

# Negative parity states in the IVBM

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**Abstract.** The low-lying spectra including the first few excited positive and negative parity bands of some heavy even-even nuclei from the rare earth and actinide mass regions are investigated within the framework of the symplectic Interacting Vector Boson Model with  $Sp(12, R)$  dynamical symmetry group. Symplectic dynamical symmetries allow the change of the number of excitation quanta or phonons building the collective states providing for larger representation spaces and richer subalgebraic structures to incorporate more complex nuclear spectra. The theoretical predictions for the energy levels and transition probabilities between the collective states are compared with experiment and some other collective models incorporating octupole degrees of freedom.

## 1. Introduction

It is well known [1],[2] that in some mass regions several bands of negative parity are observed in the low-lying nuclear spectra, like  $K^\pi = 0^-, 1^-$  and  $2^-$  bands. The most well-studied of them is the  $K^\pi = 0^-$  band, usually interpreted as an octupole vibrational band, connected to the ground state band (GSB) by enhanced  $E1$  transitions.

Negative parity states have been described within different approaches mainly by inclusion of octupole or/and dipole degrees of freedom. The bands of negative parity states are often associated with the reflection asymmetry in the intrinsic frame of reference. In the geometrical approach this is achieved by including the  $\alpha_{30} \equiv \beta_3$  deformation [3]. In the Interacting Boson Model (IBM) the description of negative states requires the introduction of  $p$  or/and  $f$  bosons with negative parity in addition to the standard  $s$  and  $d$  bosons (*spdf*-IBM) [4],[5]. An alternative interpretation of the low-lying negative parity states has been provided in different cluster models [6]-[7] in which the dipole degrees of freedom are related with the relative motion of the clusters. Based on the Bohr Hamiltonian different critical point symmetries (CPS) including quadrupole and octupole deformations have been proposed [8]-[10], [11] extending the concept of CPS introduced for the description of positive parity states.

In the present paper we give a unified description of the first few excited low-lying positive and negative parity bands in the even-even nuclei from the the rare earth and actinide mass regions within the framework of the symplectic Interacting Vector Boson Model (IVBM) with  $Sp(12, R)$  dynamical symmetry group [12]. It is shown that the negative parity bands arise along with the positive bands (ground state,  $\beta$  and  $\gamma$ ) without the introduction of any additional collective degrees of freedom.

## 2. The IVBM

It was suggested by Bargmann and Moshinsky [13] that two types of bosons are needed for the description of nuclear dynamics. It was shown there that the consideration of



only two-body system consisting of two different interacting vector particles will suffice to give a complete description of  $N$  three-dimensional oscillators with a quadrupole-quadrupole interaction. The latter can be considered as the underlying basis in the algebraic construction of the *phenomenological* IVBM [12].

The algebraic structure of the IVBM is realized in terms of creation and annihilation operators of two kinds of vector bosons  $u_m^\dagger(\alpha)$ ,  $u_m(\alpha)$  ( $m = 0, \pm 1$ ), which differ in an additional quantum number  $\alpha = \pm 1/2$  (or  $\alpha = p$  and  $n$ )—the projection of the  $T$ —spin (an analogue to the  $F$ —spin of IBM-2 or the  $I$ —spin of the particle-hole IBM). All bilinear combinations of the creation and annihilation operators of the two vector bosons generate the boson representations of the non-compact symplectic group  $Sp(12, R)$ . Its irreducible representations are infinite dimensional.

We consider the following chain [12],[14]

$$Sp(12, R) \supset U(6) \supset SU(3) \otimes U(2) \supset SO(3) \otimes U(1),$$

$$[N]_6 \quad (\lambda, \mu) \Longleftrightarrow (N, T) \quad K \quad L \quad T_0 \quad (1)$$

where below the different subgroups the quantum numbers characterizing their irreducible representations are given. We note that within the symmetric irreducible representation  $[N]_6$  of  $U(6)$  the groups  $SU(3)$  and  $U(2)$  are mutually complementary [15], i.e. the quantum numbers of the two groups are related:  $N = \lambda + 2\mu$  and  $T = \lambda/2$ . Making use of the latter we can write the basis as

$$|[N]_6; (\lambda, \mu); K, L; T_0\rangle = |(N, T); K, L; T_0\rangle \quad (2)$$

The basis states associated with the even irreducible representation of the  $Sp(12, R)$  can be constructed by the application of powers of raising generators  $F_M^L(\alpha, \beta) = \sum_{k,m} C_{1k1m}^{LM} u_k^+(\alpha) u_m^+(\beta)$  of the same group on the vacuum. Each raising operator will increase the number of bosons  $N$  by two. The resulting infinite set of basis states so obtained is denoted as (2) and is shown in Table 1.

**Table 1.** Classification of the basis states.

$N$	$T$	$T_0 \dots$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$\dots$
0	0					(0, 0)				
2	1				(2, 0)	(2, 0)	(2, 0)			
	0					(0, 1)				
4	2			(4, 0)	(4, 0)	(4, 0)	(4, 0)	(4, 0)		
	1				(2, 1)	(2, 1)	(2, 1)			
	0					(0, 2)				
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

We use the following Hamiltonian [14]:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2 \quad (3)$$

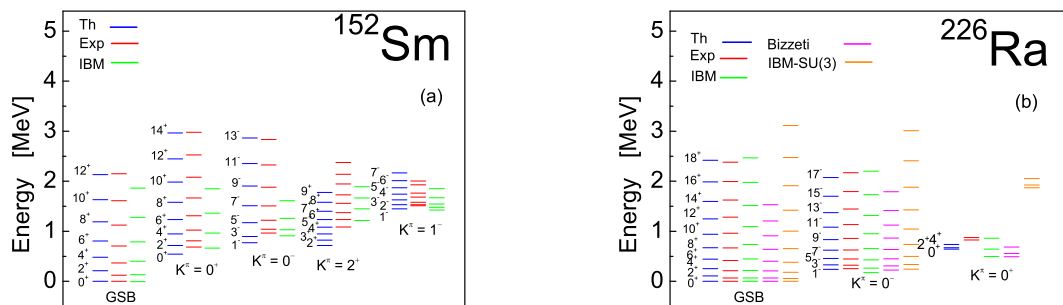
expressed in terms of the first and second order invariant operators of the different subgroups in the chain (1). It is obviously diagonal in the basis (2) and its eigenvalues are just the energies of the nuclear system:

$$E(N, L, T, T_0) = aN + bN^2 + \alpha_3 T(T + 1) + \beta_3 L(L + 1) + \alpha_1 T_0^2. \quad (4)$$

### 3. Application

In our application, the most important point is the identification of the experimentally observed states with a certain subset of the basis states (2). In this regard, the following two points are of importance. As we noted, the irreducible representations of  $Sp(12, R)$  are infinite dimensional. Then the natural question arises -how to select a certain subset of the basis states given in Table 1? It turns out that an appropriate set of states is given by the so called "stretched states" [16], which are defined as the  $SU(3)$  states of the type  $(\lambda, \mu) = (\lambda_0, \mu_0 + k)$ , where  $k = 0, 2, 4, \dots$ . The latter represent dominant  $SU(3)$  multiplets in the low-lying collective states. In the symplectic IVBM the change of the number  $k$ , which is related in the applications to the angular momentum  $L$  of the states, gives rise to the collective bands. The second point concerns the parity of the state. We assume that the one type of two vector bosons, say  $p$ -boson, transforms under space reflections as a pseudovector, while the other -  $n$ -boson - transforms as a vector. So, we define the parity of the considered collective state as  $\pi = (-1)^{N_n}$  which generalizes our previous definition of the parity  $\pi = (-1)^T$  given in Ref.[14]. This allows us to describe both positive and negative parity states in the IVBM on the same footing without introducing of any additional collective degrees of freedom.

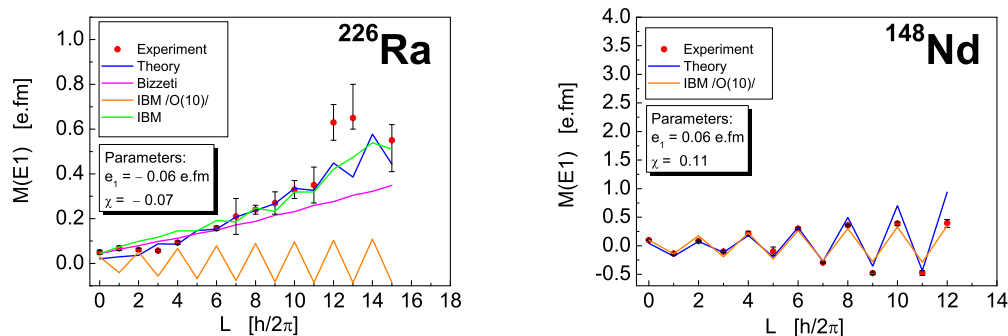
We consider the first few excited low-lying positive (ground state,  $\beta$ ,  $\gamma$ ) and negative ( $K^\pi = 0^-, 1^-$ ) parity bands. We take two examples for which there is enough experimental data on  $E1$  transitions. In Fig.1(a) we compare our theoretical predictions for the excitation energies in  $^{152}\text{Sm}$  with experiment [1] and the results obtained by the diagonalization of the  $spdf$ -IBM Hamiltonian [17]. This nucleus in the positive parity part of the spectrum is considered as an example of  $X(5)$  CPS [18]. The results obtained by the CPS approach [10] are also shown in Fig.1(a). The next example is  $^{226}\text{Ra}$  considered in the literature as possessing a stable octupole shape. In Fig.1(b) we compare the results obtained by us in the IVBM and the pure  $SU(3)$  dynamical limit of the  $spdf$ -IBM for this nucleus with experiment [19]. Calculations in the  $SU(3)$  limit of  $spdf$ -IBM are performed using the Hamiltonian and matrix elements given in [21]. For comparison, the predictions of the diagonalized  $spdf$ -IBM Hamiltonian [20] and CPS approach [9] are also shown. One sees that the IVBM describes reasonably well the low-lying excitation energies of the two nuclei under consideration. Note that in the case of  $^{226}\text{Ra}$ , the experimental data show large deviations from the rotational  $L(L+1)$  rule ( $SU(3)$  limit of the  $spdf$ -IBM) for both the ground state and  $K^\pi = 0^-$  bands despite the fact that  $R_{4/2} = 3.13$ .



**Figure 1.** (Color online) Comparison of the theoretical energies for the first few low-lying positive and negative parity bands in  $^{152}\text{Sm}$  and  $^{226}\text{Ra}$  with experiment and some other collective models incorporating octupole degrees of freedom.

It is well known that the transition probabilities are more sensitive test for each model.

Negative parity states of the  $K^\pi = 0^-$  band are characterized by the enhanced  $E1$  transitions to the GSB. Thus, we consider only the  $E1$  transitions characteristic for the octupole/dipole collectivity. In Fig.2 we compare our theoretical results for the matrix elements of the interband  $E1$  transitions between the states of the GSB and  $K^\pi = 0^-$  band in  $^{226}\text{Ra}$  and  $^{148}\text{Nd}$  with experiment [19], [1] and some other collective models. We see that IVBM describes well the experimental data.



**Figure 2.** (Color online) Theoretical and experimental values for the matrix elements of the interband  $E1$  transitions between the states of the GSB and  $K^\pi = 0^-$  band in  $^{226}\text{Ra}$  and  $^{148}\text{Nd}$ . For comparison, the theoretical predictions of some other collective models incorporating octupole degrees of freedom are also shown.

#### 4. Conclusions

In the present work the low-lying spectra including the first few excited positive and negative parity bands of some heavy even-even nuclei from the rare earth and actinide mass regions are investigated within the framework of the symplectic IVBM with  $Sp(12, R)$  dynamical symmetry group. Symplectic dynamical symmetries allow the change of the number of excitation quanta or phonons building the collective states providing for larger representation spaces and richer subalgebraic structures to incorporate more complex nuclear spectra. The theoretical predictions for the energy levels and matrix elements of the  $E1$  transitions between the states of the GSB and  $K^\pi = 0^-$  band are compared with experiment and some other collective models incorporating octupole degrees of freedom. The IVBM describes well the experimental data. The results obtained for the energy levels and the  $E1$  transitions of the considered nuclei prove the correct mapping of the basis states to the experimentally observed ones and reveal the relevance of the used dynamical symmetry of the IVBM in the description of negative parity states.

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