

# Recent progresses on the pseudospin symmetry in single particle resonant states

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**Abstract.** The pseudospin symmetry (PSS) is a relativistic dynamical symmetry directly connected with the small component of the nucleon Dirac wave function. Much effort has been made to study this symmetry in bound states. Recently, a rigorous justification of the PSS in single particle resonant states was achieved by examining the asymptotic behaviors of the radial Dirac wave functions: The PSS in single particle resonant states in nuclei is conserved exactly when the attractive scalar and repulsive vector potentials have the same magnitude but opposite sign. Several issues related to the exact conservation and breaking mechanism of the PSS in single particle resonances were investigated by employing spherical square well potentials in which the PSS breaking part can be well isolated in the Jost function. A threshold effect in the energy splitting and an anomaly in the width splitting of pseudospin partners were found when the depth of the square well potential varies from zero to a finite value.

## 1. Introduction

In 1969, the pseudospin symmetry (PSS) was found in atomic nuclei: Doublets of single particle levels with quantum numbers  $(n_r, l, j = l + 1/2)$  and  $(n_r - 1, l + 2, j = l + 3/2)$  in the same major shell are nearly degenerate [1, 2]. Since the PSS was observed, much effort has been devoted to investigate its origin [3, 4, 5, 6, 7]. Finally, it was revealed that the PSS in nuclei is a relativistic symmetry which is exactly conserved when the scalar potential  $S(r)$  and the vector potential  $V(r)$  have the same size but opposite sign, i.e.,  $\Sigma(r) \equiv S(r) + V(r) = 0$  [8], or, more generally, when  $d\Sigma(r)/dr = 0$  [9, 10]. The PSS is usually viewed as a dynamical symmetry [11, 12] because, in either limit,  $\Sigma(r) = 0$  or  $d\Sigma(r)/dr = 0$ , there are no bound states in realistic nuclei, thus in realistic nuclei the PSS is always broken.

The spin symmetry (SS) and PSS have been investigated extensively. The readers are referred to Ref. [13] for a review and to Refs. [14, 15] for overviews of recent progresses. Here we simply mention that the study of the PSS has been extended along several lines:

- (i) From single particle states in the Fermi sea to those in the Dirac sea, i.e., the SS in anti-nucleon spectra [16, 17, 18, 19].
- (ii) From normal nuclei to hypernuclei [20, 21, 22].
- (iii) From spherical systems to deformed ones [23, 24, 25, 26].
- (iv) From bound states to resonant states; next we will focus on this extension.



There has been a great interest in the exploration of continuum and resonant states in atomic nuclei [27, 28, 29]. The study of symmetries, e.g., the PSS, in resonant states is very interesting. There have been some numerical investigations of the PSS in single particle resonances [30, 31, 32, 33, 34] and the SS in single particle resonant states [35]. Recently, we gave a rigorous justification of the PSS in single particle resonant states and investigated several open problems related to the exact conservation and breaking mechanism of the PSS in single particle resonances [36, 37, 15]. In this contribution, we will present some results concerning the PSS in single particle resonant states.

## 2. Justification of the PSS in resonant states

In atomic nuclei, the equation of motion for nucleons, the Dirac equation, reads,

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + \beta(M + S(\mathbf{r})) + V(\mathbf{r})] \psi(\mathbf{r}) = \epsilon \psi(\mathbf{r}), \quad (1)$$

where  $\boldsymbol{\alpha}$  and  $\beta$  are Dirac matrices,  $M$  is the nucleon mass, and  $S(\mathbf{r})$  and  $V(\mathbf{r})$  the scalar and vector potentials.

For a spherical nucleus, the upper and lower components of the Dirac spinor are  $iF_{n\kappa}(r)/r \cdot Y_{jm}^l(\theta, \phi)$  and  $-G_{\tilde{n}\kappa}(r)/r \cdot Y_{jm}^{\tilde{l}}(\theta, \phi)$ , respectively. One can derive a Schrödinger-like equation for the lower component,

$$\left[ \frac{d^2}{dr^2} - \frac{1}{M_-(r)} \frac{d\Sigma(r)}{dr} \frac{d}{dr} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \frac{1}{M_-(r)} \frac{\kappa}{r} \frac{d\Sigma(r)}{dr} - M_+(r)M_-(r) \right] G(r) = 0, \quad (2)$$

where  $M_+(r) = M + \epsilon - \Delta(r)$  and  $M_-(r) = M - \epsilon + \Sigma(r)$ .

For the continuum in the Fermi sea,  $\epsilon \geq M$ ; the regular solution for Eq. (2) vanishes at the origin and at large  $r$ ,

$$G(r) = \frac{i}{2} \left[ \mathcal{J}_\kappa^G(p) h_{\tilde{l}}^-(pr) - \mathcal{J}_\kappa^G(p)^* h_{\tilde{l}}^+(pr) \right], \quad r \rightarrow \infty, \quad (3)$$

where  $h_{\tilde{l}}^\pm(pr)$  are the Ricatti-Hankel functions.  $\mathcal{J}_\kappa(p)$  is the Jost function which is an analytic function of  $p$ . The zeros of  $\mathcal{J}_\kappa^G(p)$  on the lower  $p$  plane and near the real axis correspond to resonant states. The resonance energy  $E$  and width  $\Gamma$  are determined by the relation  $E - i\Gamma/2 = \epsilon - M$ .

In the PSS limit, Eq. (2) is reduced as

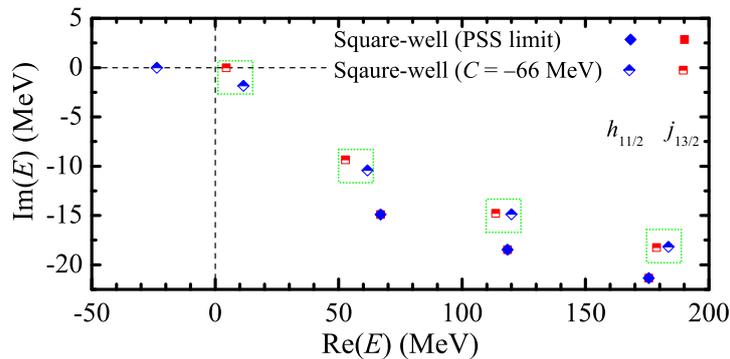
$$\left[ \frac{d^2}{dr^2} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + (\epsilon - M) M_+(r) \right] G(r) = 0. \quad (4)$$

For pseudospin doublets with  $\kappa$  and  $\kappa' = -\kappa + 1$ , the small components satisfy the same equation because they have the same pseudo orbital angular momentum  $\tilde{l}$ . For continuum states,  $G_\kappa(\epsilon, r) = G_{\kappa'}(\epsilon, r)$  is true for any energy  $\epsilon$  thus we have  $\mathcal{J}_{\kappa'}^G(p) = \mathcal{J}_\kappa^G(p)$ . Thus the zeros are the same for  $\mathcal{J}_{\kappa'}^G(p)$  and  $\mathcal{J}_\kappa^G(p)$  and the PSS in single particle resonant states in nuclei is exactly conserved [36].

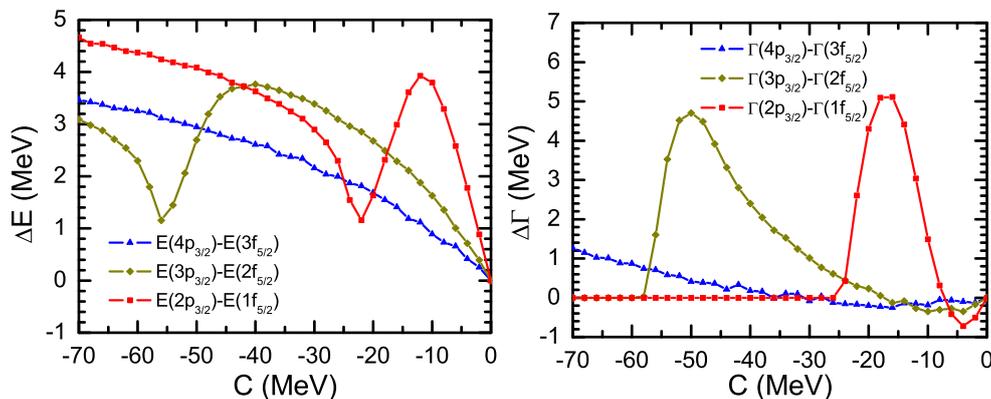
## 3. Numerical Results and Discussions

Spherical square well potentials for  $\Sigma(r)$  and  $\Delta(r)$  read,

$$\Sigma(r) = \begin{cases} C, & r < R, \\ 0, & r \geq R, \end{cases} \quad \Delta(r) = \begin{cases} D, & r < R, \\ 0, & r \geq R, \end{cases} \quad (5)$$



**Figure 1.** (Color online) The zeros of the Jost function  $\mathcal{J}_\kappa^G$  on the complex energy plane in square-well potentials (5) with  $C = 0$  (solid symbols) and  $C = -66$  MeV (half-filled symbols) for PS partners  $h_{11/2}$  (diamond) and  $j_{13/2}$  (square).



**Figure 2.** (Color online) The energy (the left panel) and width (the right panel) splitting between PSS partners with the pseudo orbital angular momentum  $\tilde{l} = 2$ ,  $p_{3/2}$  and  $f_{5/2}$ , as a function of the potential depth  $C$ . Taken from Ref. [15].

where  $C$  and  $D$  are depths and  $R$  is the width. The Jost function  $\mathcal{J}_\kappa^G(p)$  is derived as [36],

$$\mathcal{J}_\kappa^G(p) = -\frac{p^{\tilde{l}}}{2ik^{\tilde{l}+1}} \left\{ j_{\tilde{l}}(kR)ph_{\tilde{l}}^{\pm'}(pR) - kj_{\tilde{l}}'(kR)h_{\tilde{l}}^{\pm}(pR) - b \left[ kj_{\tilde{l}}'(kR) - \frac{\kappa}{R}j_{\tilde{l}}(kR) \right] h_{\tilde{l}}^{\pm}(pR) \right\}, \quad (6)$$

with  $b = C/(\epsilon - M - C)$  and  $k = \sqrt{(\epsilon - C - M)(\epsilon - D + M)}$ . By examining the zeros of the Jost function (6), one gets single particle bound states and resonant states.

Figure 1 shows solutions for PSS doublets with  $\tilde{l} = 6$ , i.e.,  $h_{11/2}$  with  $\kappa = -6$  and  $j_{13/2}$  with  $\kappa' = 7$  for square-well potentials with  $D = 650$  MeV and  $R = 7$  fm. The conservation of the PSS for single particle resonant states is seen: When  $C = 0$ , the roots for  $h_{11/2}$  and  $j_{13/2}$  are the same. When  $C = -66$  MeV, the PSS is broken.

By examining the Jost function, one can trace continuously the PSS partners from the PSS limit to the case with a finite potential depth. This has been done in Ref. [15] in which we found a threshold effect in the energy splitting and an anomaly in the width splitting of pseudospin partners when  $C$  varies from zero to a finite value. As the depth of the single particle potential becomes deeper, the PSS is broken more and a threshold effect in the energy splitting appears which can be seen in Fig. 2: The energy splitting first increases then decreases

until the pseudospin doublets encounter the threshold where one of the levels becomes a bound state and the splitting takes a minimum value; When the potential becomes even deeper, the splitting increases again. When the depth of the single particle potential increases from zero, there also appears an anomaly in the width splitting of PS partners which is shown in Fig. 2: It first decreases from zero to a maximum value with negative sign, then increases and becomes zero again; after the inversion of the width splitting, the splitting increases and reaches a maximum and positive value, then it becomes smaller and finally reaches zero. Note that for the doublet  $4p_{3/2}$  and  $3f_{5/2}$ , the threshold is much below  $-70$  MeV and in Fig. 2 the splittings in energy and width always increase with  $|C|$  increasing.

Finally we mention that there are still many open problems concerning the exact and breaking of the PSS in single particle resonant states; the readers are referred to Refs. [15, 37] for further discussions.

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