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# Probing pairing correlations in Sn isotopes using Richardson-Gaudin integrability

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**Abstract.** Pairing correlations in the even-even $A = 102 - 130$ Sn isotopes are discussed, based on the Richardson-Gaudin variables in an exact Woods-Saxon plus reduced BCS pairing framework. The integrability of the model sheds light on the pairing correlations, in particular on the previously reported sub-shell structure.

## 1. Introduction

Pairing is an important component of the correlations in atomic nuclei at low-excitation energy [1, 2, 3]. The Sn isotopes provide a unique laboratory to probe the neutron-neutron pairing correlations, because the large proton shell gap at $Z = 50$ ensures that the low-lying nuclear structure is largely unaffected by proton particle-hole excitations across the shell gap. Moreover, experimental data of the Sn isotopes in three major shells have become available in recent years thanks to intensive experimental activity with radio-active beam facilities. There exist several theoretical approaches to investigate pairing correlations in atomic nuclei, ranging from fundamental ab initio calculations to studies based on a more phenomenological footing [2]. In the present contribution, we will employ a Woods-Saxon [4] plus level-independent Bardeen-Cooper-Schrieffer (BCS) pairing Hamiltonian [5, 6] as a global probe for pairing correlations in the ground state of Sn. The level-independent, or reduced, BCS Hamiltonian has a complete basis of Bethe Ansatz eigenstates [7, 8], and belongs to the class of Richardson-Gaudin (RG) integrable models [9, 10]. Integrability offers unique opportunities to investigate pairing correlations. On the one hand, the RG variables in the pair-product structure allow for a transparent graphical representation, as well as a clear-cut connection with bosonization approximations [11] via a pseudo-deformation of the quasi spin algebra [12]. On the other hand, physical observables related to particle removal and addition properties [13] can be obtained conveniently using Slavnov's theorem for the RG model [14].

## 2. Richardson-Gaudin integrability for Sn isotopes

The reduced BCS Hamiltonian is given by [1]

$$\hat{H} = \sum_{i=1}^{m} \varepsilon_i \hat{n}_i + g \sum_{i,k=1}^{m} \hat{S}_i^\dagger \hat{S}_k, \tag{1}$$

with $\hat{S}_i^\dagger = \sum_{m_i>0} (-)^{j_i-m_i} a_{j_i m_i}^\dagger a_{j_i - mi}^\dagger$ the nucleon-pair creation operator in a single-particle level $\varepsilon_i$ with (spherical) quantum numbers $(i \equiv n_i, l_i, j_i)$ and of degeneracy $\Omega_i = 2j_i + 1$. This Hamiltonian supports a complete set of Bethe Ansatz eigenstates parametrised by the set of RG variables $\{x\}$ that are a solution of the RG equations [7, 8]. The associated eigenstate energy is then given as $E = \sum_{\alpha=1}^{N_p} x_\alpha + \sum_{i=1}^{m} \varepsilon_i v_i$, with $v_i$ the seniority [15], and $N_p$ the number of pairs.

The single-particle levels are provided by a Woods-Saxon potential [4], for which we used a recent global parametrisation [16], and the single-particle energy spectrum for $^{100}$Sn is given in Table 1. We followed a global prescription $g = g_0/\sqrt{A}$ for the pairing interaction, in order to reproduce the 3-point pairing gaps $\Delta^{(3)}(A) = (-)^A[BE(A) - 2BE(A-1) + BE(A-2)]$ [4], presented in Figure 1b. The two-neutron separation energies $S_{2n} = [BE(A) - BE(A-2)]$ [4] are

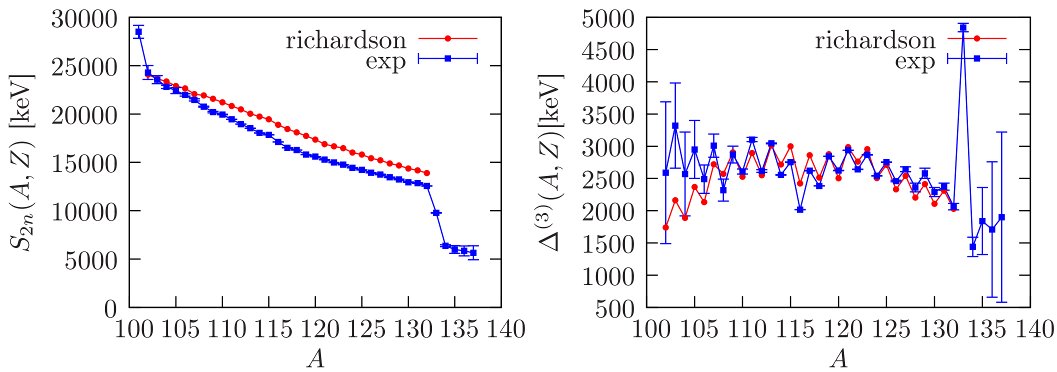


**Figure 1.** Experimental (squares) and theoretical (circles) two-neutron separation energies $S_{2n}$ (a) and three-point pairing gaps $\Delta^{(3)}$ (b). Experimental data taken from [17].

given in Figure 1a, following a general linear trend, with the exception of a small kink around mid shell, signaling a sub-shell closure. The calculated curve is smoother than the experimental values at this point, consistent with the overestimated pairing gaps $\Delta^{(3)}$ around mid shell. Recent measurements showed a decrease in the $B(E2 : 0_1^+ \to 2_1^+)$ strength around mid shell [18], which was qualitatively attributed [19] to this sub-shell effect in the seniority scheme [15]. Figure 2 depicts the RG variables for the ground state of the even-even $^{102\text{-}130}$Sn isotopes, and sheds more light on the sub-shell structure. Weakly correlated pair states give rise to a clustering of RG variables around the single-particle poles in the complex plane, whereas collective pairing states organise the RG variables along a broad arc in the complex plane [10, 12]. The pairing interaction in the lighter isotopes is strong enough to distribute the RG variables along an arc in the complex plane, however the arc only extends over the $d_{5/2}$ and $g_{7/2}$ sub-shell single particle poles. For the heavier nuclei, the pairs separate into two distinct sets, with seven RG variables clustering around the $d_{5/2}$ and $g_{7/2}$ sub-shell poles and the remaining forming a collective arc around the other poles. For medium-heavy nuclei, there is a gradual transition between both situations. This structure can be quantified using the pseudo-deformation scheme, where all RG variables can be labeled according to their collective behaviour in the Tamm-Dancoff Approximation (TDA) (see Table 1) [12]. From the table, it can be seen that the TDA structure is consistent

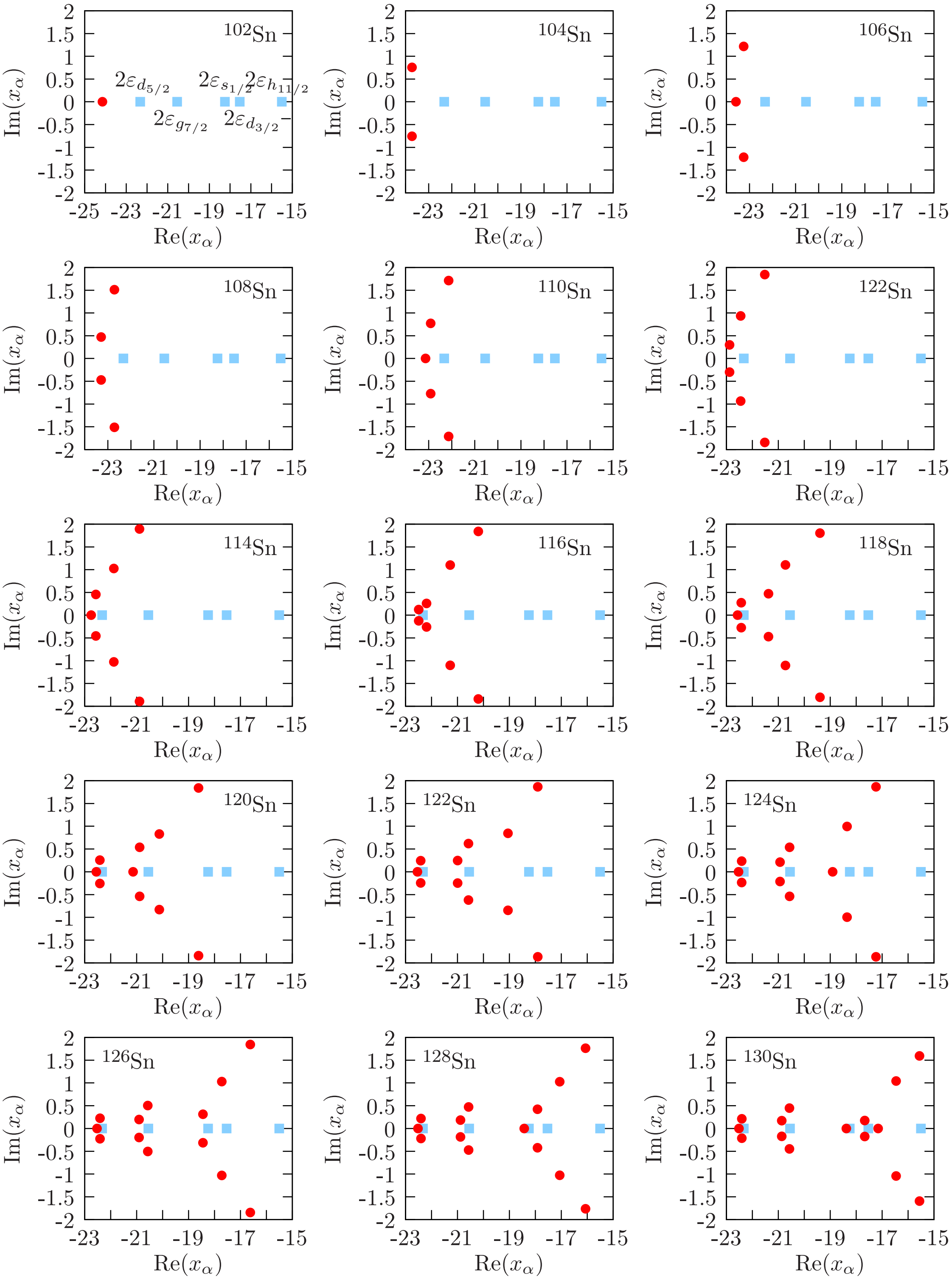


**Figure 2.** The RG variables (circles) and single-particle poles (squares) of the even-even $^{102\text{-}130}$Sn isotopes.

with the discussed sub-shell structure. The lightest isotopes are consistent with a collective TDA condensation in the $d_{5/2}$ and $g_{7/2}$ sub shell, whereas the TDA structure of the heavier isotopes points towards a normal filling of the $d_{5/2}$ and $g_{7/2}$ sub shell, with the additional pairs collectively distributed over the $s_{1/2}$, $d_{3/2}$ and $h_{11/2}$ sub shell.

| level | $\varepsilon_i$ [MeV] | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_{5/2}$ | -11.164 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $g_{7/2}$ | -10.275 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 7 | 4 | 4 | 4 | 4 | 4 |
| $s_{1/2}$ | -9.124 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 5 | 6 | 1 | 1 |
| $d_{3/2}$ | -8.766 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 2 |
| $h_{11/2}$ | -7.754 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |

**Table 1.** The single-particle energies $\varepsilon_i$ obtained from a Woods-Saxon potential [16], and the TDA eigenmode decomposition of the $0^+$ground state for the even-even isotopes $^{102\text{-}130}$Sn. The number of active pairs $N_p$ in the isotope $^{\mathrm{A}}$Sn is given in the upper row ($N_p = (A-50)/2$).

## 3. Conclusions

We have investigated pairing correlations in the Sn isotopes by inspecting the location of the RG variables with respect to the single-particle poles in the complex plane, generated by a schematic Woods-Saxon plus reduced BCS Hamiltonian. The results point towards a sub-shell structure, consistent with previous studies. We expect this structure to also be reflected in the relevant transition rates; this will be investigated in future publications

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