The second order extended Kalman filter and Markov nonlinear filter for data processing in interferometric systems

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Abstract. Recurrent stochastic data processing algorithms using representation of interferometric signal as output of a dynamic system, which state is described by vector of parameters, in some cases are more effective, compared with conventional algorithms. Interferometric signals depend on phase nonlinearly. Consequently it is expedient to apply algorithms of nonlinear stochastic filtering, such as Kalman type filters. An application of the second order extended Kalman filter and Markov nonlinear filter that allows to minimize estimation error is described. Experimental results of signals processing are illustrated. Comparison of the algorithms is presented and discussed.

1. Introduction

Interferometric methods are the most accurate among other optical methods [1, 2]. Signals obtained in interferometric systems are affected by noise. To recover information about properties of objects it is necessary to use the data processing algorithms robust to different types of noise [2].

Conventional processing methods based on Fourier transform do not consider available *a priori* information about a model of signal formation and need in acquisition of full signal realization before calculating that limits the processing speed. Recurrent stochastic data processing algorithms based on the state-space approach and formalism of stochastic differential equations perform consequent calculating of signal samples series. In this way, interferometric signals are represented as output of a dynamic system [2]. The most popular state-space estimation algorithm is linear Kalman filter (LKF). This filter is optimal in terms of minimum estimation error variance [3]. Dependence of measured interferometric signal on parameters of model is nonlinear, it is why the LKF is not applicable for such systems. To solve this problem it is necessary to use the algorithms of nonlinear stochastic filtering. Extended Kalman filter (EKF) [2, 4], the most widely used nonlinear Kalman type algorithm, uses linearization of nonlinear system equations by the first-order Taylor series expansion. To minimize approximation error one can use the second-order Taylor series expansion of state and measurement equations, which is used in the second order Kalman filter (SOEKF) [4] and Markov nonlinear filter (MNLF) [5]. In the paper, the EKF, SOEKF and MNLF algorithms that were applied to interferometric signals processing are compared and discussed.

2. Theoretical background

One can represent the state of a dynamic system by vector of parameters. In case of the interferometric signal, the vector of parameters includes background, amplitude and phase. Output of the system is measured discrete signal value.

The discrete dynamic system can be described by system and observation equations [3, 4]:

$$\boldsymbol{\theta}(k) = \mathbf{f}(\boldsymbol{\theta}(k-1)) + \mathbf{w}(k), \tag{1}$$

$$\mathbf{s}(k) = \mathbf{h}(\boldsymbol{\theta}(k)) + \mathbf{n}(k), \tag{2}$$

where $\boldsymbol{\theta}(k)$ is vector of parameters; $\mathbf{s}(k)$ is output of the dynamic system; $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are nonlinear differentiable functions; $\mathbf{w}(k)$ is system noise; $\mathbf{n}(k)$ is observation noise; $k = 0..K-1$. System equation determines vector of parameters at each discrete-time sample. Observation equation (2) produces output signal by current vector of parameters. This equation is nonlinear because of the output depends on the signal phase nonlinearly.

Data processing by EKF, SOEKF, and MNLF is divided into two steps: prediction and update. In the first step, current vector of parameters is predicted using estimation of this vector at previous discrete-time sample taking into account the system equation. At the second step the gain coefficients of the filter are computed according to approximation of nonlinear equations and stochastic properties of noise. Predicted vector of parameters is updated by using measured value of the signal on current sample and the gain of the filter. In a manner, the gain determines degree of confidence to current measurements. Updated vector is the estimate of the vector of parameters at current sample.

Approximation of nonlinear system equations by the second-order Taylor series expansion in SOEKF and MNLF is carried using additional modifications of the prediction and update formulas, which are different for proposed algorithms. The prediction and update equations are changed in SOEKF compared with EKF by addition of terms using the second derivatives of nonlinear system equations. The gain in SOEKF is calculated similarly to the EKF. MNLF uses prediction of vector of parameters like in the EKF, but the equations for gain computation and update of predicted vector differ have differences [2, 4]. Table 1 illustrates the equations, which are different for described algorithms.

Table 1. Comparison of the algorithms

	EKF	SOEKF	MNLF
Prediction	$\hat{\boldsymbol{\theta}}(k) = \mathbf{f}(\boldsymbol{\theta}(k-1))$	$\hat{\boldsymbol{\theta}}(k) = \mathbf{f}(\boldsymbol{\theta}(k-1)) + \underline{\mathbf{A}(k)}$	$\hat{\boldsymbol{\theta}}(k) = \mathbf{f}(\boldsymbol{\theta}(k-1))$
Update	$\boldsymbol{\theta}(k) = \hat{\boldsymbol{\theta}}(k) +$ $+ \mathbf{P}(k)[\mathbf{s}(k) - \mathbf{h}(\hat{\boldsymbol{\theta}}(k))]$	$\boldsymbol{\theta}(k) = \hat{\boldsymbol{\theta}}(k) +$ $+ \mathbf{P}(k)[\mathbf{s}(k) - \mathbf{h}(\hat{\boldsymbol{\theta}}) - \underline{\mathbf{B}(k)}]$	$\boldsymbol{\theta}(k) = \hat{\boldsymbol{\theta}}(k) +$ $+ \mathbf{P}(k)[\mathbf{s}(k) - \mathbf{h}(\hat{\boldsymbol{\theta}}) - \underline{\mathbf{B}(k)}]$
Gain	$\mathbf{P}(k) = \mathbf{R}(k)\mathbf{H}(k)^{\mathrm{T}} \times$ $\times \left(\mathbf{H}(k)\mathbf{R}(k)\mathbf{H}(k)^{\mathrm{T}} + \mathbf{R}_n\right)^{-1}$	$\mathbf{P}(k) = \mathbf{R}(k)\mathbf{H}(k)^{\mathrm{T}} \times$ $\times \left(\mathbf{H}(k)\mathbf{R}(k)\mathbf{H}(k)^{\mathrm{T}} + \mathbf{R}_n\right)^{-1}$	$\mathbf{P}(k) = \mathbf{R}(k)\mathbf{H}(k)^{\mathrm{T}} \times$ $\times \left(\mathbf{H}(k)\mathbf{R}(k)\mathbf{H}(k)^{\mathrm{T}} + \mathbf{R}_n + \underline{\mathbf{C}(k)}\right)^{-1}$

In this table, $\mathbf{R}(k)$ is prediction of the covariance matrix of errors; $\mathbf{R}_n$ is covariance matrix of system noise; $\mathbf{H}(k)$ is observation matrix calculated by linearization of the observation equation. Underlined terms $\mathbf{A}(k)$, $\mathbf{B}(k)$, and $\mathbf{C}(k)$, which additionally use the second partial derivatives of equations (1) and (2) [2, 4, 5], represent difference between the algorithms. Computation of these terms requires additional operation time with respect to the computational time of the EKF algorithm.

3. Results

Figure 1 shows experimental results of interferometric signal processing by the MNLF algorithm (as an example). Signal amplitude varies in accordance with normal distribution with expected value at the 500$^{\text{th}}$ sample. Phase of signal is determined as $\Phi(k) = 2\pi(ak - bk^2)$, where $a = 10^{-1}$, $b = 200^{-2}$. The signal is affected by additive Gaussian noise with standard deviation equal to 5% of maximal amplitude. Phase variations are expected *a priori* as linear, i.e. $\Phi(k) = 2\pi ak$, where $a = 70^{-1}$. It has been found that the distribution of phase estimation error is close to Gaussian function. Maximal phase estimation error did not exceed $\pi/6$ rad.

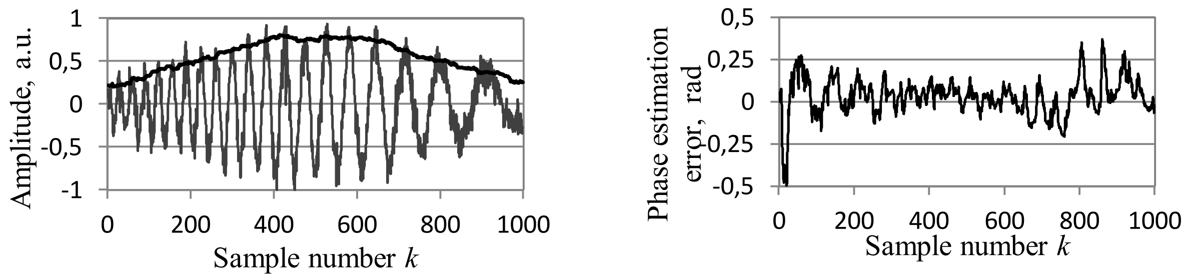

Figure 1. The results of signal processing by MNLF: the normalized signal and estimated amplitude (left plot) and phase estimation error (right plot).

Table 2 shows the obtained RMS errors of signal parameters and signal-to-noise ratio (SNR) of the restored signal. Initial SNR is equal to 10 dB.

Table 2. Standard deviations of parameters estimation errors of the algorithms

	EKF	SOEKF	MNLF
Background, %	1.11	1.12	1.10
Amplitude, %	4.45	4.46	4.23
Phase, rad	0.14	0.14	0.15
SNR, dB	17.90	17.92	17.84

The SOEKF and MNLF estimation errors differ little from the EKF estimation error. It shows that additional terms of the SOEKF and MNLF have small values in this case. Consequently using the high-order Taylor series expansion of nonlinear equations is not expedient for interferometric systems.

4. Conclusion

Considered algorithms are applicable for parameters estimation of interferometric signals with original SNR less than or equal to 10 dB. Typically maximal phase estimation error does not exceed $\pi/6$ rad. Comparison of the algorithms shows that quality of estimation by the algorithms, using the second-order Taylor series expansion of nonlinear system equations, differs little from the EKF using linearization for the considered kind of signal. Therefore, using of EKF for interferometric data processing is enough in terms of computational power minimization.

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