Spiral light beams: characteristics and applications

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Abstract. The report contains discussions of theoretical and experimental results of the study on the light beams that retain their intensity structure under propagation and focusing. We describe the behavior of a family of such laser beams called spiral beams, keeping their intensity invariant within scale and rotation, under their propagation. These very features of spiral beams are of a high practical interest for those occupied with the laser technologies and image processing. Some of the applications, in particular their use for the retrieval of the one-dimensional signals phase and contour analysis are demonstrated.

1. Spiral beams

There exists a class of coherent light fields, referred to as spiral beams, of the following form

$$F(x,y,l)=\frac{1}{\sigma}\exp\left(-\frac{x^2+y^2}{\rho^2\sigma}\right)f\left(\frac{x\pm \mathrm{i}y}{\rho\sigma}\right), \tag{1}$$

that retain their structure within scale and rotation. Here, $f(z)$ is an arbitrary entire analytic function, $\sigma=1+\frac{2il}{k\rho^2}$ l is the distance along the beam propagation direction, k is the wavenumber, ρ is const, and the sign in the argument of $f(z)$ determines the direction of beam rotation during propagation [1].

This class of the light field contains the beams in the form of arbitrary curves $\zeta(t)$ [2]:

$$\mathrm{S}\left(z,\bar{z}\middle|\zeta(t),\ t\in[0,T]\right)=$$
$$=\exp\left(-\frac{z\bar{z}}{\rho^2}\right)\int_0^T\exp\left[-\frac{\zeta(t)\bar{\zeta}(t)}{\rho^2}+\frac{2z\bar{\zeta}(t)}{\rho^2}+\frac{1}{\rho^2}\int_0^t\left(\bar{\zeta}(\tau)\zeta'(\tau)-\zeta(\tau)\bar{\zeta}'(\tau)\right)d\tau\right]\left|\zeta'(t)\right|dt. \tag{2}$$

The mentioned features are very useful for a number of applications described below.

Journal of Physics: Conference Series **536** (2014) 012001 doi:10.1088/1742-6596/536/1/012001

2. Spiral beams applications

2.1. The analytic continuation for one-dimensional phase retrieval problem [3]

The astigmatic transformation of the light field $U(\xi)exp\left(-\dfrac{\eta^2}{8\rho^2}\right)$ looks like the following:

$$\begin{aligned} F(x,y) &= \iint_{\mathbb{R}^2} \exp\left(-\frac{ik(x\xi+y\eta)}{f}+\frac{i\xi\eta}{\rho^2}\right)U(\xi)exp\left(-\frac{\eta^2}{8\rho^2}\right)d\xi d\eta = \\ &= 2\rho\sqrt{2\pi}\exp\left(-\frac{k^2\rho^2(x^2+y^2)}{f^2}\right)\exp\left(-\frac{2ik^2\rho^2xy}{f^2}\right)F_0(x+iy) \end{aligned}, \quad (3)$$

where the value $F_0(z)=\exp\left(-\dfrac{k^2\rho^2|z|^2}{f^2}\right)\displaystyle\int_{\mathbb{R}}\exp\left(-\frac{ikz\xi}{f}-\frac{\xi^2}{8\rho^2}\right)U(\xi)d\xi$, f is the focal distance.

It's easy to see that $F_0(z)$ is the analytic continuation of $F_0(x)$ and is determined by means of its zeroes. All the zeroes have the same sign hence it's probable to restore the initial one-dimensional field $U(\xi)$ by a single intensity measurement. In figure 1 the numerical simulation result is presented.

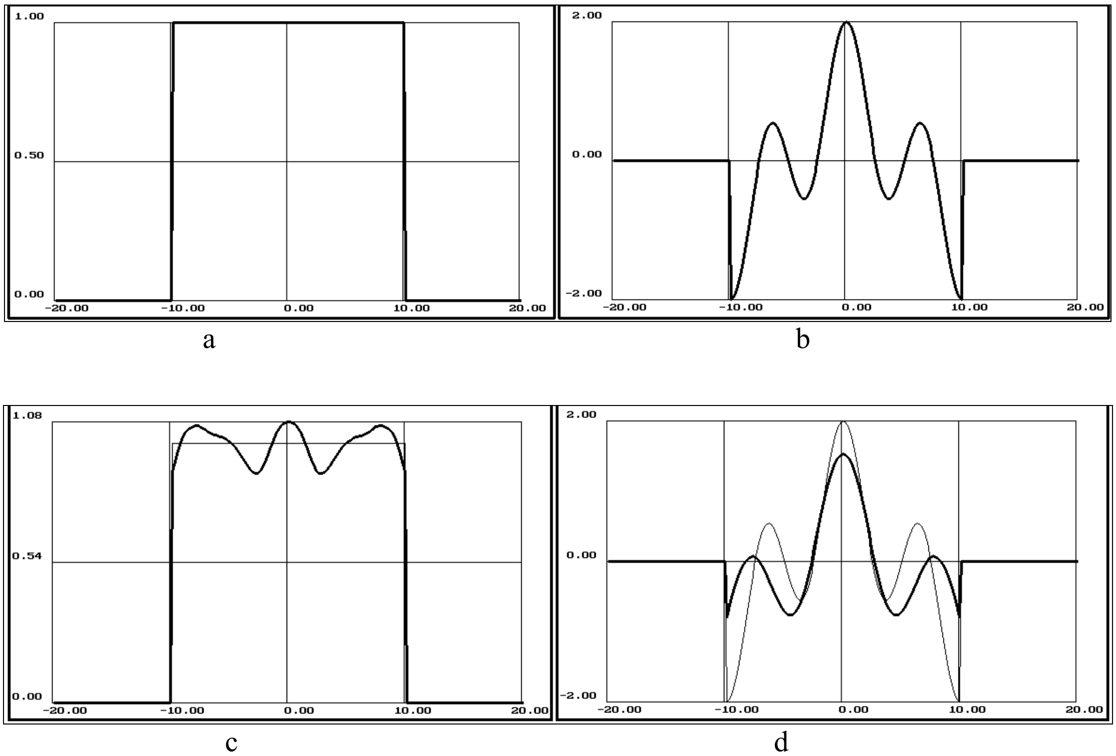

Figure1. The initial light field (intensity (a) and phase (b)) and the result of the retrieval (c, d)

2.2. Contour analysis [4]

In the spiral beams optics a so called quantization condition takes place:

$$S_{curve} = \frac{1}{2}\pi\rho^2 N,\ N = 0,\ 1,\ 2\ldots, \quad (4)$$

where S_{curve} is the area inside a closed curve. If this condition is valid, then the intensity distribution does not depend on the initial point on the curve. It's obvious, that the same property is typical for any finite sum of the corresponding Taylor series S_N. Note that for the initial contour the dependence of the finite sums on the initial point choice is crucial and represents a serious problem for the methods of the contour image processing.

These circumstances give new possibilities for the contour analysis owing to, the representation of the rotated expansion in the following form:

$$S_N(ze^{i\alpha}, \bar{z}e^{-i\alpha} \mid \zeta(t), t \in [a, a+T]) = \left(e^{-\frac{z\bar{z}}{\rho^2}} \sum_{n=0}^{N} \left(c_n e^{i\alpha n}\right) z^n \right) e^{i\psi(a)} = \left(e^{-\frac{z\bar{z}}{\rho^2}} \sum_{n=0}^{N} c'_n z^n \right) e^{i\psi(a)}, \tag{5}$$

here c_n are coefficients of the Taylor series, $\psi(a)$ is the phase shift.

Thus, the intensity of the rotated beam will be of the same form, and angle of the contour rotation is determined by the following expressions:

$$\forall n \in \overline{0, N},\ \frac{|c'_n|}{|c_n|} = 1,\ \varphi_n = \frac{1}{in} \ln \frac{c'_n}{c_n} \tag{6}$$

If $\varphi_n = \text{const}$, then this is the angle of contour rotation. Thus the representation of the initial contour as a spiral beam allows us to easily define the angle of rotation irrespective of the initial point of the contour. The outlined method is still being developed now.

References

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