as  $I(J^P) = 0(3^+)$ .

Such inevitable symmetry in the process of  $pn \rightarrow d^* \rightarrow \Delta\Delta \rightarrow d\pi^0\pi^0$  led to the conclusion[4] that  $d^*$  must be an unconventional origin, possibly indicating a genuine six-quark nature, since the totally symmetric [6] orbital state corresponds to dominantly the “hidden color” state rather than the “deltaron” state which has the [42] orbital symmetry. From Eqs.(2), (3) and (4), one



may note that there are only two possible quark structures for an  $I(J^P) = 0(3^+)$  resonance in the two-baryon system given by[4]

$$\begin{aligned} |\psi_{d^*}\rangle &= \sqrt{\frac{1}{5}}|\Delta\Delta\rangle + \sqrt{\frac{4}{5}}|6Q\rangle \quad \text{and} \\ |\psi_{d^*}\rangle &= \sqrt{\frac{4}{5}}|\Delta\Delta\rangle - \sqrt{\frac{1}{5}}|6Q\rangle, \end{aligned} \quad (15)$$

where  $\Delta\Delta$  means the asymptotic  $\Delta\Delta$  configuration and  $6Q$  is the genuine hidden color six-quark configuration. The first solution denotes a  $S^6$  quark structure (all six quarks in the  $S$ -shell), or the totally symmetric  $[6]$  orbital state, and the second one a  $S^4P^2$  configuration (4 quarks in the  $S$ -shell and 2 quarks in the  $P$ -shell), or the  $[42]$  orbital state. As discussed in Ref.[4], since the quark structure with the large  $\Delta\Delta$  coupling would correspond to a “deltaron” state and can be excluded, it may be natural to assign the observed  $d^*$  resonance to the  $S^6$  six-quark predominantly hidden color state, thus providing an explanation for its narrow decay width as discussed in Ref.[4].

It is also interesting to note that such a resonance seems to couple with the anomalous enhancement in the low-mass region of the  $\pi\pi$ -invariant mass spectrum known as the ABC effect[53] and the  $\Delta\Delta$  system could well serve as a doorway state to an exotic excitation as discussed by Clement[51]. Such anomalous enhancement has been pointed out already more than fifty years ago in  $p + d$  collisions[53]. In the reaction  $pd \rightarrow {}^3\text{He}X$ , an anomalous enhancement in the low-mass region of the  $\pi\pi$ -invariant mass spectrum was observed[55]. In particular, the missing mass of the detected  ${}^3\text{He}$  ejectile that corresponds kinematically to the production of a pion pair, i.e.  $X = \pi\pi$ , exhibited an excess above phase space starting right at threshold at low  $\pi\pi$ -masses. In subsequent bubble-chamber[56, 57] and single-arm magnetic spectrometer measurements[58, 59, 60, 61, 62, 63, 64], this enhancement was found in fusion reactions leading to  $d$ ,  ${}^3\text{He}$  and  ${}^4\text{He}$ , when an isoscalar pion pair was produced. However, no enhancement was found in the fusion reaction to tritium, where an isovector pion pair was produced. These results suggested that the effect only appears in double-pionic fusion reactions in combination with the production of an isoscalar pion pair. In the absence of any consistent explanation, this enhancement effect in the low-mass spectrum was coined as the ABC effect following the initials of the authors of Ref.[53]. For a detailed understanding of the two-pion production process in  $pp, pd$  and  $dd$  collisions systematic experimental studies were initiated in the nineties at CELSIUS and continued later on at COSY[51]. According to Ref.[51], these exclusive and kinematically complete high-statistics measurements have been carried out at the detector installations PROMICE/WASA[65, 66, 67], CELSIUS/WASA[68, 69, 70, 71, 72, 73, 74], COSY-TOF[75, 76], WASA-at-COSY[77, 78] and COSY-ANKE[79] covering the energy range from the two-pion production threshold up to  $\sqrt{s} = 2.5$  GeV (corresponding to a nucleon beam energy of 1.4 GeV).

Since most of these experiments were done at beam energies, which energetically allow the mutual excitation of two colliding nucleons into their first excited state, the  $\Delta(1232)P_{33}$ , theoretical attempts were undertaken to understand the ABC effect by the excitation of a  $\Delta\Delta$  system via  $t$ -channel meson exchange between the two colliding nucleons or by variations of this scenario[80, 81, 82, 83, 84]. Such calculations, indeed, predicted a low-mass enhancement, however, always also a high-mass enhancement. Surprisingly enough, the inclusive single-arm measurements at that time seemed to support such a high-mass enhancement. And it was only recently that the first exclusive and kinematically complete measurements of solid statistics[68, 77, 85, 86] over practically the full phase space revealed that the high-mass enhancement observed in the single-arm measurements rather had resulted from other reactions like  $3\pi$  and  $\eta$  production[63]. However, the new generation of measurements confirmed the

existence of a very pronounced low-mass enhancement. More detailed discussion of the recent theoretical calculations and the experimental strategies for definitive experimental confirmation or exclusion can be found from Ref.[4]. Investigations of the basic double-pionic fusion in the region of the ABC effect has also been discussed in contrast to the conventional fusion processes in the primordial nucleosynthesis and those taking place in star burning[87].

#### 4. Reduced Nuclear Amplitudes

One of the basic problems in the analysis of nuclear scattering amplitudes is how to consistently account for the effects of the underlying quark/gluon component structure of nucleons. Traditional methods based on the use of an effective nucleon/meson local Lagrangian field theory are not really applicable, giving the wrong dynamical dependence in virtually every kinematic variable for composite hadrons. The inclusion of ad hoc vertex form factors is unsatisfactory since one must understand the off-shell dependence in each leg while retaining gauge invariance; such methods have little predictive power. On the other hand, the explicit evaluation of the multi-quark hard-scattering amplitudes needed to predict the normalization and angular dependence for a nuclear process, even at the leading order of QCD coupling constant  $\alpha_s$ , requires the consideration of millions of Feynman diagrams. Beyond leading order one must include contributions of non-valence Fock states wave functions, and a rapidly expanding number of radiative corrections and loop diagrams. The reduced amplitude method[88], although not an exact replacement for a full QCD calculation, provides a simple method for identifying the dynamical effects of nuclear substructure, consistent with covariance, QCD scaling laws and gauge invariance. The basic idea has already been introduced for the reduced deuteron form factor. More generally, if the nuclear binding is neglected, then the light-cone nuclear wave-function can be written as a cluster decomposition of collinear nucleons:  $\psi_{q/A} = \psi_{N/A} \Pi_N \Psi_{q/N}$  where each nucleon has  $1/A$  of the nuclear momentum. A large momentum transfer nucleon amplitude then contains as a factor the probability amplitude for each nucleon to remain intact after absorbing  $1/A$  of the respective nuclear momentum transfer. Each probability amplitude can be identified with the respective nucleon form factor  $F(\hat{t}_i = \frac{1}{A^2} t_A)$ , where  $t_A$  is the square of the transferred momentum to the nucleus with mass number  $A$ . Thus for any exclusive nuclear scattering process, the reduced nuclear amplitude can be defined as

$$m = \frac{\mathcal{M}}{\Pi_{i=1}^A F_N(\hat{t}_i)}. \quad (16)$$

The predictions of pQCD for pion photoproduction on the deuteron  $\gamma D \rightarrow \pi^0 D$  have been given[89] at large momentum transfer using the reduced amplitude formalism. The cluster decomposition of the deuteron wave function at small binding only allows the nuclear coherent process to proceed if each nucleon absorbs an equal fraction of the overall momentum transfer. Furthermore, each nucleon must scatter while remaining close to its mass shell. Thus, the nuclear photoproduction amplitude  $\mathcal{M}_{\gamma D \rightarrow \pi^0 D}(u, t)$  factorizes as a product of three factors: (1) the nucleon photoproduction amplitude  $\mathcal{M}_{\gamma N_1 \rightarrow \pi^0 N_1}(u/4, t/4)$  at half of the overall momentum transfer, (2) a nucleon form factor  $F_{N_2}(t/4)$  at half the overall momentum transfer, and (3) the reduced deuteron form factor  $f_d(t)$ , which according to pQCD, has the same monopole fall-off as a meson form factor. A comparison with the recent JLAB data for  $\gamma D \rightarrow \pi^0 D$  of Meekins *et al* [90] and the available  $\gamma p \rightarrow \pi^0 p$  data [91, 92, 93, 94, 95] shows good agreement between the pQCD prediction and experiment over a large range of momentum transfers and center-of-mass angles. The reduced amplitude prediction is consistent with the constituent counting rule  $p_T^{11} \mathcal{M}_{\gamma D \rightarrow \pi^0 D} \rightarrow F(\theta_{cm})$  at large momentum transfer. This is found to be consistent with measurements for photon lab energies  $E_\gamma > 3$  GeV at  $\theta_{cm} = 90^\circ$  and  $136^\circ$ .

The predictions of QCD for nuclear reactions are most easily described in terms of light-front (LF) wave functions defined at equal LF time  $\tau = t + z/c$  [96]. The deuteron eigenstate can be

projected on the complete set of baryon number  $B = 2$ , isospin  $I = 0$ , spin  $J = 1$ ,  $J_z = 0, \pm 1$  color-singlet eigenstates of the free QCD Hamiltonian, beginning with the six-quark Fock states. Each Fock state is weighted by an amplitude which depends on the LF momentum fractions  $x_i = k_i^+/p^+$  and on the relative transverse momenta  $\vec{k}_{\perp i}$ . There are five different linear combinations of six color-triplet quarks which make an overall color-singlet, only one of which corresponds to the conventional proton and neutron three-quark clusters. Thus, the QCD decomposition includes four six-quark unconventional states with “hidden color” [28]. Nevertheless, the cluster decomposition theorem [97] states that in the zero binding limit ( $B.E. \rightarrow 0$ ), the LF wave function of the deuteron must reduce to a convolution of on-shell color-singlet nucleon wave functions:

$$\begin{aligned} \lim_{B.E. \rightarrow 0} \psi_{uudd}^D(x_i, \vec{k}_{\perp i}, \lambda_i) &= \int_0^1 dz \int d^2 \ell_{\perp} \psi^d(z, \ell_{\perp}) \\ &\times \psi_{uud}^p(x_i/z, \vec{k}_{\perp i} + (x_i/z)\ell_{\perp}, \lambda_i) \\ &\times \psi_{udd}^n(x_i/(1-z), \vec{k}_{\perp i} - [x_i/(1-z)]\ell_{\perp}, \lambda_i), \end{aligned} \quad (17)$$

where  $\psi^d(z, \ell_{\perp})$  is the reduced “body” LF wave function of the deuteron in terms of its nucleon components. Applying this cluster decomposition to an exclusive process involving the deuteron, one can derive a corresponding reduced nuclear amplitude (RNA) [26, 88, 98]. Moreover, at zero binding, one may take  $\psi^d(z, \ell_{\perp}) \rightarrow \delta(z - m_p/(m_p + m_n)) \times \delta^2(\ell_{\perp})$ . In effect, each nucleon carries half of the deuteron’s four-momentum.

Thus, in the weak nuclear binding limit, the deuteron form factor reduces to the overlap of nucleon wave functions at half of the momentum transfer, and  $F_D(Q^2) \rightarrow f_d(Q^2)F_N^2(Q^2/4)$  where the reduced form factor  $f_d(Q^2)$  is computed from the overlap of the reduced deuteron wave functions [98]. The reduced deuteron form factor resembles that of a particle spin-one meson form factor since its nucleonic substructure has been factored out. The pQCD predicts the nominal scaling  $Q^2 f_d(Q^2) \sim \text{const}$  [26]. The measurements of the deuteron form factor show that this scaling is in fact well satisfied at spacelike  $Q^2 \geq 1 \text{ GeV}^2$  [99].

Considering a similar analysis of pion photoproduction on the deuteron  $\gamma D \rightarrow \pi^0 D$  at weak binding, we obtained the reduced amplitude scaling [89]

$$\mathcal{M}_{\gamma D \rightarrow \pi^0 D}(u, t) = C' f_d(t) \mathcal{M}_{\gamma N_1 \rightarrow \pi^0 N_1}(u/4, t/4) F_{N_2}(t/4), \quad (18)$$

where the constant  $C'$  is expected to be close to unity. A comparison with elastic electron scattering then yields the following proportionality of amplitude ratios:

$$\frac{\mathcal{M}_{\gamma D \rightarrow \pi^0 D}}{\mathcal{M}_{e D \rightarrow e D}} = C' \frac{\mathcal{M}_{\gamma p \rightarrow \pi^0 p}}{\mathcal{M}_{e p \rightarrow e p}}. \quad (19)$$

The new factored form, Eq. (18), differs significantly from the older reduced nuclear amplitude factorization [88], for which

$$\mathcal{M}_{\gamma D \rightarrow \pi^0 D}^{\text{older}}(u, t) \simeq m_{\gamma d \rightarrow \pi^0 d}(u, t) F_N^2(t/4). \quad (20)$$

Here,  $m_{\gamma d \rightarrow \pi^0 d}$  is the reduced amplitude; it scales the same as  $m_{\gamma p \rightarrow \pi^0 p}$  at fixed angles since the nucleons of the reduced deuteron  $d$  are effectively point-like. The advantages of this reduction are that some nonperturbative physics is included via the nucleon form factors and that systematic extension to many nuclear processes is possible [88]. The new factorization given by Eq. (18) is an improvement because it includes nonperturbative effects in the pion production process itself.

JLAB experimental data [90] on  $\pi^0$  photoproduction from a deuteron target, up to a photon lab energy  $E_{\text{lab}} = 4$  GeV, were presented as an example inconsistent with both constituent-counting rules (CCR) [100, 101] and old RNA [88] predictions. One potential explanation for this disagreement has been odderon exchange [102, 103]. Because the odderon has zero isospin and is odd under charge conjugation, such an exchange is allowed in the  $t$  channel of  $\pi^0$  photoproduction. However, it is shown in Ref. [89] that the improved factorization given by Eqs. (18) and (19) is in good agreement with the JLAB data [90] for  $\gamma D \rightarrow \pi^0 D$  and the available  $\gamma p \rightarrow \pi^0 p$  data [91, 92, 93, 94, 95] as well as the existing  $eD \rightarrow eD$  and  $ep \rightarrow ep$  data. There is thus no need to invoke any additional anomalous contribution to understand the data [90].

## 5. Summary and Conclusion

In summary, we have indicated a few phenomena that go beyond what can be described in terms of the baryon-meson picture of nuclear physics. We have stressed that if one accounts for the substructure of hadrons in terms of quarks and gluons, new phenomena may occur.

We discussed [50] some ideas concerning a modification of the nucleon-meson picture of the nuclear forces. The occurrence of colored multi-quark substructures in nuclei, was found to have consequences for the understanding of the nucleon-nucleon potential at short distances, which means distances of the same order of the size of the nucleons themselves. Two main points were found: (i) the composite nature of the nucleons, which naturally leads to the idea of strong form factors for the meson-nucleon vertices, can be used to estimate these form factors in the quark-gluon picture, and (ii) the occurrence of “exotic” channels, say the colorless combination of two colored clusters,  $CC$ , leads in a natural way to an energy-dependent effective, “optical”, potential which incorporates the short-range effective repulsion that is responsible for the saturation of the nuclear force as an effect of the leaking of the nucleon-nucleon channel into the  $CC$  channel.

Such multi-quark states must have their own evolution as discussed in Sec. 2. Combined with the idea of reduced nuclear amplitudes, this work leads to predictions of the asymptotic behavior of nuclear observables like electro-magnetic form factors, which can be and have been tested in experiments. The deuteron tensor structure function  $b_1$  could be sensitive to hidden color degrees of freedom at large  $x$ . The order of magnitude increase in the nine-quark hidden color degrees of freedom may be behind the significant enhancement of the  $A/{}^3\text{He}$  ratio as  $Q^2$  gets larger. Recent observation of  $d^*$  resonance raises the possibility of producing other novel color-singlet six-quark dibaryon configurations allowed by QCD as discussed in Sec. 3.

The link between the traditional nuclear physics and the quark-gluon picture may be provided by the reduced nuclear amplitudes. An elaboration of this idea was given in Sec. 4.

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