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Supersymmetry: superfield equations of motion

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Abstract. Supersymmetry and superfields are considered in connection with Poincaré superalgebras. A formalism of projection operators for deriving wave equations for ordinary fields and superfields is developed. Superfield equations of motion in the case of massive and massless fields are presented together with an application in linear supergravity.

1. Supersymmetry, superfields, Poincaré superalgebra

Supersymmetry has been known more than few decades, but it is still alive despite the absence of experimental verification. Supersymmetry offers several relevant theoretical solutions for modern physics.

1.1. Supersymmetry (Bose-Fermi symmetry)

Let us consider a general $N = 1$ superfield

$$\phi_i(x,\theta) = A_i(x) + \bar{\theta}\psi_i(x) + \bar{\theta}\theta F_i(x) + \bar{\theta}\gamma^5\theta G_i(x) + \bar{\theta}i\gamma^\mu\gamma^5\theta A_{\mu i} + \bar{\theta}\theta\bar{\theta}\chi_i(x) + (\bar{\theta}\theta)^2 D_i(x)$$

where θ_α is a four-component anticommuting Majorana spinor and i is a Lorentz index.

Now we introduce the most general Poincaré superalgebra [1]. The generators of the Poincaré group P^μ and $M^{\mu\nu}$, and n supergenerators S_α $(\alpha = 1, 2, \ldots, n)$ satisfy the following relations:

$$\begin{aligned}
\left[P^\mu, P^\nu\right] &= 0 \\
\left[M^{\mu\nu}, P^\rho\right] &= \eta^{\nu\rho}P^\mu - \eta^{\mu\rho}P^\nu \\
\left[M^{\mu\nu}, M^{\rho\sigma}\right] &= \eta^{\nu\rho}M^{\mu\sigma} + \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho} \\
\left[S_\alpha, P^\mu\right] &= 0 \\
\left[S_\alpha, M^{\mu\nu}\right] &= B^{\mu\nu}_{\alpha\beta}S_\beta
\end{aligned}$$

where $\eta^{\mu\nu} = \mathrm{diag}(+ - - -)$.

The simple $N = 1$ supersymmetry algebra is related to the bispinor representation of the Lorentz group. We have four bispinor generators S_α, $B^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu}$ and $A_\mu = \gamma_\mu C$. The N-extended Poincaré superalgebra is related to the direct sum of N bispinor representations.

1.2. New possibilities [2]

- In the most general case the matrices A_μ are related to the β-matrices of an invariant first order wave equation

$$(i\partial_\mu\beta^\mu - m)\,\psi(x) = 0$$

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in the following way: $A_\mu = \beta_\mu C$. Here $\psi(x)$ corresponds to a representation of the Lorentz group and $B^{\mu\nu}$ are the Lorentz generators of the corresponding representation.

- If we consider new possibilities for $N = 1$ superalgebras, we can take the Rarita-Schwinger equation for a vector-bispinor field and use the β-matrices of the Rarita-Schwinger equation

$$(\beta^\mu)^\rho_\sigma = \gamma^\mu \eta^\rho_\sigma + \left(\frac{a}{\sqrt{3}} - \frac{1}{2}\right)\eta^{\mu\rho}\gamma_\sigma + \left(\frac{b}{\sqrt{3}} - \frac{1}{2}\right)\gamma^\rho \eta^\mu_\sigma + \left(\frac{a+b}{4\sqrt{3}} + \frac{c}{4} + \frac{3}{8}\right)\gamma^\rho\gamma^\mu\gamma_\sigma$$

- If we take an irreducible representation for generators S_α (vector representation $(1/2,\, 1/2)$, e.g), then there is no first order wave equation and we get $\{S_\alpha, S_\beta\} = 0$.
- If we consider the Kemmer-Duffin equation for spin 0, $\psi(x)$ is a direct sum of vector and scalar fields and therefore 5 generators S_α also correspond to vector and scalar representations. As a result we get the Bose superalgebra.

Let us turn to the $(N = 1)$ Poincaré supergroup and algebra. It is well known that there are the following physically interesting irreducible representations: massive representations (m, Y), where superspin Y gives Poincaré spins $s = Y + 1/2$, Y , $Y - 1/2$, and the massless representations $(0, Y)$, where we have the helicities $s = Y + 1/2$ and Y.

The problem is: if we consider different Lorentz superfields $\varphi_i(x, \theta)$, we need additional conditions to guarantee that our field or combination of fields describe physical states with a given mass and superspin. Such additional conditions are relativistic wave equations. The most familiar of them for ordinary fields is the Dirac equation, which describes a fermion field with a given rest mass m and spin $1/2$.

We have developed a quite powerful method of constructing and analyzing relativistically invariant wave equations for fields and superfields to describe states with a certain mass and spin (superspin), and also wave equations for massless fields. Our method is based on spin projection and superspin projection operators. In order to understand the main principles of our method, we first describe its ideas used in the case of ordinary fields and then generalize them to the case of superfields.

2. Ordinary Lorentz fields, wave equations, spin projection operators

We consider a finite dimensional Lorentz field $\psi(x)$ and demand, that $\psi(x)$ is a solution of the n-th order wave equation [3]

$$i\partial_{\mu_1} \ldots i\partial_{\mu_n} \beta^{\mu_1 \ldots \mu_n}\psi(x) = m^n \psi(x)$$

where the matrices $\beta^{\mu_1 \ldots \mu_n}$ satisfy the commutation relations

$$\left[S^{\mu\nu}, \beta^{\mu_1 \ldots \mu_n}\right] = \sum_l \left(\eta^{\nu\mu_l}\beta^{\mu_1 \ldots \mu \ldots \mu_n} - \eta^{\mu\mu_l}\beta^{\mu_1 \ldots \nu \ldots \mu_n}\right)$$

The most simple example of equations is the well-known Dirac equation.

It is easy to verify that the wave equation determines the mass of a given physical state. If we take the momentum representation and consider a $\hat{p}$-system with momentum $\hat{p}^\mu = (\epsilon m_i, \vec{0})$ (the rest system of a particle), the wave equation reduces to the eigenvalue problem of the matrix $\beta^{0 \ldots 0}$

$$\beta^{0 \ldots 0}\psi = \left(\frac{\epsilon m}{m_i}\right)^n \psi \quad \text{or} \quad \beta^{0 \ldots 0}\psi = \lambda_i \psi$$

Now it follows that to each nonzero eigenvalue λ_i there corresponds a nontrivial solution ψ_i, which describes masses $m_i = \epsilon m (\lambda_i)^{-1/n}$.

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When dealing with equations it is natural to suppose that no other subsidiary conditions are imposed, because it is well known that wave equations with additional restrictions on fields are in general inconsistent when interactions with other fields are introduced. Also it follows from the above given relation for masses that only the first and second order equations may have a physical (real) mass spectrum. Using relativistic wave equations we define the proper Poincaré basis. It follows that $\beta^{0\ldots 0}$ commutes with the Lorentz generators S^{kl}, $(k,l = 1,2,3)$ (spin generators). The Poincaré basis $\psi_{\mu_i s\sigma}$ at rest is defined as the eigenfunction of three operators $\beta^{0\ldots 0}$, $\vec{S}^2$ and S^3:

$$\beta^{0\ldots 0}\psi_{m_i s\sigma} = \lambda_i \psi_{m_i s\sigma}, \quad \vec{S}^2\psi_{m_i s\sigma} = s(s+1)\psi_{m_i s\sigma} \quad S^3\psi_{m_i s\sigma} = \sigma\psi_{m_i s\sigma}$$

The state of an arbitrary momentum $p = L\hat{p}$ is obtained from the rest state $\psi_{m_i s\sigma}$ via the boost transformation.

Dealing with irreducible Lorentz fields $\psi_i, \psi_j, \ldots$, we introduce covariant spin projection operators P^s_{ij}, satisfying (*no sum* over j)

$$P^s_{ij}P^{s'}_{jk} = \delta_{ss'}P^s_{ik}$$

where P^s_{ii} are ordinary projection operators that extract spin s from ψ_i, i.e. $\psi^s_i = P^s_{ii}\psi_i$, and P^s_{ij} $(i \neq j)$ are spin transition operators, i.e. $\psi^s_i = P^s_{ij}\psi_j$. Now the n-th order wave equation previously introduced is presented in a general matrix form

$$(-\Box)^{n/2}\beta^{0\ldots 0}(\partial)\psi(x) = m^n\psi(x)$$

where

$$\beta^{0\ldots 0}(\partial) = \begin{pmatrix} a_{11}P_{11} & \ldots & a_{1n}P_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1}P_{n1} & \ldots & a_{nn}P_{nn} \end{pmatrix}, \quad \psi(x) = \begin{pmatrix} \psi_1(x) \\ \vdots \\ \psi_n(x) \end{pmatrix}$$

Here a_{ij} are arbitrary numerical coefficients and operators P_{ij} are

$$P_{ij} = \sum_s a_{ij}(s)P^s_{ij}$$

In the first order equations case $a_{ij}(s)$ are uniquely determined, in the second order case there is some freedom of choice for $a_{ij}(s)$.

Example 2.1 (spin 3/2)**.** Spin 3/2 equations for a vector-bispinor field $\psi_{\mu\alpha}(x)$ may be represented in the following *two-component* form:

$$i\,\Box^{1/2}\begin{pmatrix} P^{3/2}_{11} + \frac{1}{2}P^{1/2}_{11} & aP^{1/2}_{12} \\ bP^{1/2}_{21} & cP^{1/2}_{22} \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = m\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

where 1 is a representation $1 = (1,\ 1/2)\oplus(1/2,\ 1)$ and $2 = (1/2,\ 0)\oplus(0,\ 1/2)$. Depending on the choice of free real parameters a, b and c we get a lot of different equations. For example, if $ab = -1/4$ and $c = -1/2$ we have a single spin 3/2 equation (Rarita-Schwinger equation). In the case of $ab = c/2$ we have an equation which describes fields of spin 3/2 with mass m and of spin 1/2 with mass $m' = m/|c + 1/2|$ (the latter equations are important in the massless case). There are also equations describing spin 3/2 and two spin 1/2 fields with different masses. The

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properties of spin 1/2 are determined by the eigenvalues of the 2×2 spin 1/2 matrix $\begin{pmatrix} 1/2 & a \\ b & c \end{pmatrix}$. The covariant form of the above given equation is

$$(i\partial - m)\psi^\mu + i\left(\frac{a}{\sqrt{3}} - \frac{1}{2}\right)\partial^\mu\gamma_\nu\psi^\nu + i\left(\frac{b}{\sqrt{3}} - \frac{1}{2}\right)\gamma^\mu\partial_\nu\psi^\nu + i\left(\frac{3}{8} + \frac{c}{4} - \frac{a+b}{4\sqrt{3}}\right)\gamma^\mu\partial\gamma_\nu\psi^\nu = 0$$

At that the invariant bilinear form and Lagrangian are given, which means that one can develop the usual Lagrangian formalism for free and interacting fields, as well as derive Green's functions.

Example 2.2 (spin 2)**.** In the spin 2 case the second order wave equation has a matrix form

$$\Box\begin{pmatrix} P_{11}^2 - \frac{1}{2}P_{11}^0 & aP_{12}^0 \\ bP_{21}^0 & cP_{00}^0 \end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} + m^2\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} = 0$$

Its covariant form for a symmetric tensor field is

$$\begin{aligned}\Box h^{\mu\nu} \quad &- \quad \partial^\mu\partial_\rho h^{\rho\nu} - \partial^\nu\partial_\rho h^{\rho\mu} + \left(\frac{1}{2} - \frac{a}{\sqrt{3}}\right)\partial^\mu\partial^\nu h^\rho_\rho + \left(\frac{1}{2} - \frac{b}{\sqrt{3}}\right)\eta^{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} \\ &+ \quad \left(\frac{a+b}{4\sqrt{3}} + \frac{c}{4} - \frac{3}{8}\right)\eta^{\mu\nu}h^\rho_\rho + m^2 h^{\mu\nu} = 0\end{aligned}$$

Depending on the choice of a, b and c we get different equations, describing either a single spin 2, or spin 2 and spin 0.

Similarly one can derive equations for higher spins: 5/2, 3 etc. These are of course more complex.

3. Massless gauge invariant equations

Let us consider the problem of massless fields [4]. It turns out that the same formalism of spin projection operators is useful to derive massless gauge invariant equations for a given helicity. Moreover, we also get additional restrictions on external sources when interactions with an external source are included.

There are different possibilities to describe massless fields, but the most common is to use gauge invariant equations. If we use equations for massive fields, treated above, it appears that some of them are invariant under gauge transformations and describe massless fields with a given helicity. If we write the relativistically invariant wave equation in the massless case ($m = 0$) as $\pi\psi(x) = 0$ and assume that there is a gauge transformation $\delta\psi = Q^g\epsilon$, where Q^g is an operator of gauge transformation, then for certain types of equations in addition to Q^g there also exists an operator Q^z, satisfying $Q^z\pi = 0$. The latter means that if we consider interactions with some external source $\pi\psi(x) = J$ we get a source constraint $Q^zJ = 0$.

Example 3.1 (helicity 3/2)**.** If $ab = c/2$, the equation

$$i\not{\partial}\psi^\mu + i\left(\frac{a}{\sqrt{3}} - \frac{1}{2}\right)\partial^\mu\gamma_\nu\psi^\nu + i\left(\frac{b}{\sqrt{3}} - \frac{1}{2}\right)\gamma^\mu\partial_\nu\psi^\nu + i\left(\frac{3}{8} + \frac{ab}{4} - \frac{a+b}{4\sqrt{3}}\right)\gamma^\mu\not{\partial}\gamma_\nu\psi^\nu = 0$$

is gauge invariant under the following gauge transformation

$$\delta\psi^\mu = \partial^\mu\epsilon - \frac{1}{4}\left(1 + \frac{\sqrt{3}}{2a}\right)\gamma^\mu\not{\partial}\epsilon$$

where ϵ is an arbitrary bispinor field. In our formalisms there are no additional conditions on ϵ. The source constraint is

$$\partial_\mu J^\mu - \frac{1}{4}\left(1 + \frac{\sqrt{3}}{2b}\right)\not{\partial}\gamma_\mu J^\mu = 0$$

Example 3.2 (helicity 2)**.** If $ab = -c/2$, the equation

$$\Box\, h^{\mu\nu} - \partial^\mu \partial_\rho h^{\rho\nu} - \partial^\nu \partial_\rho h^{\rho\mu} + \left(\frac{1}{2} - \frac{a}{\sqrt{3}}\right) \partial^\mu \partial^\nu h^\rho_\rho + \left(\frac{1}{2} - \frac{b}{\sqrt{3}}\right) \eta^{\mu\nu} \partial_\rho \partial_\sigma h^{\rho\sigma}$$
$$+ \left(\frac{a+b}{4\sqrt{3}} - \frac{ab}{2} - \frac{3}{8}\right) \eta^{\mu\nu} h^\rho_\rho = 0$$

is invariant under the gauge transformation

$$\delta h^{\mu\nu} = \partial^{(\mu} \epsilon^{\nu)} - \frac{1}{4}\left(1 + \frac{\sqrt{3}}{2a}\right) \eta^{\mu\nu} \partial_\rho \epsilon^\rho$$

The source constraint reads

$$\partial_\mu J^{\mu\nu} - \frac{1}{4}\left(1 + \frac{\sqrt{3}}{2b}\right) \partial^\nu J^\mu_\mu = 0$$

It should be mentioned that in our approach no other subsidiary conditions are needed. Usually the gauge transformation is written as $\delta h^{\mu\nu} = \partial^{(\mu} \epsilon^{\nu)}$ with the additional restriction $\partial_\rho \epsilon^\rho = 0$. Such additional restriction usually leads to inconsistencies when interactions are included.

It is interesting to note that the massive equations we obtain from gauge invariant equations adding the mass term are not single particle equations. In the $s = 3/2$ case, for example, spin $1/2$ is also present. The same is true in the $s = 2$ case, where spin 0 is also present.

4. Superfield equations of motion, superspin projection operators

Analogous considerations are applicable in the superfield case [5, 6, 7]. To consider irreducible Lorentz superfields $\psi_i(x,\theta), \psi_j(x,\theta), \ldots,$ we introduce superspin projection operators E^Y_{ij}, satisfying (no sum over j)

$$E^Y_{ij} E^{Y'}_{jk} = \delta_{YY'} E^Y_{ik}$$

E^Y_{ii} are ordinary projection operators that separate superspin Y from ψ_i, i.e. $\psi^Y_i = E^Y_{ii} \psi_i$, and E^Y_{ij} $(i \neq j)$ are spin transition operators, i.e. $\psi^Y_i = E^Y_{ij} \psi_j$. However, the calculation of superprojectors is rather more complicated than in the ordinary field case. We have developed the formalism of calculation of projection operators in the cases of $N = 1$ and N-extended superfields.

Now the n-th order wave equation, previously introduced, is presented in a general matrix form

$$(-\Box)^{n/2} \Pi(\partial, D) \psi(x,\theta) = m^n \psi(x,\theta)$$

where

$$\Pi(\partial, D) = \begin{pmatrix} a_{11}E_{11} & \ldots & a_{1n}E_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1}E_{n1} & \ldots & a_{nn}E_{nn} \end{pmatrix}, \quad \psi(x,\theta) \begin{pmatrix} \psi_i(x,\theta) \\ \vdots \\ \psi_n(x,\theta) \end{pmatrix}$$

D is a four component superderivative. Here a_{ij} are arbitrary numerical coefficients and operators E_{ij} are

$$E_{ij} = \sum_Y a_{ij}(Y) E^Y_{ij}$$

In most cases $a_{ij}(Y)$ are uniquely determined.

5. Massive superfield equation ($Y = 3/2$)

Here we consider, as an example, only the superspin 3/2 case which is mostly used in the $N = 1$ linear massive supergravity [8, 9]. It appears that if we use only the vector superfield $h^\mu(x,\theta)$ we obtain the equation

$$\Box \left(E_{11}^{3/2} - \frac{2}{3} E_{11}^{0} \right)^{\mu}_{\nu} h^\nu(x,\theta) + m^2 h^\mu(x,\theta) = 0$$

which describes superspin 3/2 and also superspin 0 with nonphysical mass $im\sqrt{3/2}$. For that reason some other additional restrictions must be added to eliminate nonphysical superspin 0.

Therefore it is useful to add scalar superfield $\varphi(x,\theta)$ and use two superfields. Let us consider the following equation:

$$\Box \begin{pmatrix} E_{11}^{3/2} - \frac{2}{3}E_{11}^0 & aE_{12}^0 \\ bE_{21}^0 & cE_{00}^0 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} + m^2 \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = 0$$

where φ_1 is a vector superfield and φ_2 is a scalar superfield.

The superspin 0 depends on the eigenvalues of the matrix $\begin{pmatrix} -2/3 & a \\ b & c \end{pmatrix}$. A single superspin 3/2 is described if $ab = -4/9$ and $c = 2/3$, in other cases one or two superspin 0 fields are also present. In the covariant form the equation reads

$$\frac{2}{3}\left[\left(\Box + \frac{d^2}{4}\right) h^\mu - \partial^\mu \partial_\nu h^\nu\right] - \frac{1}{6}\epsilon^{\mu}{}_{\nu\rho\sigma}\partial^\rho d^\sigma h^\nu + \frac{a}{2} d\partial^\mu \varphi + m^2 h^\mu = 0, \; -\frac{b}{2} d\partial_\nu h^\nu - \frac{c}{4} d^2\varphi + m^2\varphi = 0$$

where

$$d = \bar{D}D, \quad d^\mu = i\bar{D}\gamma^\mu\gamma^5 D$$

Different approaches to massive equations are presented in Ref. [10].

6. Massless superfield equation for $Y = 3/2$

Let us consider the gauge invariant massless equations presented in Ref. [11]. If we substitute $m = 0$ in the massive superfield equation and write it as $\pi\psi(x,\theta) = 0$ we get massless equations only if there exists a gauge transformation $\delta\psi = Q^g\epsilon$, where Q^g is an operator and ϵ a superfield. Moreover, in addition to Q^g which satisfies $\pi Q^g = 0$ there also exists an operator Q^z, satisfying $Q^z\pi = 0$. The latter means that if we consider interactions with an external source

$$\pi\psi(x,\theta) = J(x,\theta)$$

we get a source constraint $Q^z J(x,\theta) = 0$. Gauge transformations also set conditions on free coefficients $a_{11}, a_{12}, \ldots$. As a matter of fact, the operators Q^g and Q^z are uniquely expressed via the superspin projection operators.

Example 6.1 (massless superspin 3/2)**.** The above mentioned superspin equation is gauge invariant only if $ab = -2c/3$. Take for simplicity $a = b = -2/3$, then

$$\frac{2}{3}\left[\left(\Box + \frac{d^2}{4}\right) h^\mu - \partial^\mu \partial_\nu h^\nu\right] - \frac{1}{6}\epsilon^{\mu}{}_{\nu\rho\sigma}\partial^\rho d^\sigma h^\nu - \frac{1}{3} d\partial^\mu \varphi = 0, \quad \frac{1}{3} d\partial_\nu h^\nu - \frac{1}{6} d^2\varphi = 0$$

The gauge transformation is now

$$\delta h^\mu = \bar{D}\gamma^\mu\epsilon(x,\theta), \quad \delta\varphi = \bar{D}\epsilon(x,\theta)$$

where $\epsilon(x,\theta)$ is a spinor superfield.

If we consider interaction with an external source

$$\frac{2}{3}\left[\left(\Box+\frac{d^2}{4}\right)h^\mu-\partial^\mu\partial_\nu h^\nu\right]-\frac{1}{6}\epsilon^\mu{}_{\nu\rho\sigma}\partial^\rho d^\sigma h^\nu-\frac{1}{3}d\partial^\mu\varphi=J^\mu(x,\theta)$$

$$\frac{1}{3}d\partial_\nu h^\nu-\frac{1}{6}d^2\varphi=J(x,\theta)$$

we have the source constraint

$$(\gamma_\mu D)_\alpha J^\mu(x,\theta)-D_\alpha J(x,\theta)=0$$

From the last constraint it follows that

$$2i\partial_\mu J^\mu=dJ,\quad \bar{D}(d)DJ=0$$

In $N=1$ supergravity models $J^\mu(x,\theta)$ is connected with an axial source superfield (supercurrent), $J(x,\theta)$ is a scalar source superfield, and $h^\mu(x,\theta)$ is called the metric superfield. It should be mentioned that the latter conditions were posed out of some physical considerations. But adding to $h^\mu(x,\theta)$ a scalar superfield $\varphi(x,\theta)$, we obtain normal gauge invariant second order equation, which in the free field case describes massless superspin $3/2$. Including interaction with an external source we obtain a unique source constraint, which has previously been used in a more complicated and artifical form.

7. Conclusions

We have developed an interesting approach to fields and superfields of arbitrary spin and superspin. At present the field equations for higher spins and superfield equations may seem uninteresting, since they have no useful applications in modern field theory, but this need not remain so. For example, it is possible to analyze different string and superstring models via the fields and superfields. We have analyzed a few string models, writing down the equations for the component fields, and it appears that these equations are not consistent if interactions are introduced.

References

[1] Loide R-K 1984 *Phys. Lett.* B **141** 61
[2] Loide R-K and Suurvarik P 2010 *Proc. Estonian Acad. Sci.* **59** 332
[3] Loide R-K 1985 *J. Phys. A: Math. Gen.* **18** 2833
[4] Loide R-K, Ots I and Saar R 1992 *Proc. Estonian Acad. Sci.* **41** 270
[5] Loide R-K and Suurvarik P 1985 *Proc. Acad. Sci. of ESSR* **34** 248
[6] Loide R-K, Ots I, Saar R and Suurvarik P 2004 *Hadronic J.* **27** 151
[7] Suurvarik P and Loide R-K 2006 *Czech. J. Phys.* **56** 1299
[8] Hinterbichler K 2012 *Rev. Mod. Phys.* **84** 671
[9] Gregoire T, Schwartz M D and Shadmi Y 2004 *JHEP* **0407** 029
[10] Buchbinder I L, James Gates Jr S, Kuzenko S M and Phillips J 2005 *JHEP* **0502** 056
[11] Suurvarik P and Loide R-K 2005 *Czech. J. Phys.* **55** 1521