Approximate Solution of Schrodinger Equation in D-Dimensions for Scarf Hyperbolic plus Non-Central Poschl-Teller Potential Using Nikiforov-Uvarov Method

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Abstract. The approximate analytical solution of Schrodinger equation in D-Dimensions for Scarf hyperbolic plus non-central Pocshl-Teller potential were investigated using Nikiforov-Uvarov method. The approximate bound state energy are given in the close form and the corresponding approximate wave function for arbitary l-state in D-dimensions are formulated in the form of generalized Jacobi Polynomials. Special case is given for the ground state in 3 dimensions. The existence of arbitrary dimensions increase bound state energy system. In the other hand, the existence of arbitrary dimensions decreases the amplitude of wave function. The effect of Scarf Hyperbolic potential increases the bound state energy of system. The effect of non central Poschl-Teller potential decreases the bound state energy of system.

1. Introduction

The analytical solutions of Schrodinger equations for some physical potentials are very essential since they provide the important information of the quantum system. Recently, considerable efforts have been paid to obtain the exact solution of the shape invariant potentials. These potentials include Coulomb, Morse, Poschl-Teller, Hulten, inverted generalized hyperbolic potentials. The bound state energy spectra of these potentials have been investigated by various techniques such as Coulomb potential using Laplace transformation [1], Morse potential using series expansion method [2], Modified Poschl-Teller, Hulten, and Scarf hyperbolic and Scarf hyperbolic potential using NU method [3-6]. However, some shape invariant potentials in D-dimensions resolved only in radial part [1-6]. The angular part of wave function which have some potentials still unresolved.

The exact solution of Schrodinger equation is obtained if the angular momentum $l = 0$ in 1-dimension. Nevertheless, for $l \neq 0$ and $D > 1$, the Schrodinger equation can only be solved approximately for

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different suitable approximation scheme. One of the suitable approximation scheme is conventionally proposed by Greene and Aldrich [7,8].

In this paper we will attempt to solve the Schrodinger equation in D-Dimensions using Nikivorof Uvarov method. The NU method which was developed by Nikiforov-Uvarov [9]. This method based on solving the second order linear differential equations by reducing it to a generalized equation of hypergeometric type by a suitable change of variable.

Recently, the radial part of Schrodinger equation in D-Dimensions have been solved with various method. The combination between radial plus angular potential still haven't been studied yet. One of the combination shape invariant potential in D-Dimensions that remain unresolved is Scarf hyperbolic plus non-central Poschl-Teller potential. The Scarf potential describe particles which periodically arranged such as a crystal [5,6,10,11]. The application of solution in this potential are crystal model in solid state physics [5,6]. The Poschl-Teller potential describe the particle in Poschl-Teller Oscilattor. Moreover, the modified Poschl-Teller potential can been used to derive the well-known SO(2) spectrum generating algebra foran infinite square well problem [12].

The solution Scarf hyperbolic and Poschl-Teller potential in D-Dimensions still remain in radial part [3,5,6]. In the combined potential in D-Dimensions with the centrifugal term, this potential is separable potential, therefore, it can be solved using variable separation method. The radial part of Schrodinger equation have one solution. Nevertheless, the angular part of Schrodinger equation have D-1 solutions.

This paper is organized as follows. In section 2, we review the Nikiforov-Uvarov (NU) method briefly. The Scarf Hyperbolic plus Non-Central Pocshl-Teller Potential and their solution described in the section 3. In the section 4, special case given for the ground and first excited state in 3 dimensions. A brief results and conclusion in section 5.

2. Review of Nikiforov-Uvarov Method

The D-dimensional Schrodinger equation of any shape invariant potential can be reduced into hypergeometric or confluent hypergeometric type differential equation by suitable variable transformation [13-16]. The hypergeometric type differential equation, which is solved using Nikiforov-Uvarov method, is presented as:

$$\frac{\partial^2\Psi(s)}{\partial s^2}+\frac{\bar{\tau}(s)}{\sigma(s)}\frac{\partial\Psi(s)}{\partial s}+\frac{\bar{\sigma}(s)}{\sigma^2}\Psi(s)=0 \tag{1}$$

where $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials at most in the second order, and $\bar{\tau}(s)$ is first order polynomial. Equation (1) can be solved using separation of variable method which is expressed as:

$$\Psi=\phi(s)y(s) \tag{2}$$

By inserting equation (2) into equation (1) we get hypergeometric type equation, that is:

$$\sigma\frac{\partial^2 y}{\partial s^2}+\tau\frac{\partial y}{\partial s}+\lambda y=0 \tag{3}$$

ϕ (s) is a logarithmic derivative whose solution obtained from condition:

$$\frac{\phi'}{\phi}=\frac{\pi}{\sigma} \tag{4}$$

while the function π (s) and the parameter λ are defined as:

$$\pi=\left(\frac{\sigma'-\bar{\tau}}{2}\right)\pm\sqrt{\left(\frac{\sigma'-\bar{\tau}}{2}\right)^2-\bar{\sigma}+k\sigma} \tag{5}$$

$$\lambda=k+\pi' \tag{6}$$

The value of k in equation (5) can be found from the condition that the expression under the square root of equation (5) must be square of polynomial which is mostly first degree polynomial and therefore the discriminate of the quadratic expression is zero. A new eigenvalue of equation (3) is:

$$\lambda = \lambda_n = -n\pi' - \frac{n(n-1)}{2}\sigma''; \; n = 0, 1, 2, \ldots \tag{7}$$

where

$$\tau = \overline{\tau} + 2\pi \tag{8}$$

The new bound state energy is obtained using equation (6) and (7). To generate the bound state energy and the corresponding eigenfunction, the condition that $\tau' < 0$ is required. The solution of the second part of the wave function, $y_n(s)$, which is connected to Rodrigues relation [17], is given as:

$$y_n(z) = \frac{C_n}{\rho(z)} \frac{d^n}{dz^z}\left\{\sigma^n(z)\rho(z)\right\} \tag{9}$$

where C_n is normalization constant, and the weight function $\rho(s)$ must satisfies the condition:

$$\frac{\partial(\sigma\rho)}{\partial s} = \tau(s)\rho(s) \tag{10}$$

The wave function of the system is therefore obtained from equation (4), (9) and (10).

3. The Scarf Hyperbolic plus Non-Central Pocshl-Teller Potential

The Scarf Hyperbolic potential can be expressed as:

$$V = \frac{\hbar^2}{2m}\left\{\frac{b^2 + a(a+1)}{\sinh^2 r} - \frac{2b\left(a + \frac{1}{2}\right)\cosh r}{\sinh^2 r}\right\} \tag{11}$$

The visualization of this potential ($a = 2$ and $b = 2$) in 1 Dimension is:

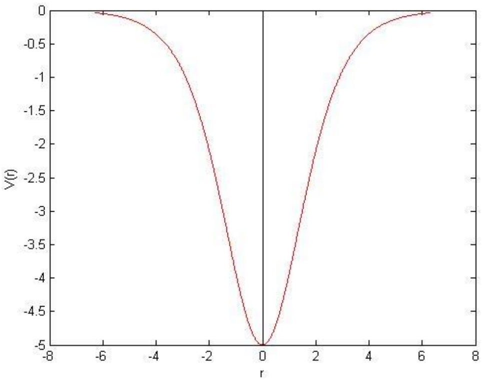

Figure 1. Visualization of Scarf Hyperbolic Potential for $a = 2$ and $b = 2$

The Scarf hyperbolic potential is symetric. This potential have a symetrical axis at $r = 0$. The Scarf potential describe particles which periodically arranged such as a crystal [11]. The application of solution in this potential are crystal model in solid state physics [5,6].

The Non Central Poschl-Teller potential can be expressed as:

$$V = \frac{\hbar^2}{2mr^2}\left\{\frac{\kappa(\kappa-1)}{\sin^2 \Omega_D} + \frac{\lambda(\lambda-1)}{\cos^2 \Omega_D}\right\} \tag{12}$$

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The visualization of this potential (κ= 2 and λ = 2) in 1 Dimension is:

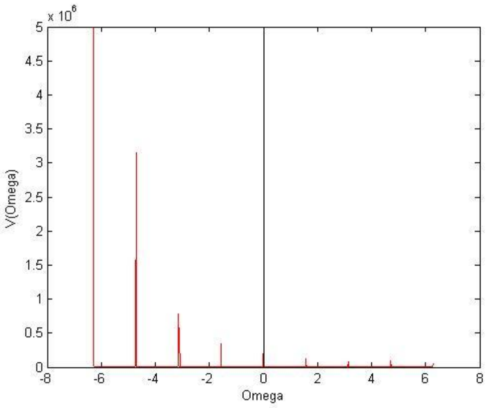


Figure 2. Visualization of Poschl-Teller Potential for κ= 2 and λ = 2

The Poschl-Teller potential like spikes which their peak decrease exponentially according to the Omega (Ω). This potential describe the particle in Poschl-Teller Oscillator. Moreover, the modified Poschl-Teller potential can been used to derive the well-known SO(2) spectrum generating algebra for an infinite square well problem [12].

The Scrodinger Equation in D-Dimensions for Scarf Hyperbolic plus Non-Central Pocshl-Teller Potential can be expressed as:

$$-\frac{\hbar^2}{2m}\nabla_D{}^2\Psi(r,\Omega)+\left[\begin{array}{l}\frac{\hbar^2}{2m}\left\{\frac{b^2+a(a+1)}{\sinh^2 r}-\frac{2b\left(a+\frac{1}{2}\right)\cosh r}{\sinh^2 r}\right\}\\ +\frac{\hbar^2}{2mr^2}\left\{\frac{\kappa(\kappa-1)}{\sin^2\Omega_D}+\frac{\lambda(\lambda-1)}{\cos^2\Omega_D}\right\}\end{array}\right]\Psi(r,\Omega)=E\Psi(r,\Omega) \tag{13}$$

where

$$\nabla_D{}^2=\frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right)-\frac{\Lambda_D{}^2\left(\Omega_D\right)}{r^2} \tag{14}$$

With $\Lambda_D{}^2\left(\Omega_D\right)$ is hypersperical harmonics, that is, D-Dimensional angular momentum operator. For $2 \le k \le D-1$, we have:

$$\Lambda_D{}^2\left(\Omega_D\right)=L_k{}^2=\sum_{a<b=2}^{k-1}L_{ab}{}^2=-\frac{1}{\sin^{k-1}\theta_k}\frac{\partial}{\partial\theta_k}\left(\sin^{k-1}\theta_k\frac{\partial}{\partial\theta_k}\right)+\frac{L_{k-1}{}^2}{\sin^2\theta_k} \tag{15}$$

and for k = 1, we have:

$$L_1{}^2=-\frac{\partial^2}{\partial\theta_1^2} \tag{16}$$

Subtitute (14), (15), (16), using variable subtitution, and separation variabel, that is,

$$\varepsilon^2=-\frac{2m}{\hbar^2}E \tag{17}$$

$$\Psi\left(r,\Omega_D=\theta_1,\theta_2,\ldots\theta_{D-1}\right)=\Psi(r)\Phi\left(\theta_1=\varphi\right)H\left(\theta_2,\ldots\theta_{D-1}\right) \tag{18}$$

we have the radial, polar, and azimuth part of Schrodinger equation are:

$$\frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right)\Psi(r)-\left\{\frac{b^2+a(a+1)}{\sinh^2 r}-\frac{2b\left(a+\frac{1}{2}\right)\cosh r}{\sinh^2 r}+\varepsilon^2\right\}\Psi(r)=\frac{\lambda^*}{r^2}\Psi(r) \tag{19}$$

$$\frac{1}{\sin^{k-1}\theta_k}\frac{\partial}{\partial\theta_k}\left(\sin^{k-1}\theta_k\frac{\partial}{\partial\theta_k}H\right)-\frac{l_{k-1}\left(l_{k-1}+k-2\right)}{\sin^2\theta_k}H+l_k\left(l_k+k-1\right)H$$
$$-\left\{\frac{\kappa(\kappa-1)}{\sin^2\theta_k}+\frac{\lambda(\lambda-1)}{\cos^2\theta_k}\right\}H=0 \tag{20}$$

for $k = 2, 3, 4, \ldots, D-1$.

$$-\frac{1}{\Phi}\frac{d^2}{d\theta_1^2}\Phi=L_k^{\ 2} \tag{21}$$

3.1. Solution of Azimuth Part of Schrodinger Equation in D-Dimensions

The azimuth part of Schrodinger equation in D-Dimensions (equation 20) is the ordinary differential equation. The unnormalized solution of azimuth part of Schrodinger equation in D-Dimensions (21) is:

$$\Phi = Ae^{im\varphi} \tag{22}$$

where A is normalization constant.

3.2. Solution of Radial Part of Schrodinger Equation in D-Dimensions

The radial part of Schrodinger equation in D-Dimensions (equation 19) is hypergeometry differential equation. Equation (19) can be solved using some approximation and variable subtitution, that is,

$$\Psi(r)=r^{-\frac{D-1}{2}}R(r) \tag{23}$$

$$r\approx\sinh r \tag{24}$$

$$\cosh r = s \tag{25}$$

Subtitute equation (23), (24), and (25) to equation (19), we have:

$$\frac{d^2}{ds^2}R(s)+\frac{s}{\left(s^2-1\right)}\frac{d}{ds}R(s)$$
$$-\left\{\frac{b^2+a(a+1)-2b\left(a+\frac{1}{2}\right)s+\varepsilon^2\left(s^2-1\right)+A_{D-1}+\frac{(D-1)(D-3)}{4}}{\left(s^2-1\right)^2}\right\}R(r)=0 \tag{26}$$

By comparing equation (1), (26), and using eigenvalue of Nikivorof-Uvarov method, we obtain the energy spectra of Scarf Hyperbolic plus Non-Central Poschl-Teller potential in D-Dimensions is:

$$E=-\frac{\hbar^2}{2m}\left(n-p+\tfrac{1}{2}\right)^2 \tag{27}$$

where

$$p=\sqrt{\frac{\left\{b^2+\left(a+\frac{1}{2}\right)^2+A_{D-1}+\frac{(D-1)(D-3)}{4}\right\}-\sqrt{\left[\left\{b+\left(a+\frac{1}{2}\right)\right\}^2+A_{D-1}+\frac{(D-1)(D-3)}{4}\right]\left[\left\{b-\left(a+\frac{1}{2}\right)\right\}^2+A_{D-1}+\frac{(D-1)(D-3)}{4}\right]}}{2}} \tag{28}$$

and $A_{D\text{-}1}$ is centrifugal term and depend on the eigenvalue of polar part of Schrodinger equation in D-Dimensions.

By using eigenfunction of NU method, we obtain:

$$R_n = B_n\left(s^2-1\right)^{\left(\frac{1}{4}-\frac{1}{2}p\right)}\left(s+1\right)^{-\frac{b\left(a+\frac{1}{2}\right)}{2p}}\left(s-1\right)^{\frac{b\left(a+\frac{1}{2}\right)}{2p}}P_n^{(\alpha,\beta)}\left(s\right) \tag{29}$$

where $P_n^{(\alpha,\beta)}$ is Jacobi Polynomial, that is,

$$P_n^{(\alpha,\beta)}\left(s\right)=\frac{\left(-1\right)^n}{2^n n!}\left(1-s\right)^{-\alpha}\left(1+s\right)^{-\beta}\frac{d^n}{dz^n}\left\{\left(1-s\right)^{\alpha}\left(1+s\right)^{\beta}\left(1-s^2\right)^n\right\} \tag{30}$$

and B_n is normalization constant.

Finally, from equation (23), (25), and (29), we have the radial wave function of Schrodinger equation for Scarf Hyperbolic plus Non-Central Pocshl-Teller Potentialin D-Dimensions is:

$$\Psi\left(r\right)=B_n r^{-\frac{D-1}{2}}\left(\sinh r\right)^{\frac{1}{2}-p}\left(\cosh r+1\right)^{-\frac{b}{2p}\left(a+\frac{1}{2}\right)}\left(\cosh r-1\right)^{\frac{b}{2p}\left(a+\frac{1}{2}\right)}P_n^{(\alpha,\beta)}\left(\cosh r\right) \tag{31}$$

The effect of extra dimensions on bound state energy and radial wave function can be visualized as

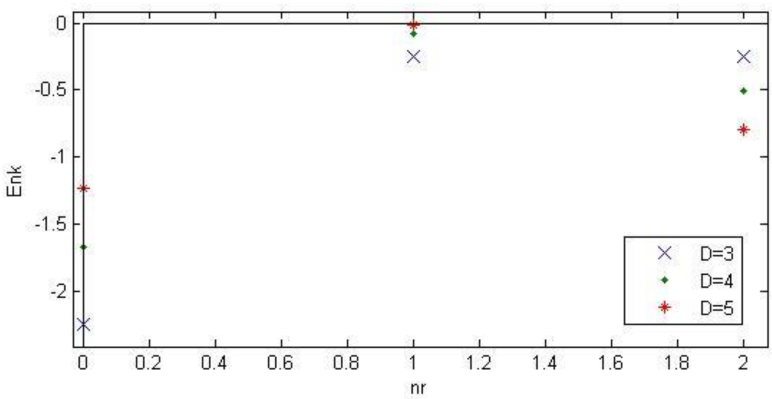


Figure 3. Bound State Energy of Scarf Hyperbolic Potential (a = 2 and b = 2)

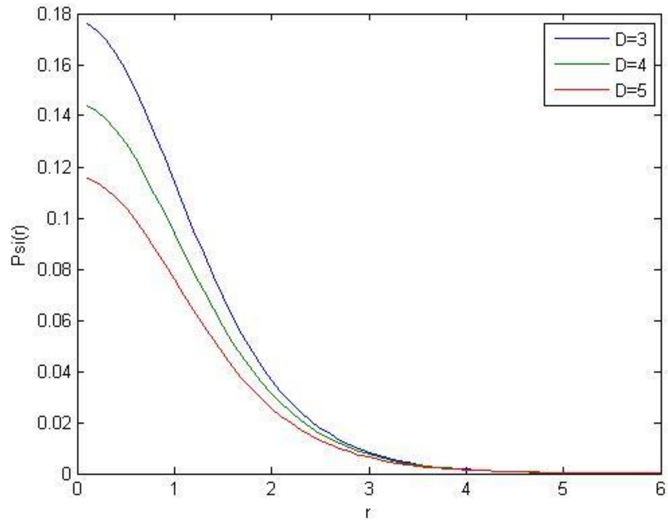


Figure 2. Radial Wave Function on Ground State
for Scarf Hyperbolic Potential (a = 2 and b = 2)

From Figure 3, we have the effect of extra dimensions increase the bound state energy of system. The effect of extra dimensions in radial wave function is decrease their amplitude shown in Figure 4. It was found to agree with previous work [5,6]. Based on superstring theory [18], the amount of dimensions in the universe restricted to 10 spatial dimensions and 1 time dimension. If the amount of spatial dimension more than 10, the universe unstable and collapse. Thus, the maximum value of D is limited to 10 spatial dimensions.

3.3. Solution of Polar Part of Schrodinger Equation in D-Dimensions

According to the value of k, there are two equations at polar part of Schrodinger equation in D-Dimensions. First, for $k = 2, 3, 4, \ldots, D-1$, we solve equation (20) using variable subtitution, that is,

$$A_k = l_k\left(l_k + k - 1\right) \tag{32a}$$

$$A_{k-1} = l_{k-1}\left(l_{k-1} + k - 2\right) \tag{32b}$$

$$s = \cos 2\theta_k \tag{33}$$

Subtitute equation (32a), (32b), and (33) to (21a), we have

$$\frac{d^2}{ds^2}H - \frac{\frac{1}{2}\left[(k-1)+(k+1)s\right]}{\left(1-s^2\right)}\frac{d}{ds}H - \left\{\frac{\frac{1}{2}\kappa(\kappa-1)(1+s)+\frac{1}{2}\lambda(\lambda-1)(1-s)+\frac{1}{2}A_{k-1}(1+s)-\frac{1}{4}A_k\left(1-s^2\right)}{\left(1-s^2\right)^2}\right\}H = 0 \tag{34}$$

By comparing equation (1), (34), andusing eigenvalue of Nikivorof-Uvarov method, we obtain the eigenvalue of Scarf Hyperbolic plus Non-Central Pocshl-Teller potential in D-Dimensions:

$$A_k = 4\left(n+p+\tfrac{1}{2}\right)^2 - \tfrac{1}{4}(k-3)^2 - (k-2) \tag{35}$$

where

$$p = \sqrt{\tfrac{1}{2}(q+t) + \tfrac{1}{2}\sqrt{4qt-(k-2)^4-2(k-2)^2(q-t)}} \tag{36}$$

$$q = \tfrac{1}{2}\left[\kappa(\kappa-1) + A_{k-1}\right] \tag{37}$$

$$t = \tfrac{1}{2}\lambda(\lambda-1) + \tfrac{1}{16}(k-1)^2 + \tfrac{1}{16}(k-3)^2 \tag{38}$$

By using eigenfunction of NU method, we obtain:

$$H\left(s_k\right) = B_n\left(1-s^2\right)^{-\frac{1}{2}\left(\frac{1}{4}(k-3)-p\right)}(1+s)^{\frac{1}{2}\left\{\frac{1}{4}(k-1)-\frac{1}{2p}\left[(k-2)^2+q-t\right]\right\}}(1-s)^{-\frac{1}{2}\left\{\frac{1}{4}(k-1)-\frac{1}{2p}\left[(k-2)^2+q-t\right]\right\}}P_n^{(\alpha,\beta)}(s) \tag{39}$$

Finally, from equation (33), (36), (37), (38), and (39), we have the first equation ofpolar wave function Schrodinger equation for Scarf Hyperbolic plus Non-Central Pocshl-Teller Potentialin D-Dimensions ($k = 2, 3, 4, \ldots, D-1$), that is:

$$H\left(\cos 2\theta_k\right) = B_{n_{\theta_k}}\left(\sin 2\theta_k\right)^{-\frac{1}{2}\left(\frac{1}{4}(k-3)-p_{\theta_k}\right)}\left(\cos\theta_k\right)^{\left\{\frac{1}{4}(k-1)-\frac{1}{2p_{\theta_k}}\left[(k-2)^2+q_{\theta_k}-t_{\theta_k}\right]\right\}}\left(\sin\theta_k\right)^{-\left\{\frac{1}{4}(k-1)-\frac{1}{2p_{\theta_k}}\left[(k-2)^2+q_{\theta_k}-t_{\theta_k}\right]\right\}}P_{n_{\theta_k}}^{\left(\alpha_{\theta_k},\beta_{\theta_k}\right)}\left(\cos 2\theta_k\right) \tag{40}$$

4. Special Case for the Ground and First Excited State in 3 Dimensions

Special case for 3 Dimensional problem for Scarf Hyperbolic plus Non-Central Pocshl-Teller Potential, that is:

$$\Psi\left(r,\Omega_3\right) = \Psi(r)Y\left(\Omega_3\right) \tag{41}$$

with solution of radial part of 3 dimensional Schrodinger equation:

$$\Psi(r)=B_{n_r} r^{-1}(\sinh r)^{\frac{1}{2}-p_r}(\cosh r+1)^{-\frac{b}{2p_r}\left(a+\frac{1}{2}\right)}(\cosh r-1)^{\frac{b}{2p_r}\left(a+\frac{1}{2}\right)} P_{n_r}^{(\alpha_r,\beta_r)}(\cosh r) \tag{42}$$

$$B_{n_r}=(-1)^{\frac{b\left(a+\frac{1}{2}\right)}{2p_r}-\frac{1}{2}p_r-\frac{1}{2}}\sqrt{\frac{2n+\alpha_r+\beta_r+1}{2^{\alpha_r+\beta+1}}\frac{\Gamma(n_r+\alpha_r+\beta_r+1)n_r!}{\Gamma(n_r+\alpha_r+1)\Gamma(n_r+\beta_r+1)}} \tag{43}$$

$$P_{n_r}^{(\alpha_r,\beta_r)}(\cosh r)=\frac{(-1)^{n_r}}{2^n n_r!}(1-\cosh r)^{-\alpha_r}(1+\cosh r)^{-\beta_r}\frac{d^{n_r}}{d(\cosh r)^{n_r}}\left\{\begin{matrix}(1-\cosh r)^{\alpha_r}(1+\cosh r)^{\beta_r}\\ (\sinh^2 r)^{n_r}\end{matrix}\right\} \tag{44}$$

$$\alpha_r=\frac{b\left(a+\frac{1}{2}\right)}{p_r}-p_r \tag{45}$$

$$\beta_r=-\frac{b\left(a+\frac{1}{2}\right)}{p_r}-p_r \tag{46}$$

$$p_r=\sqrt{\frac{\left\{b^2+\left(a+\frac{1}{2}\right)^2+l(l+1)\right\}-\sqrt{\begin{matrix}\left[\left\{b+\left(a+\frac{1}{2}\right)\right\}^2+l(l+1)\right]\\ \left[\left\{b-\left(a+\frac{1}{2}\right)\right\}^2+l(l+1)\right]\end{matrix}}}{2}} \tag{47}$$

From equation (42) to (47), we can make a visualization of radial wave function with Matlab, that is:

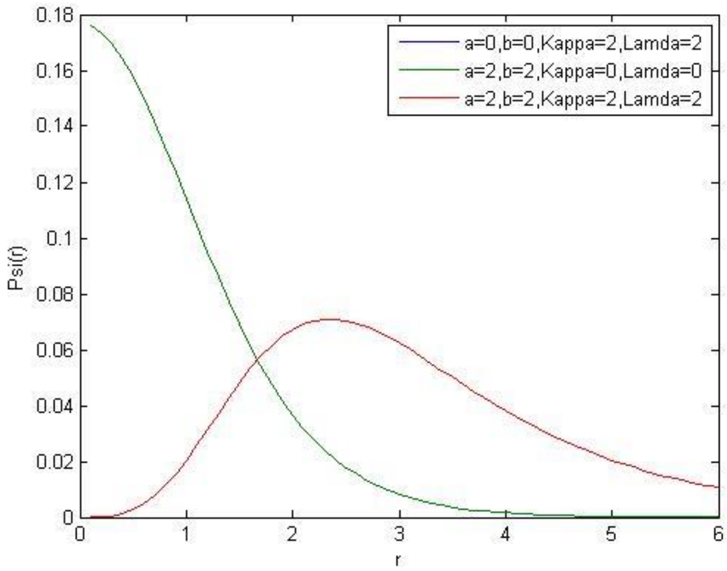


Figure 5. Radial Ground State Wave Function in 3 Dimension for Scarf Hyperbolic plus Non-Sentral Poschl-Teller Potential

From Figure 5, we have the effect of each potential parameters. The effect of parameter Non Central Poschl-Teller won't appear if the parameters of Scarf hyperbolic is zero. It causes the radial wave function vanish. If the Scarf hyperbolic parameters not zero, the Non Central Poschl-Teller parameters decrease the radial wave function and move wave function to right.

Solution of Angular part devided into 2 parts, that is:

$$Y_{l_1}^{(l)}\left(\Omega_D=\theta_1,\theta_2\right)=\Phi\left(\theta_1=\varphi\right)H\left(\theta_2\right) \tag{48}$$

Solution of azimuth part equation:

$$\Phi = Ae^{im\varphi} \tag{49}$$

$$A=\frac{1}{\sqrt{2\pi}} \tag{50}$$

Solution of polar part equation:

$$H\left(\cos 2\theta_2\right)=B_n\left(1+\cos 2\theta_2\right)^{\frac{\sqrt{t_{\theta_2}}}{\sqrt{2}}+\frac{1}{4}}\left(1-\cos 2\theta_2\right)^{\frac{\sqrt{q_{\theta_2}}}{\sqrt{2}}}P_n^{(\alpha,\beta)}\left(\cos 2\theta_2\right) \tag{51}$$

with

$$P_{n_{\theta_2}}^{\left(\alpha_{\theta_2},\beta_{\theta_2}\right)}\left(\cos 2\theta_2\right)=\frac{(-1)^{n_{\theta_2}}}{2^{n_{\theta_2}+\alpha_{\theta_2}+\beta_{\theta_2}}n_{\theta_2}!}\left(\sin^2\theta_2\right)^{-\alpha_{\theta_2}}\left(\cos^2\theta_2\right)^{-\beta_{\theta_2}}$$
$$\frac{d^{n_{\theta_2}}}{d\left(\cos 2\theta_2\right)^{n_{\theta_2}}}\left\{\left(\sin^2\theta_2\right)^{\alpha_{\theta_2}+n_{\theta_2}}\left(\cos^2\theta_2\right)^{\beta_{\theta_2}+n_{\theta_2}}\right\} \tag{52}$$

$$p_{\theta_2}=\tfrac{1}{\sqrt{2}}\left(\sqrt{q_{\theta_2}}+\sqrt{t_{\theta_2}}\right) \tag{53}$$

$$q_{\theta_2}=\tfrac{1}{2}\left[\kappa\left(\kappa-1\right)+m^2\right] \tag{54}$$

$$t_{\theta_k}=\tfrac{1}{2}\lambda\left(\lambda-1\right)+\tfrac{1}{8} \tag{55}$$

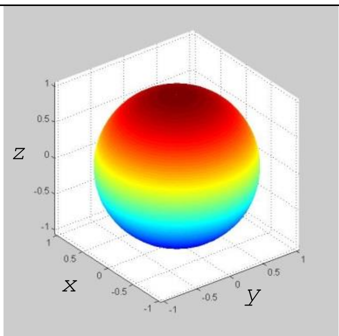
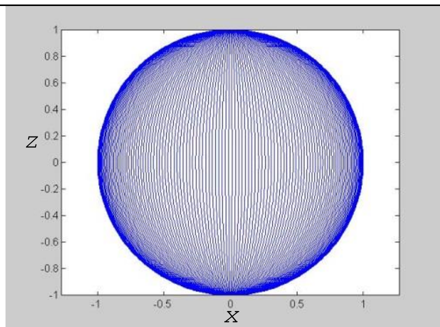
$$\alpha=\sqrt{2t_{\theta_2}} \tag{56}$$

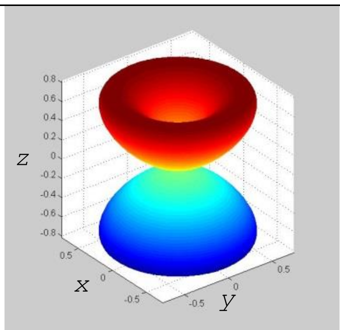
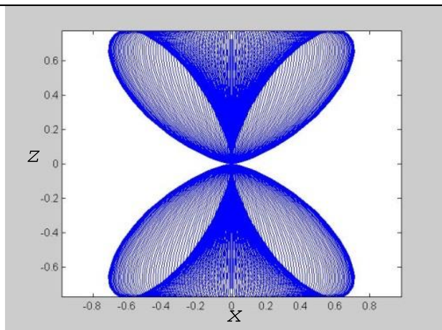
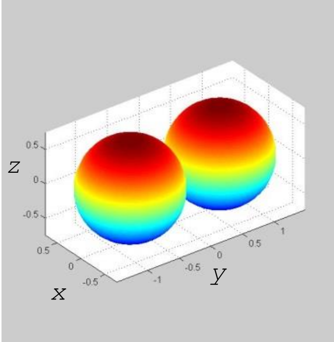
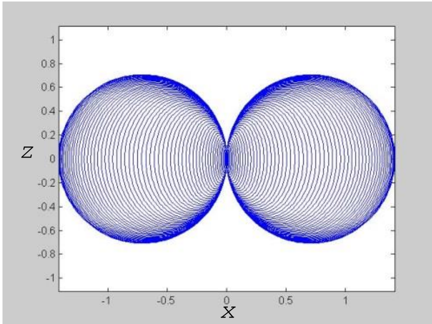
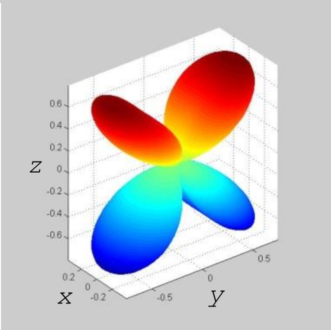
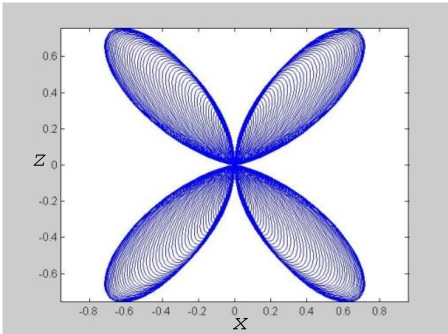
$$\beta=\sqrt{2q_{\theta_2}} \tag{57}$$

$$l=2\left(n_{\theta_k}+p_{\theta_k}+\tfrac{1}{4}\right) \tag{58}$$

Using subtitution of all equation (48) to (58), we get unnormalized solution of 3 dimensional Schrodinger equation for Scarf hiperbolik plus non sentral Poschl-Teller potential, for example:

Table 1. Visualization of angular part wave function for Non-Sentral Poschl-TellerPotential

No	$H\left(\cos 2\theta\right)_{n_l m\kappa\lambda}$	3 Dimension	2 Dimensional Projection at *x*-*z* axis
1	$H\left(\cos 2\theta_2\right)_{0000}$		

No	$H(\cos 2\theta)_{n_l m\kappa\lambda}$	3 Dimension	2 Dimensional Projection at *x-z* axis
2	$H(\cos 2\theta)_{0033}$		
3	$H(\cos 2\theta)_{0100}$		
4	$H(\cos 2\theta)_{0133}$		

The effect of polar potential parameters in 3-dimensions Non-Central Poschl-Teller potential on orbital (sub shell) electrons can be described in Table 1. Without polar potential (κ and λ is zero), the orbital electron will be changed into orbitals in spherical harmonics or hydrogen-like atom. This phenomena figured in the number 1and 3, which corresponds to an orbital in spherical harmonics in the same parameters [19].

The effect of wave function parameter (λ) make the wave function pulled to z axis and having a reflection of the x-y plane so that the wave function looks like 2 pieces of adjacent balloons. The effect of κ parameter rotate wave function with direction φ with the rotary axis at the origin so that the wave function looks like a donut. When these two parameters given, the effect of each parameter affects the wave function. The combination of these effects result in Table 3 number 2 and 4.

The energy spectra of 3 dimensional Schrodinger equation for Scarf hyperbolic plus non central Poschl-Teller potential, that is:

$$E = -\frac{\hbar^2}{2m}\left(n_r - p_r + \tfrac{1}{2}\right)^2 \qquad (59)$$

with

$$p_r = \sqrt{\frac{\left\{b^2 + \left(a+\frac{1}{2}\right)^2 + l\left(l+1\right)\right\} - \sqrt{\left[\left\{b+\left(a+\frac{1}{2}\right)\right\}^2 + l\left(l+1\right)\right]}\sqrt{\left[\left\{b-\left(a+\frac{1}{2}\right)\right\}^2 + l\left(l+1\right)\right]}}{2}} \tag{60}$$

$$l\left(l+1\right) = 4\left(n_{\theta_2} + p_{\theta_2} + \frac{1}{2}\right)^2 - \frac{1}{4} \tag{61}$$

$$p_{\theta_2} = \frac{1}{\sqrt{2}}\left(\sqrt{q_{\theta_2}} + \sqrt{t_{\theta_2}}\right) \tag{62}$$

$$q_{\theta_2} = \frac{1}{2}\left[\kappa\left(\kappa-1\right) + m^2\right] \tag{63}$$

$$t_{\theta_2} = \frac{1}{2}\lambda\left(\lambda-1\right) + \frac{1}{8} \tag{64}$$

From equation (59) to (64), we can make a visualization of bound state energy with Matlab, that is:

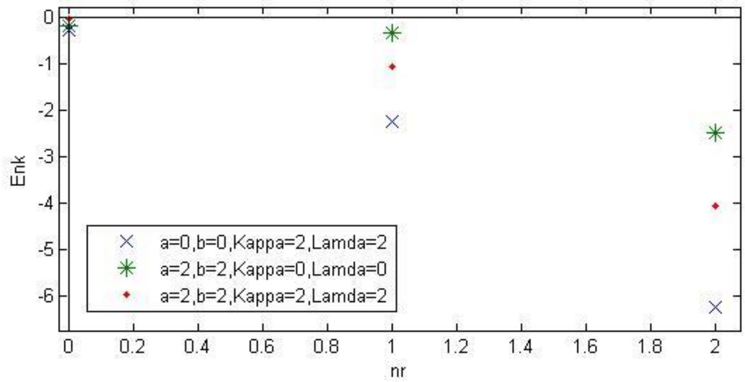


Figure 6 Bound State Energy in 3 Dimension for Scarf Hyperbolic plus Non-Sentral Poschl-Teller Potential

From figure 6, we have the bound state energy for each value of potential parameters. The effect of Scarf Hyperbolic potential increases the bound state energy of system. In the other hand, the effect of non central Poschl-Teller potential decreases the bound state energy of system

5. Concluding Remarks

In this paper, we present the solutions of Schrodinger equation in D-dimension for Scarf hyperbolic plus non central Poschl-Teller potential within the framework of an approximation to the centrifugal and high dimension term. The bound state energy were obtained in D-dimensions using the Nikiforov-Uvarov method, and it was found to agree with previous works [5,10-11]. The corresponding wave function of the Scarf hyperbolic plus non central Poschl-Teller potential were obtained in terms of the Jacobi polynomials. The example of bound state energy and wave function in 3 dimensions presented in condition of ground state. The existence of arbitrary dimensions increase bound state energy system. In the other hand, the existence of arbitrary dimensions decreases the amplitude of wave function. The effect of Scarf Hyperbolic potential increases the bound state energy of system. The effect of non central Poschl-Teller potential decreases the bound state energy of system.

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