

# Experimental comparison of phase retrieval methods which use intensity distribution at different planes

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**Abstract.** Performance of the three phase retrieval methods that use spatial intensity distributions was investigated in dealing with a task of reconstruction of the amplitude characteristic of the test object. These methods differ both by mathematical models and order of iteration execution. The single-beam multiple-intensity reconstruction method showed the best efficiency in terms of quality of reconstruction and time consumption.

## 1. Introduction

Full wavefront reconstruction from intensity distributions is receiving a lot of attention lately. This is due to the fact that the methods to the solution of this problem, though usually more complex data processing algorithm, but impose significantly lower requirements for the experimental setup, as opposed to the more traditional holographic approaches. The first phase retrieval iterative algorithm was proposed by Gerchberg and Saxton [1]. They used spatial amplitude distribution in the plane of the object and in the Fourier plane. Its demerits as stagnating and sensitivities to the introduction of errors and noise have been overcome by use of additional datasets in iterative procedure [2]. Most simple example of obtaining such datasets is the registration of intensity distributions in different planes [3-5]. These methods differ by mathematical models and how to use the intensity distributions in the iterative algorithm. The purpose of this work is experimental comparison of efficiency of these methods for amplitude object.

## 2. Description of the methods

1) Fresnel Iterative Method (FRIM) [3] uses the 2D Fresnel transform in mathematical model:

$$U(x, y, l) = \frac{\exp(ikl)}{i\lambda l} \iint_{-\infty}^{+\infty} \exp\left(\frac{ik}{2l}[(x-x')^2 + (y-y')^2]\right) u(x', y') dx' dy' \quad (1)$$

Here  $U(x, y, l)$  is a complex-valued wave field at the distance  $l$ ;  $x, y$  are coordinates in the image plane;  $u(x', y')$  is the complex characteristic of the object;  $x', y'$  are coordinates in the object plane;  $\lambda$  is the wavelength,  $k = 2\pi/\lambda$  is the wave vector. The order of the use of measured intensity distributions in the FRIM is following:  $I_1 \rightarrow O \rightarrow I_2 \rightarrow O \rightarrow I_3 \rightarrow O \rightarrow \dots \rightarrow O \rightarrow I_n \rightarrow O \rightarrow I_1 \dots$ . Here  $I_1, I_2, I_3, \dots, I_n$  are spatial intensity distributions recorded at different planes on various distances from the object; arrows denotes wavefront propagation at free space;  $O = v(x', y')$  is the object approximation function at the object plane. When convergence will take place this function coincides with  $u(x', y')$ .

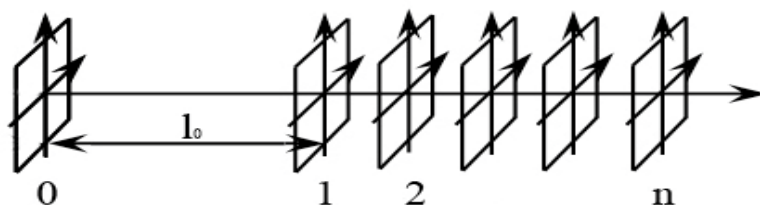


2) The single-beam multiple-intensity reconstruction (SBMIR) method [4] uses the angular spectrum approach in solving the Rayleigh–Sommerfeld equation, which simulates wave propagation between the adjacent measurement planes. The first iteration includes one complete set of sequential propagations over the  $n$  planes in the forward direction. Before the 2nd iteration is carried out, the wavefront at the last measurement plane is backpropagated directly to the 1st plane, i.e.  $I_1 \rightarrow I_2 \rightarrow \dots \rightarrow I_n \rightarrow I_1 \dots$ . When convergence will be reconstructed wavefront is propagated to the object plane to obtain object function  $u(x', y')$ .

3) Forward-Backward (FB) SBMIR method [5] differ from conventional SBMIR method by the iterative calculation procedure:  $I_1 \rightarrow I_2 \rightarrow I_3 \rightarrow \dots \rightarrow I_{n-1} \rightarrow I_n \rightarrow I_{n-1} \rightarrow \dots \rightarrow I_3 \rightarrow I_2 \rightarrow I_1 \dots$ .

### 3. The experiment

Figure 1 shows the object plane and the intensity measurement planes. As radiation source we use a single-mode laser Lasos RLD F-638-50-pvc,  $\lambda = 634.9$  nm. The test object logo "ITMO" was made on high-resolution film (depicted at figure 2). The size of the object  $0.5 \times 0.6$  mm, letters height 0.2 mm. To register intensity distributions the CMOS matrix "VEI - 830" with resolution  $2048 \times 1536$  and pixel size  $\Delta x = 2.8 \mu\text{m}$  was used. For precise displacements of the sensor, a motorized micrometer stage Standa 8MT175 was used. Three intensity measurement were used in the experiment for each method. Distance  $l_0 = 17.8$  mm, and distances between measurement planes were chosen in according to the criterion that gives the best quality of reconstruction as defined in [6]. Therefore the distance between the first and the second planes was 0.81 mm, the distance between second and third planes was 0.90 mm.



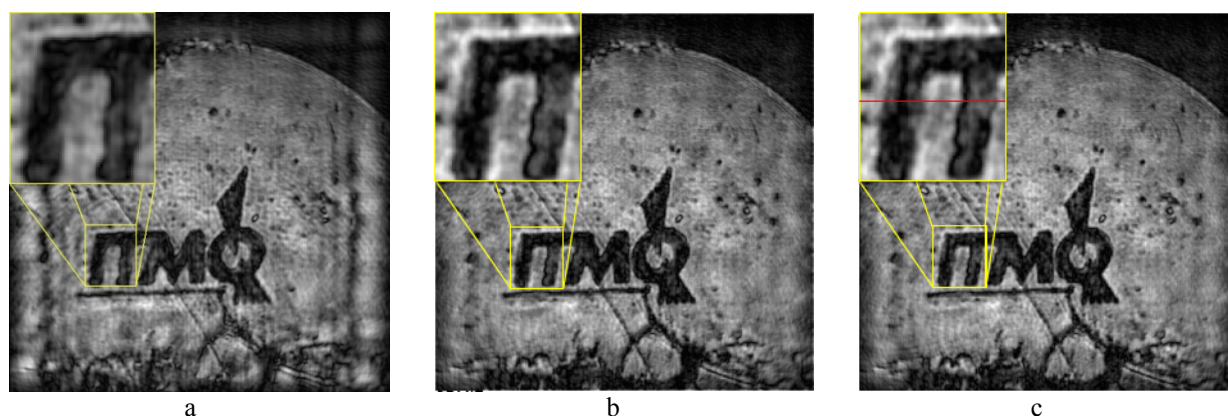
**Figure 1.** Diagram for measurements datasets from spatial intensities distributions.



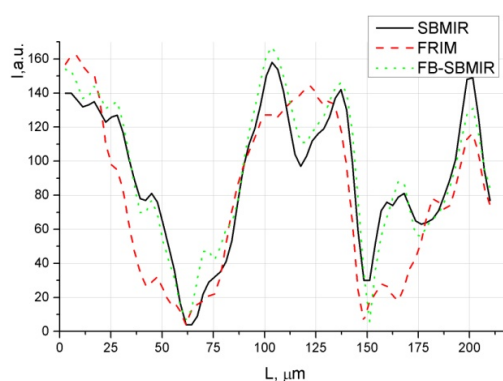
**Figure 2.** Test object used in the experiment.

### 4. Obtained results

Figure 3 shows the reconstructed images of the object "ITMO" by three different methods: FRIM (a), SBMIR (b), FB-SBMIR (c). Due to the features of FRIM iterative procedure, associated with alternating calculation of wavefront propagation from registration planes to the object plane and back, diffraction artifacts on the borders of the reconstructed image (figure 3a) appears. Comparing the results on the figure 3, it can be seen that the reconstructed images are almost identical for all three methods. This is well illustrated in Figure 4, which shows the cross section of the enlarged section letters "IT", indicated by the red line on figure 3c. As can see from figure 4, SBMIR and FB-SBMIR showed very similar results, but the method FB-SBMIR more time consuming.



**Figure 3.** Reconstructed images: a – FRIM, b – SBMIR, c – FB-SBMIR



**Figure 4.** Transverse object amplitude distribution, reconstructed by different methods: FRIM(solid black line), SBMIR(dashed red line) and FB-SBMIR(dotted blue line).

## 5. Conclusion

The comparison of the effectiveness of three iterative phase retrieval methods in relation the reconstruction of complex characteristics of the test object was made. On the reconstructed image by FRIM methods the diffraction artifacts appears. The reason is iteratively repeating process of the wavefront propagation between object and registration planes. A similar process take place inside laser cavity, which leads to the formation of high-order transverse electromagnetic modes described by Hermite-Gaussian functions. The results, obtained in the experiment by SBMIR and FB-SBMIR methods are similar. But FB-SBMIR method was developed for dynamic wavefront sensor setup with stack of beamsplitters providing instant registration if all intensity distributions. In this case the criterion of the registration of datasets defined in [6] that gives the best quality of reconstruction is difficult to apply in practice. Therefore, we can conclude that the best performance demonstrates the SBMIR method.

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