

Analysis of Energy and Wave Function For Manning-Rosen Plus Scarf Potential D-Dimension With Nikiforov Uvarov Method

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Abstract. Approximate analysis of D-dimensional Schrödinger equation for Manning Rosen Plus Scarf potential investigated by Nikiforov-Uvarov method. The approximate energy of Manning Rosen plus Scarf is expressed of closed form. The approximate wave function is expressed of general Jacobi polynomial.

1. Introduction

Quantum mechanics always use a different approach to determine magnitude relevant with the motion of the particles is to use the wave function for the dynamics of particles moving represent to obtained from solution of the Schrödinger equation relating particle. In quantum mechanics, exactly solutions of the wave equation with certain physical potentials were of great interest. Schrodinger equation is a partial differential equation that describes the quantum state of a physical system. Solution of the Schrödinger equation is exact on some potentially be applied in a physical system that is very important because it can be used to determine the wave functions and energy levels of a system of particles.

Several methods can be used to solve the equations Schrodinger, among other SUSY [3], WKB approach [4], Romanovsky [5], and Nikiforov-Uvarov [6]. Describe the potential in the quantum dynamics of particles in quantum mechanics. Some examples of potential in quantum mechanics, among others, potential Coloumb, Rosen Morse, Manning-Rosen, Scarf, Poschl Teller Eckart and others.

In this work used the NU method to obtain the energy and the wave function of the non-central Manning Rosen potential plus Scarf. NU method has been used to complete the analysis of some of potential, such of Scarf Hiperbolik potential, Deng Fan deform plus Eckart deform potential, Klatzer deform plus Ring Shaped deform potential.



2. Schrodinger equation in D-Dimension

Schrodinger equations in D dimension based on the use of D-dimensional polar coordinate with polar variable r (hyper radius) and the angular momentum variables $\theta_1, \theta_2, \theta_3, \dots, \theta_{D-1}, \phi$ (hyper angle). For D-dimensional space, the Schrodinger equation can be write :

$$-\frac{\hbar^2}{2m} \nabla_D^2 \Psi(r, \Omega_D) + V(r, \Omega_D) \Psi(r, \Omega_D) = E \Psi(r, \Omega_D) \quad (1)$$

$$\text{Where the Laplacian operator is } \nabla_D^2 = \frac{1}{r^{D-1}} \frac{\partial}{\partial r} \left(r^{D-1} \frac{\partial}{\partial r} \right) - \frac{\Lambda_D^2(\Omega_D)}{r^2} \quad (2)$$

The second term on the right-hand side of Eq. (2) is the multidimensional space centrifugal term, and Ω_D represents the angular coordinates. In this case, the operator Λ_D^2 yields a hyperspherical harmonic as its eigenfunction, mathematically

$$\Lambda_D^2(\Omega_D) = L_k^2 = \sum_{a < b=2}^{k-1} L_{ab}^2 = -\frac{1}{\sin^{k-1} \theta_k} \left(\sin^{k-1} \theta_k \frac{\partial}{\partial \theta_k} \right) + \frac{L_{D-1}^2}{\sin^2 \theta_k} \quad (3)$$

The eq.(2) helps us to write the wave function as

$$\Psi_{nlm}(r, \Omega_D) = R_{n,l}(r) Y_l^m(\Omega_D) \quad (4)$$

where $R_{n,l}(r)$ is radial part of equation and $Y_l^m(\Omega_D)$ is the angular part called hyperspherical harmonics. The $Y_l^m(\Omega_D)$ obeys the eigenvalue equation

$$\Lambda_D^2 Y_l^m(\Omega_D) = l(l + D - 2) Y_l^m(\Omega_D) \quad (5)$$

3. Nikiforov-Uvarov (NU) Method

Nikiforov Uvarov method is based on the solution of a second order differential equation by using an intermediary hypergeometry equation. The Schrodinger equation $\Psi''(r) + [E - V(r)]\Psi(r) = 0$ (6) can be solved by transforming it into a hypergeometric-type equation through using the transformation, $s = s(x)$ and its resulting equation is expressed as

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)} \psi'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)} \psi(s) = 0 \quad (7)$$

where $\sigma(s)$ and $\bar{\sigma}(s)$ are second order polynomial, $\bar{\tau}(s)$ is first order polynomial, and $\psi(s)$ is a function of hypergeometry. To solve eq.(7), we set the wave function as

$$\Psi(s) = \varphi(s) \chi_n(s) \quad (8)$$

Substituting eq.(8) into eq.(7) reduce eq.(7) into a hypergeometric type equation

$$\sigma(s) \chi_n''(s) + \tau(s) \chi_n'(s) + \lambda \chi_n(s) = 0 \quad (9)$$

where the wave function $\varphi(s)$ is defined as a logarithmic derivative. The first part of the wave function can be describe as

$$\frac{\phi'}{\phi} = \frac{\pi}{\sigma} \quad (10)$$

and the other wave function $\chi_n(s)$ is the hypergeometric-type function whose polynomials are given by the Rodrigues relation

$$\chi_n(s) = \frac{B_n}{\rho(s)} \frac{d^n}{ds^n} [\sigma^n(s) \rho(s)] \quad (11)$$

where B_n is the normalization constant and weight function $\rho(s)$ satisfies the Pearson equation is expressed as:

$$\frac{d}{ds} (\sigma(s) \rho(s)) = \tau(s) \rho(s) \quad (12)$$

The function $\pi(s)$ and the parameter λ required for NU method are defined as follows:

$$\pi = \left(\frac{\sigma' - \bar{\tau}}{2} \right) \pm \sqrt{\left(\frac{\sigma' - \bar{\tau}}{2} \right)^2 - \bar{\sigma} + k\sigma} \quad (13)$$

$$\lambda = k + \pi' \quad (14)$$

It is necessary that the term under the square root sign in Eq. (13) is the square of a polynomial. To calculate k from Eq. (13), the discriminant of the quadratic term must vanish. The eigenvalues in Eq. (14) take the form

$$\lambda = \lambda_n = -n\tau' - \frac{n(n-1)}{2} \sigma'', \quad n = 0, 1, 2, \dots \quad (15)$$

Where

$$\tau(s) = \bar{\tau}(s) + 2\pi(s) \quad (16)$$

and its derivative is less than zero, which is the necessary condition for bound state solutions. The energy eigenvalues are obtained by comparing Eq. (14) with Eq. (15).

4. Analysis Manning Rosen Plus Scarf Potential D dimension with Nikiforov-Uvarov Method

The non-central potential which is combination of Manning Rosen potential and Scarf non-central potential given as

$$V(r, \Omega_D) = \frac{\hbar^2}{2m} \left(\frac{v(v-1)}{\sinh^2 r} - 2q \coth r \right) + \frac{\hbar^2}{2mr^2} \left(\frac{b^2 + a(a-1)}{\sin^2 \Omega_D} - \frac{2b(a - \frac{1}{2}) \cos \Omega_D}{\sin^2 \Omega_D} \right) \quad (17)$$

where $b^2 + a(a-1) > 0$ and $2b(a - \frac{1}{2}) > 0$.

Substitution of Manning Rosen plus Scarf potential in Schrodinger equation D dimension, then separation of variables to obtained

$$\frac{1}{R(r)} r^{D-1} \frac{\partial}{\partial r} \left(r^{D-1} \frac{\partial R(r)}{\partial r} \right) r^2 - \left(\frac{v(v-1)}{\sinh^2 r} - 2q \coth r + \varepsilon^2 \right) r^2 = l(l + D - 2) \quad (18)$$

$$\frac{1}{Y(\Omega_D)} \Delta_D^2(\Omega_D) Y(\Omega_D) + \left(\frac{b^2 + a(a-1)}{\sin^2 \Omega_D} - \frac{2b(a - \frac{1}{2}) \cos \Omega_D}{\sin^2 \Omega_D} \right) = l(l + D - 2) \quad (19)$$

Eq (18) and eq(19) are radial part and polar part respectively.

Solution of the radial part in equation (18), performed by intermediaries hypergeometry equation can write as $\frac{d^2 \chi(r)}{dr^2} - \left[\frac{(l + \frac{D-1}{2})(l + \frac{D-3}{2})}{r^2} \right] \chi(r) - \left(\frac{v(v-1)}{\sinh^2 r} - 2q \coth r + \varepsilon^2 \right) \chi(r) = 0$ (20)

If $\coth r = s$ and $\frac{1}{r^2} = \left(d_0 + \frac{1}{\sinh^2 r} \right)$ the equation (20) can rewritten as

$$\frac{d^2 \chi(s)}{ds^2} - \frac{2s}{(1-s^2)} \frac{d\chi(s)}{ds} - \left[\frac{(l + \frac{D-1}{2})(l + \frac{D-3}{2}) d_0}{(1-s^2)^2} - \frac{(l + \frac{D-1}{2})(l + \frac{D-3}{2})(1-s^2)}{(1-s^2)^2} - \frac{v(v-1)(1-s^2)}{(1-s^2)^2} - \frac{2qs}{(1-s^2)^2} + \frac{\varepsilon^2}{(1-s^2)^2} \right] \chi(s) = 0 \quad (21)$$

Equation (21) is differential equations that can be solved by Nikiforov-Uvarov method. By substituting each of parameter NU, we obtained:

$$\pi = \pm \sqrt{\left(\left(l + \frac{D-1}{2} \right) \left(l + \frac{D-3}{2} \right) d_0 + v(v-1) - k \right) s^2 - 2qs + \left(l + \frac{D-1}{2} \right) \left(l + \frac{D-3}{2} \right) d_0 + \varepsilon^2 - \left(\left(l + \frac{D-1}{2} \right) \left(l + \frac{D-3}{2} \right) d_0 + v(v-1) - k \right)} \quad (22)$$

Energy eigen values and eigen functions are

$$\lambda_1 = \lambda_2 = \left(l + \frac{D-1}{2}\right) \left(l + \frac{D-3}{2}\right) d_0 + v(v-1) - p^2 - p \quad (23)$$

$$\text{and new eigen value is } \lambda_n = 2n(p+1) + n(n+1) = 2np + n^2 + n \quad (24)$$

$$\text{where } p = \sqrt{\left(l + \frac{D-1}{2}\right) \left(l + \frac{D-3}{2}\right) d_0 + \left(v - \frac{1}{2}\right)^2} - \left(n + \frac{1}{2}\right) \quad (25)$$

From eq(19), separation variable azimuth and polar, we obtained

$$\Phi = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \quad (26)$$

$$\text{and } \frac{\partial^2 H}{\partial \theta_k^2} + (k-1) \cot \theta_k \frac{\partial H}{\partial \theta_k} - \left(\frac{A_{k-1}}{\sin^2 \theta_k} - A_k \right) H - \left(\frac{b^2 + a(a-1)}{\sin^2 \theta_k} - \frac{2b(a-\frac{1}{2}) \cos \theta_k}{\sin^2 \theta_k} \right) H = 0 \quad (27)$$

where eq.(26) and eq(27) are solution of azimuth part and polar part equation.

To solving eq.(27), we substituted if $\cos \theta_k = s$, then eq.(27) can rewritten as

$$\frac{d^2}{ds^2} H - \frac{ks}{(1-s^2)} \frac{d}{ds} H - \left\{ \frac{b^2 + a(a-1)}{(1-s^2)^2} - \frac{2b(a-\frac{1}{2})s}{(1-s^2)^2} + \frac{A_{k-1}}{(1-s^2)^2} - \frac{A_k(1-s^2)}{(1-s^2)^2} \right\} H = 0 \quad (28)$$

Equation (28) is differential equations that can be solved by Nikiforov-Uvarov method.

By substituting each of parameter NU, we obtained:

$$\pi = \left(\frac{(k-2)s}{2} \right) \pm \sqrt{\left(\frac{(k-2)^2}{4} + A_k - k^* \right) s^2 - 2b \left(a - \frac{1}{2} \right) s + b^2 + a(a-1) + A_{k-1} - A_k + k^* - \frac{(k-2)^2}{4} + \frac{(k-2)^2}{4}} \quad (29)$$

Energy eigen values and eigen functions are

$$\lambda_1 = \lambda_2 = \frac{(k-2)^2}{4} + A_k - p^2 + p - \frac{(k-2)}{2} \quad (30)$$

and new eigen value is

$$\lambda_n = 2np + 2n + n(n-1) = 2np + n^2 + n \quad (31)$$

where

$$p = \sqrt{\frac{\left(b^2 + a(a-1) + A_{k-1} + \frac{(k-2)^2}{4} \right) + \sqrt{\left(b^2 + a(a-1) + A_{k-1} + \frac{(k-2)^2}{4} \right)^2 - 4b^2 \left(a - \frac{1}{2} \right)^2}}{2}} \quad (32)$$

The energy of Manning Rosen plus Scarf potential D dimension is

$$E_n = -\frac{\hbar^2}{2m} \left[\frac{\left\{ \sqrt{\left(l + \frac{D-1}{2}\right) \left(l + \frac{D-3}{2}\right) d_0 + \left(v - \frac{1}{2}\right)^2} - \left(n + \frac{1}{2}\right) \right\}^2}{q^2} - \left(l + \frac{D-1}{2}\right) \left(l + \frac{D-3}{2}\right) d_0 \right] \quad (33)$$

In the special condition, where $(l = 0)$ and $(D = 3)$, we obtained

$$E_n = -\frac{\hbar^2}{2m} \left\{ (v-1-n)^2 + \frac{q^2}{(v-1-n)^2} \right\} \quad (34)$$

Angular momentum parameter of Manning Rosen plus Scarf potential D dimension in the general condition obtained by $\lambda_1 = \lambda_{n_l}$ and we obtained a relationship that

$$A_k = \left((n_l + p) + \frac{1}{2} \right)^2 - \left(\frac{(k-2)}{2} + \frac{1}{2} \right)^2 \quad (35)$$

The wave function expressed by generally Jacobi polynomial. The wave function of radial part is $R_n = B_n r^{-\left(\frac{D-1}{2}\right)} (1 - \coth^2 r)^{\frac{p}{2}} (1 + \coth r)^{\frac{q}{2p}} (1 - \coth r)^{\frac{q}{2p}} P_n^{\alpha, \beta}(\coth r)$ and the wave function of polar part is

$$H = B_n (1 + \cos \theta)^{\frac{b\left(a-\frac{1}{2}\right)}{2p} - \frac{(k-p-2)}{2}} (1 - \cos \theta)^{-\frac{b\left(a-\frac{1}{2}\right)}{2p} - \frac{(k-p-2)}{2}} P_n^{\alpha, \beta}(\cos \theta) \quad (37)$$

5. Conclusion

In this work we present the approximate solution of Schrodinger equation for Manning Rosen plus Scarf potential in D dimension with Nikiforov Uvarov method. The energy of Manning Rosen plus Scarf obtained by equating the eigenvalues. The wave function expressed by generally Jacobi polynomial.

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