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Shock wave propagation of circular dam break problems

Sudi Mungkasi

Department of Mathematics, Faculty of Science and Technology, Sanata Dharma University, Mrican, Tromol Pos 29, Yogyakarta 55002, Indonesia

E-mail: `sudi@usd.ac.id`

Abstract. We examine the behavior of shock wave propagation of circular (radial) dam break problems. A dam break problem represents a reservoir having two sides of water at rest initially with different depth separated by a wall, then water flows after the wall is removed. The behavior of shock wave propagation is investigated with respect to water levels and with respect to the speeds of the shock waves. To the author's knowledge, such investigation for circular dam break problems had never been done before. Therefore, this new work shall be important for applied computational mathematics and physics communities as well as fluid dynamic researchers. Based on our research results, the propagation speed of shock wave in a circular dam break is lower than that of shock wave in a planar dam break having the same initial water levels as in the circular dam break.

1. Introduction

Water can flow in either a closed or open space. An example of water motion in a closed channel is pipe flows. An example of water motion in open space is flood flows. Studies of water flows are important, as they occur in many situations or conditions (see References [1]-[6]).

This paper considers water flows in an open channel. We solve a circular dam break problem. It is also known as a radial dam break problem. A circular dam represents a water reservoir having a wall with circle in shape, where the depth of water inside the circle wall is greater than that of water outside. Then the circular dam break problem means that we need to find the properties (water surface, momentum, velocity, energy, etc.) of water after the circular dam wall is removed completely at an instant of time. We assume that initially the area outside of the circular wall has a positive constant depth. Therefore when dam break happens, a shock wave appears and propagates radially [3].

The circular dam break problem can be modelled by the the one-dimensional shallow water equations with varying width as well as the standard two-dimensional shallow water equations. A simulation of the problem through the one-dimensional shallow water equations with varying width was conducted by Roberts and Wilson [5]. Some simulation results of the problem through the standard two-dimensional shallow water equations was presented by Mungkasi [4]. Shallow water flows was modelled mathematically for initial stages by Saint-Venant [2].

Our goal is to research on the shock wave propagation of the circular dam break problem. We implement the one-dimensional shallow water equations with varying width following Roberts and Wilson [5]. An advantage of using these equations is that the numerical method (used to solve these equations) is simpler than the two dimensional version. It is because the one-dimensional shallow water equations with varying width have the same form as the standard one-dimensional shallow water equations.

The rest of this paper is organized as follows. Governing equations are recalled in Section 2. The numerical method of Roberts and Wilson [5] is briefly presented in Section 3. Then Section 4 presents and discusses numerical results on the shock wave propagation. Finally we conclude our presentation with some remarks in Section 5.

2. Governing equations

The one-dimensional shallow water equations with varying width are [5]

$$\frac{\partial}{\partial t}(bh) + \frac{\partial}{\partial x}(bhu) = 0\,, \tag{1}$$

$$\frac{\partial}{\partial t}(bhu) + \frac{\partial}{\partial x}(bhu^2 + \frac{1}{2}gh^2 b) = -gh\frac{dz}{dx}b + \frac{1}{2}gh^2\frac{db}{dx}\,. \tag{2}$$

Here $h(x,t)$ is water depth, $u(x,t)$ is horizontal velocity, $z(x)$ is the topography, $b(x)$ is the channel varying width and g is the accelleration due to gravity. The free variables are time t and space x. The absolute water level is called stage and defined extensively as $w = h + z$.

Some notes are as follows. When the width $b(x)$ is constant, then the shallow water equations (1) and (2) are simplified to the standard one-dimensional shallow water equations. When we have horizontal topography, the source term $-gh\frac{dz}{dx}b$ disappears as $\frac{dz}{dx} = 0$.

3. Numerical method

The finite volume method of Roberts and Wilson [5] is used to solve the shallow water equations (1) and (2). The method is briefly described as follows.

Consider equations (1) and (2). These equations are conservation laws of the form

$$\frac{\partial}{\partial t}\mathbf{q} + \frac{\partial}{\partial x}\mathbf{f}(\mathbf{q}) = \mathbf{s}\,. \tag{3}$$

Here $\mathbf{q}$ is the vector of conserved quantities, $\mathbf{f}(\mathbf{q})$ is the vector of fluxes and $\mathbf{s}$ is the vector of sources.

Assume that we are given a space domain. A discretization of the space domain leads equation (3) to the semi-discrete finite volume scheme

$$\frac{d}{dt}\mathbf{q}_i + \frac{1}{\Delta x_i}\left(\mathbf{f}_{i+\frac{1}{2}} - \mathbf{f}_{i-\frac{1}{2}}\right) = \mathbf{s}_i\,. \tag{4}$$

The accuracy of the finite volume method is then dependent on the accuracy of the numerical fluxes $\mathbf{f}_{i+\frac{1}{2}}$ and $\mathbf{f}_{i-\frac{1}{2}}$ as well as on the accuracy of the solver of the ordinary differential equation (4). For our simulations we use second order finite volume method. It is second order accurate in space and second order accurate in time. We implement the minmod limiter to overcome artificial oscillation of numerical solutions.

For more details on this finite volume method for solving the shallow water equations (1) and (2), we refer to Roberts and Wilson [5].

4. Numerical results

To achieve the goal of this paper we consider a circular dam break problem. All quantities are given in SI units, so we omit the writing of units as they are already clear.

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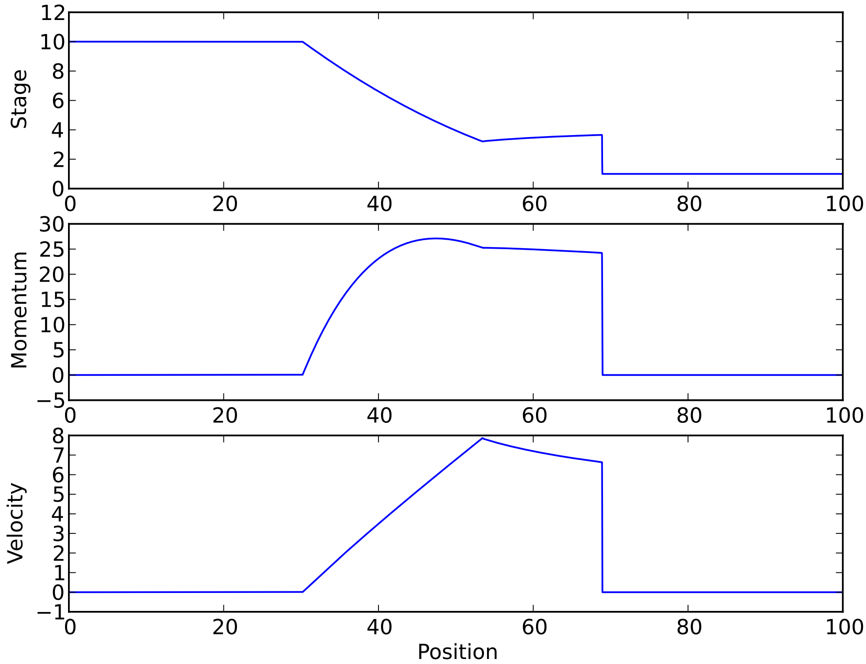

Figure 1. Simulation results for time $t = 2$.

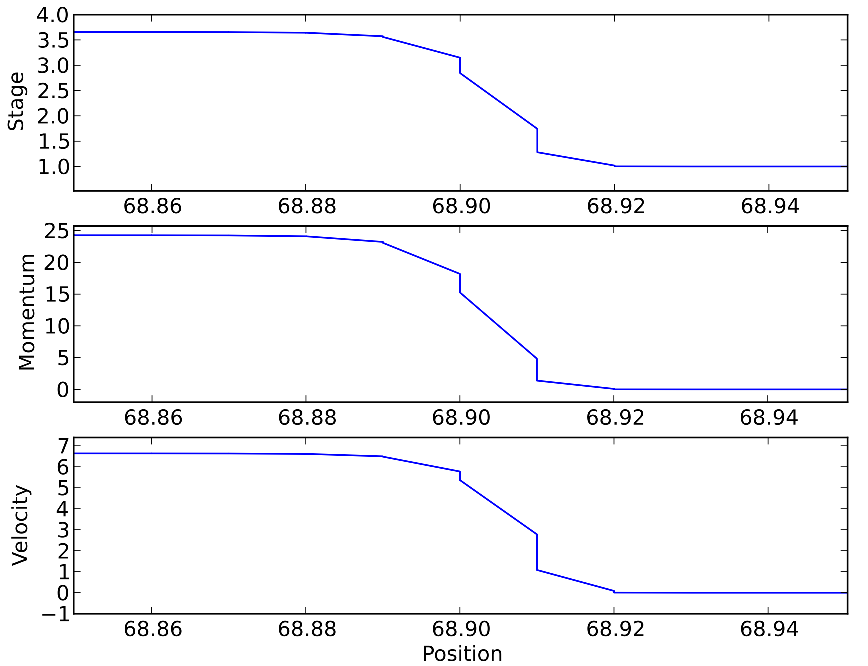

Figure 2. A magnification around shock position for time $t = 2$.

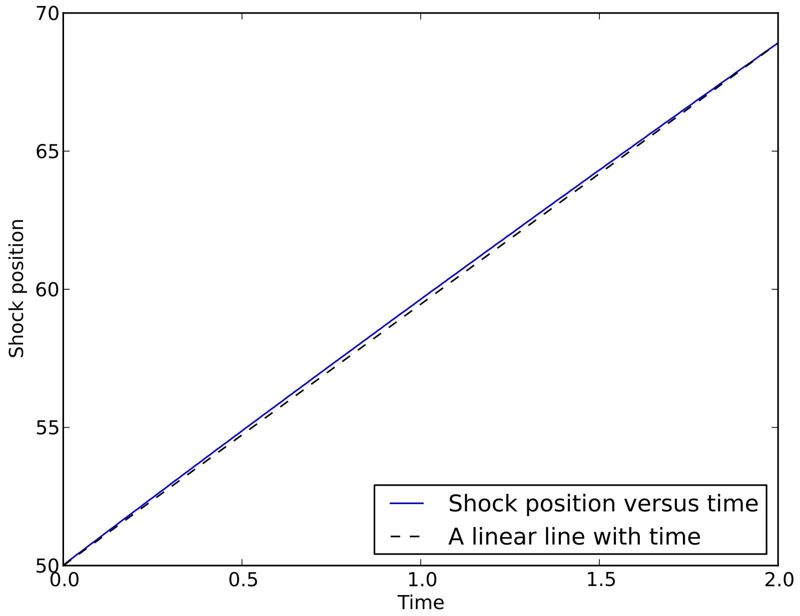

Figure 3. A track of the shock front with respect to time for $t \in [0, 2]$. The solid line shows the shock position versus time. The dashed line shows a linear line connecting the initial point $(0, 50)$ and the final point $(2, 68.915)$ of the shock positions.

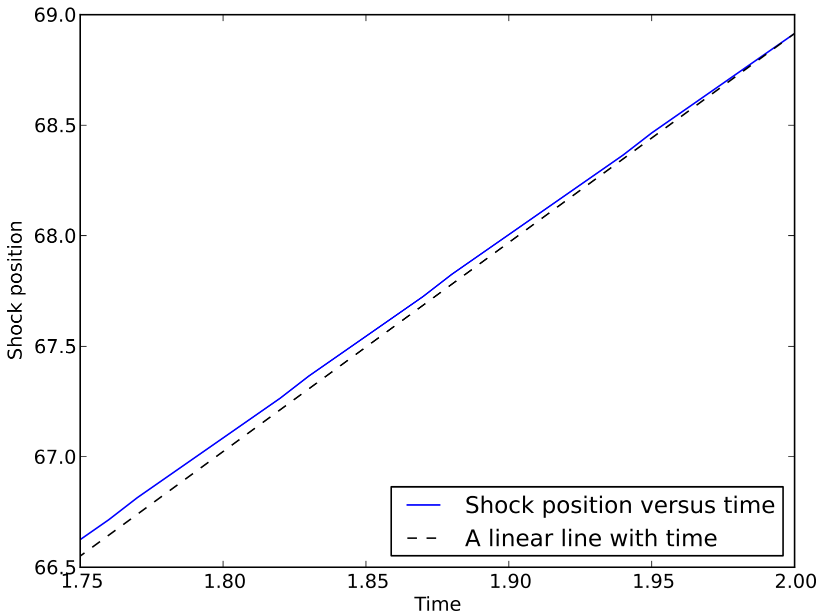

Figure 4. A track of the shock front with respect to time for $t \in [1.75, 2]$. The solid line shows the shock position versus time. The dashed line shows a linear line connecting the initial point $(0, 50)$ and the final point $(2, 68.915)$ of the shock positions.

Table 1. Track of the shock front with respect to time for $t \in [0,1]$.

time	position	time	position	time	position	time	position
0.01	50.115	0.26	52.555	0.51	54.975	0.76	57.365
0.02	50.215	0.27	52.655	0.52	55.065	0.77	57.455
0.03	50.315	0.28	52.745	0.53	55.165	0.78	57.555
0.04	50.415	0.29	52.845	0.54	55.255	0.79	57.645
0.05	50.505	0.3	52.945	0.55	55.355	0.8	57.745
0.06	50.605	0.31	53.045	0.56	55.455	0.81	57.835
0.07	50.705	0.32	53.135	0.57	55.545	0.82	57.935
0.08	50.805	0.33	53.235	0.58	55.645	0.83	58.025
0.09	50.895	0.34	53.335	0.59	55.735	0.84	58.125
0.1	50.995	0.35	53.425	0.6	55.835	0.85	58.215
0.11	51.095	0.36	53.525	0.61	55.925	0.86	58.315
0.12	51.195	0.37	53.625	0.62	56.025	0.87	58.405
0.13	51.295	0.38	53.715	0.63	56.125	0.88	58.505
0.14	51.385	0.39	53.815	0.64	56.215	0.89	58.595
0.15	51.485	0.4	53.915	0.65	56.315	0.9	58.695
0.16	51.585	0.41	54.005	0.66	56.405	0.91	58.785
0.17	51.685	0.42	54.105	0.67	56.505	0.92	58.885
0.18	51.775	0.43	54.205	0.68	56.595	0.93	58.975
0.19	51.875	0.44	54.295	0.69	56.695	0.94	59.075
0.2	51.975	0.45	54.395	0.7	56.795	0.95	59.165
0.21	52.065	0.46	54.485	0.71	56.885	0.96	59.255
0.22	52.165	0.47	54.585	0.72	56.985	0.97	59.355
0.23	52.265	0.48	54.685	0.73	57.075	0.98	59.445
0.24	52.365	0.49	54.775	0.74	57.175	0.99	59.545
0.25	52.455	0.5	54.875	0.75	57.265	1	59.635

Similar to Birman and Falcovitz [1], we consider the shallow water equations (1) and (2) with varying width $b(x) = 2\pi x$. The length of the channel is 100 . Stage is 10 for $x \in [0, 50]$. However stage is 1 for $x \in [50, 100]$. (Roberts and Wilson [5] set stage to be 2 for $x \in [50, 100]$.) This problem mimics the two-dimensional circular dam break problem. We can solve this problem using equations (1) and (2), as we exploit the symmetry of the water motion after dam break. The acceleration due to gravity is set to 9.81 . The space domain $[0, 100]$ is discretized into 1000 cells uniformly.

Figure 1 shows the simulation results at time $t = 2$. The first subfigure is the stage. The second and third subfigures are momentum and velocity respectively. We can see that there is no constant velocity across the moving water. To see the shock front clearer, we magnify Figure 1 around the shock position. The magnification is shown in Figure 2.

Moreover, the track of the shock front is summarized in Table 1 for time $t \in [0, 1]$ and Table 2 for time $t \in [1, 2]$. To see the relationship between the shock track with respect to time, we plot the data of Tables 1 and 2 into Figure 3. A magnification of Figure 3 for time $t \in [1.75, 2]$ is given in Figure 4. From Figures 3 and 4 we conclude that the relationship between the shock wave propagation with respect to time is nonlinear. This means that the shock speed of the circular dam break problem is not constant.

This is different phenomena from a corresponding planar dam break problem. Recall that for the planar dam break problem, the relationship between the shock wave propagation with respect to time is linear. That is due to the constant shock wave propagation.

We note that after 2 seconds of the circular dam break, the shock front travels about 18.915 m.

Table 2. Track of the shock front with respect to time for $t \in [1, 2]$.

time	position	time	position	time	position	time	position
1.01	59.735	1.26	62.075	1.51	64.405	1.76	66.715
1.02	59.825	1.27	62.175	1.52	64.505	1.77	66.815
1.03	59.925	1.28	62.265	1.53	64.595	1.78	66.905
1.04	60.015	1.29	62.365	1.54	64.685	1.79	66.995
1.05	60.105	1.3	62.455	1.55	64.775	1.8	67.085
1.06	60.205	1.31	62.545	1.56	64.875	1.81	67.175
1.07	60.295	1.32	62.645	1.57	64.965	1.82	67.265
1.08	60.395	1.33	62.735	1.58	65.055	1.83	67.365
1.09	60.485	1.34	62.825	1.59	65.145	1.84	67.455
1.1	60.575	1.35	62.925	1.6	65.245	1.85	67.545
1.11	60.675	1.36	63.015	1.61	65.335	1.86	67.635
1.12	60.765	1.37	63.105	1.62	65.425	1.87	67.725
1.13	60.865	1.38	63.205	1.63	65.515	1.88	67.825
1.14	60.955	1.39	63.295	1.64	65.615	1.89	67.915
1.15	61.045	1.4	63.385	1.65	65.705	1.9	68.005
1.16	61.145	1.41	63.475	1.66	65.795	1.91	68.095
1.17	61.235	1.42	63.575	1.67	65.885	1.92	68.185
1.18	61.335	1.43	63.665	1.68	65.985	1.93	68.275
1.19	61.425	1.44	63.755	1.69	66.075	1.94	68.365
1.2	61.515	1.45	63.855	1.7	66.165	1.95	68.465
1.21	61.615	1.46	63.945	1.71	66.255	1.96	68.555
1.22	61.705	1.47	64.035	1.72	66.345	1.97	68.645
1.23	61.795	1.48	64.135	1.73	66.445	1.98	68.735
1.24	61.895	1.49	64.225	1.74	66.535	1.99	68.825
1.25	61.985	1.5	64.315	1.75	66.625	2	68.915

However if we set a corresponding planar dam break problem (having the same initial water levels as in the circular dam break), based on the analytical solution of Stoker [6], the shock speed of the corresponding planar dam break is 9.8193. Therefore after 2 seconds of the planar dam break, the shock travels a distance of 19.6386. That is, the propagation speed of shock wave in a circular dam break is lower than that of shock wave in a planar dam break having the same initial water levels as in the circular dam break. Note that a corresponding planar dam break for our simulation means that we have a space $x \in [-100, 100]$, where the initial still water depth is 10 for $x \in [-100, 0]$ and 1 for $x \in [0, 100]$.

5. Conclusions

We have presented research results on the shock wave propagation of a circular dam break problem. We find that the relationship between the shock front of the circular dam break problem with respect to time is nonlinear. This nonlinear phenomenon is due to the nonconstant shock speed of the circular dam break problem. In addition, we find that a corresponding planar dam break problem having the same initial water levels as in the circular dam break produces a faster shock propagation.

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