

Pionic and hidden-color, six-quark contributions to the deuteron b_1 structure function

G A Miller

Univ. of Washington, Seattle, WA 98195, USA

E-mail: miller@uw.edu

Abstract. The b_1 deep-inelastic structure function is an observable feature of a spin-1 system that is sensitive to non-nucleonic components of the target nuclear wave function. The contributions of exchanged pions in the deuteron are estimated and found to be of measurable size for values of x of about 0.1. A simple model for a hidden-color, six-quark configuration (with only about 0.15% probability to exist in the deuteron) is proposed and found to give substantial contributions for values of x greater than about 0.2. Good agreement with the only existing (HERMES) data is obtained. Predictions are made for an upcoming JLab experiment. The Close and Kumano sum rule is investigated and found to be a useful guide to understanding various possible effects that may contribute.

1. Introduction

This conference proceeding is based entirely on previously published work [1].

Consider lepton-nucleus deep-inelastic scattering, with the direction of the photon as the spin quantization axis. The cross section for an unpolarized lepton but polarized deuteron (D) target depends on the value m of the magnetic quantum number of the target:

$$d^2\sigma^{(m)} \propto l^{\mu\nu} W_{\mu\nu}^{(m)}, \quad (1)$$

where $l^{\mu\nu}$ is the standard lepton tensor and

$$W_{\mu\nu}^{(m)} = \int d^4r e^{iq \cdot r} \langle D, J=1, J_z=m | [j^\mu(r), j^\nu(0)] | D, J=1, J_z=m \rangle. \quad (2)$$

The standard structure function F_1 is given by

$$F_1(x) = \frac{1}{3} \sum_m W_{11}^{(m)}, \quad (3)$$

and we are interested in

$$b_1(x) = W_{11}^{(1)} - W_{11}^{(0)}, \quad (4)$$

with x as the Bjorken variable, and we ignore the Q^2 dependence for simplicity. The function b_1 has been measured by the HERMES collaboration using a tensor polarized deuteron target [2] for values of Bjorken $0.01 < x < 0.45$. The function b_1 takes on its largest



value of about 10^{-2} at the lowest measured value of x (0.012), decreases with increasing x through zero and takes on a minimum value of roughly -4×10^{-3} .

The function b_1 nearly vanishes if the spin-one target is made of constituents in a relative s -state, and is very small for a target of spin 1/2 particles moving non-relativistically in higher angular momentum states [3, 4]. Thus one expects [3] that a nuclear b_1 may be dominated by non-nucleonic components of the target nuclear wave function. Consequently, a Jefferson Laboratory experiment [5] is planned to measure b_1 for values of x in the range $0.16 < x < 0.49$ and $1 < Q^2 < 5 \text{ GeV}^2$ with the aim of reducing the error bars.

2. General Remarks

The quark distribution function (related to F_1) can be written as

$$q^{(m)}(x) = \langle D, J = 1, m | O | D, J = 1, m \rangle. \quad (5)$$

If $q^{(m)}(x)$ is to have any dependence on m , the operator O can not be scalar. The Wigner Eckart theorem then dictates the O is a tensor of rank 1 or 2, if b_1 is to be different from zero. But invariance under parity says that $q^{(1)} = q^{(-1)}$. Thus O can not be of rank 1 and give a non-zero b_1 .

This means that O is a rank 2 tensor, so that b_1 measures tensor effects. A consequence is that the S wave component of the deuteron gives no contribution to b_1 because the value of m is not relevant. In general, the nucleon contributions do not give anything large enough to be observable [3, 4].

3. Pionic Contribution

This contribution is detailed in [1], which displays the relevant figures. We summarize here. The pionic contribution to b_1 is given by

$$b_1^\pi(x) = \frac{1}{2} \int_x^2 \frac{dy}{y} q^\pi(x/y) \delta f_\pi(y), \quad (6)$$

where

$$\delta f_\pi(y) \equiv f_\pi^{(0)}(y) - f_\pi^{(1)}(y) \quad (7)$$

with

$$f_\pi^{(m)}(y_A) = \int \frac{d\xi^-}{2\pi} e^{-iy_A P_D^+ \xi^-} \langle D, m | \phi_\pi(\xi^-) \phi_\pi(0) | D, m \rangle_c \quad (8)$$

gives the probability for the pion to have a light cone momentum fraction of y . The function $q^\pi(x/y)$ is the quark distribution function of the pion.

The function $\delta f_\pi(y)$ has some interesting properties. It is independent of the deuteron wave function and has a double node structure which is a consequence of the tensor nature of b_1 . Indeed, we find

$$\int_0^2 dy \frac{f_{abL}(y)}{y} = 0. \quad (9)$$

The pionic effects on b_1 are found to be substantial for x less than about 0.2, and for those values of x are large enough to account for the HERMES data. See Table I and Fig. 3 of Ref. [1].

Table 1. Measured values (in 10^{-2} units) of the tensor structure function b_1 . Both the statistical and systematic uncertainties are listed. The numbers in parenthesis refer to the structure function modes of Ref. [6].

$\langle x \rangle$	$\langle Q^2 \rangle$ [GeV ²]	b_1 [10 ⁻²]	$\pm \delta b_1^{\text{stat}}$ [10 ⁻²]	$\pm \delta b_1^{\text{sys}}$ [10 ⁻²]	b_1^π [7] [10 ⁻²]	b_1^π [6] (1) [10 ⁻²]	b_1^π [6] (3) [10 ⁻²]	b_1^{6q} [10 ⁻²]
0.012	0.51	11.20	5.51	2.77	10.5	15.5	24.1	0.00
0.032	1.06	5.50	2.53	1.84	5.6	6.8	8.9	0.00
0.063	1.65	3.82	1.11	0.60	4.2	3.7	4.1	0.00
0.128	2.33	0.29	0.53	0.44	1.6	1.3	1.3	0.01
0.248	3.11	0.29	0.28	0.24	-0.55	.13	0.12	0.41
0.452	4.69	-0.38	0.16	0.03	-0.02	-0.02	-0.022	-0.38

4. Hidden color, 6 quark states

The HERMES experimental result [2] presents an interesting puzzle because it observed a significant negative value of b_1 for $x = 0.45$. At such a value of x , any sea quark effect such as arising from double-scattering or virtual pions is completely negligible. Furthermore, the nucleonic contributions are computed to be very small [3, 4], so one must consider other possibilities. We therefore take up the possibility that the deuteron has a six-quark component that is orthogonal to two nucleons. Such configurations are known to be dominated by the effects of so-called hidden-color states in which two color-octet baryons combine to form a color singlet [8]. Such configurations can be generated, for example, if two nucleons exchange a single gluon leading to a quantum fluctuation involving a color octet and color anti-octet baryon.

In particular, a component of the deuteron in which all 6 quarks are in the same spatial wave function ($|6q\rangle$) can be expressed in terms on nucleon-nucleon NN , delta-delta $\Delta\Delta$ and hidden color components CC as [8]:

$$|6q\rangle = \sqrt{1/9}|N^2\rangle + \sqrt{4/45}|\Delta^2\rangle + \sqrt{4/5}|CC\rangle. \quad (10)$$

This particular state has an 80% probability of hidden color and only an 11% probability to be a nucleon-nucleon configuration. The 80 % cited here is a purely algebraic number that applies only for completely overlapping nucleons. The real question is the probability that the deuteron consists of 6 quarks are in the same spatial wave function, which is denoted here as P_{6q} . A recent review of hidden color phenomena is presented in [9]. In the following, the term $|6q\rangle$ is referred to interchangeably as either a six-quark or hidden color state.

The discovery of the EMC effect [10] caused researchers to consider the effects of such six-quark states [11] in a variety of nuclear phenomena [12, 13, 14]. Furthermore, the possible discovery of such a state as a di-baryon resonance has drawn recent interest [15]. Therefore we propose a model of a hidden-color six-quark components of the s and d -states of the deuteron. We also note that including a six-quark hidden color component of the deuteron does not lead to a conflict with the measured asymptotic d to s ratio of the deuteron [16]. The EMC effect remains the only nuclear effect that has not been explained using conventional (non-quark) dynamics [18, 17, 19].

Our calculation is exploratory, so we use the simplest model possible. We postulate that the S state of the deuteron has a component with 6 quarks in an s state with total angular momentum 1 and isospin 0. Then the D state has a 6-quark component with any one quark in a $d_{3/2}$ state. We define these states by combining 5 s -state quarks into a spin 1/2 component, which couples with either the $s_{1/2}$ or $d_{3/2}$ single-quark state to make a total angular momentum

of 1. We therefore write the wave functions of these states for a deuteron of $J_z = H$ as

$$\psi_{j,l,H}(\mathbf{p}) = \sqrt{N_l} f_l(p) \sum_{m_s, m_j} \mathcal{Y}_{jlm_j} \langle jm_j, \frac{1}{2} m_s | 1H \rangle, \quad (11)$$

where $l, j = s_{1/2}$ or $d_{3/2}$, N_l is a normalization constant chosen so that $\int d^3p \bar{\psi}_{j,l,H}(\mathbf{p}) \gamma^+ \psi_{j,l,H}(\mathbf{p}) = 1$, and \mathcal{Y}_{jlm_j} is a spinor spherical harmonic. The matrix element for transition between the $l = 0$ and $l = 2$ states is given by the light-cone distribution

$$F_H(x_{6q}) = \frac{1}{2} \int d^3p \bar{\psi}_{1/2,0,H}(\mathbf{p}) \gamma^+ \psi_{3/2,2,H}(\mathbf{p}) \delta\left(\frac{p \cos \theta + E(p)}{M_{6q}} - x_{6q}\right), \quad (12)$$

where $E(p) = \sqrt{p^2 + m^2}$ with m as the quark mass, and M_{6q} is the mass of the six-quark bag, x_{6q} is the momentum fraction of the six-quark bag carried by a single quark and $x_{6q} M_{6q} = xM$ [11]. Note that $p \cos \theta$ is the third (z) component of the momentum, so that the plus component of the quark momentum is $E(p) + p \cos \theta$. We take $M_{6q} = 2M$ (its lowest possible value) to make a conservative estimate.

The term of interest $b_1(x)$ is given by

$$b_1^{6q}(x) = \frac{1}{2} (2) (F_0(x) - F_1(x)) P_{6q}, \quad (13)$$

where P_{6q} is the product of the probability amplitudes for the 6-quark states to exist in the deuteron, and the factor of 2 enters because either state can be in the d -wave. Evaluation of F_H using Eq. (11) leads to the result:

$$b_1^{6q}(x) = -\sqrt{\frac{N_0 N_2}{2}} \frac{3}{4\pi} \int d^3p f_0 f_2 (3 \cos^2 \theta - 1) \delta\left(\frac{p \cos \theta + E(p)}{M} - x\right) P_{6q}. \quad (14)$$

To proceed further we, specify the wave functions to be harmonic oscillator wave functions. We take $f_2(p) = -p^2 R^2 e^{-p^2 R^2/2}$, $f_0(p) = e^{-p^2 R^2/2}$, where R is the radius parameter. This model is specified by only three parameters: R , the quark mass m , and P_{6q} . The key question is whether such a model can reproduce the HERMES data point at $x = 0.452$ without using a value of P_{6q} large enough to conflict with conventional nuclear physics calculations that do not require a non-zero value. In other words, we ask if the hidden color states provide a substantial mechanism to make b_1 non-zero at large values of x .

We adjust the value of P_{6q} to reproduce the data at that point and see how large a value is needed. Here we use a quark mass of 338 MeV [20]. We expect that the 6-quark state should be somewhat larger than that of a nucleon, and therefore choose R to be 1.2 fm. The results of the calculations are not very sensitive to the exact value of R [1].

The main result is that we can reproduce the value of the HERMES high- x point with value of $P_{6q} = 0.15\%$. This is shown in Table I and Figs. 4 and 5 of Ref. [1]. Combining the effects of pions and hidden color states leads to a reasonable reproduction of the HERMES data for its entire range of x . Furthermore, no mechanism other than hidden color is known to contribute to b_1 at large values of x . Given this, reasonable predictions for the JLab experiment are presented in Ref. [1].

5. Sum rule of Close & Kumano [21]

This sum rule states that $\int dx b_1(x) = 0$, and is derived by assuming that b_1 is carried entirely by valence quarks. This is analogous to the Gottfried sum rule for the integral of $F_{2p} - F_{2n}$, which assumed that $\bar{u}(x) = \bar{d}(x)$. Various effects of the sea violate the sum rule, and the violations may be more interesting than the sum rule.

Ref. [1] shows that nucleonic, pion and shadowing effects all violate the sum rule, and for nucleonic and pionic contributions by an infinite amount. However, integration of Eq. (14) does yield a zero due to the tensor nature of the integrand. This means that b_1^{6q} must have regions of x for which it is positive and regions of x for which it is negative.

6. Summary

We find that pionic effects are sizable for values of $x < 0.2$ and reproduces the HERMES data in that region. Furthermore, we find that 6-quark hidden color effects can enter at larger values of x , and that the combination of pionic and hidden color effects reproduces HERMES data. Predictions are made for future JLab data. The Close & Kumano sum rule does not hold for all mechanisms other than 6-quark effects. If the sum rule holds, b_1 must take on both positive and negative values. Observing such would provide evidence for 6-quark hidden-color components of the deuteron.

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