

Application of Fast Fourier Transform in thermo-magnetic convection analysis

L Pyrda

AGH University of Science and Technology, Faculty of Energy and Fuels,
Department of Fundamental Research in Energy Engineering, Poland

E-mail: pyrda@agh.edu.pl

Abstract. Application of Fast Fourier Transform in thermo-magnetic convection is reported. Cubical enclosure filled with paramagnetic fluid heated from below and placed in the strong magnetic field gradients was investigated. The main aim of study was connected with identification of flow types, especially transition to turbulence. For this purpose the Fast Fourier Transform (FFT) analysis was applied. It was followed by the heat transfer characteristic for various values of magnetic induction gradient. The analysis was done at two Rayleigh numbers $7.89 \cdot 10^5$ and $1.86 \cdot 10^6$ with thermo-magnetic Rayleigh numbers up to $1.8 \cdot 10^8$ and $4.5 \cdot 10^8$ respectively. The presented results clearly indicate flow types and also demonstrate augmented heat transfer in dependence on magnetic induction gradient. Detailed analysis of flow transition to turbulent state was compared with transition line for natural convection reported in literature. The transition to turbulence in the case of thermo-magnetic convection of paramagnetic fluid was in very good agreement with transition in the case of natural convection.

1. Introduction

Since first quantitative studies on the natural convection reported by Bénard, natural convection in a configuration heated from below (also called Rayleigh-Bénard configuration) was subject of research in many experimental and theoretical studies. In later studies most of scientific interest was devoted to description of fluid behavior, quantitative and qualitative visualization techniques, as well as presentation of dimensionless parameters which could characterize such phenomena. Most of previous studies defined flow as turbulent as soon as its behavior was non-periodic. In year 1987, after statistical analysis, Heslot [1] proposed three types of turbulence: transitional regime, soft turbulence ($10^5 < Ra < 4 \cdot 10^7$) and hard turbulence ($4 \cdot 10^7 < Ra < 6 \cdot 10^{11}$). In recent experiments Niemela et al. [2] with cryogenic helium as a working fluid presented results for hard turbulence up to $10^6 < Ra < 10^{17}$. All these studies addressed natural convection phenomena and Prandtl number close to 1.

In 1991 year Braithwaite et al. [3] presented for the first time natural convection of paramagnetic non-electrically conducting fluid in connection with the magnetizing force effect. They reported the enhancement or suppression of heat transfer rate through shallow paramagnetic fluid layer in Rayleigh-Bénard configuration. After that some of researchers extended this work applying the different geometry, flow visualization techniques and dimensionless parameters description i.e.: [4],[5],[6]. All this studies were connected with magnetic field effect on the paramagnetic or diamagnetic fluids considered steady laminar flow regimes. Most recent studies were connected with transition from laminar to turbulent flow regimes which could be obtained by application of high



gradient magnetic field (up to 900 [T²/m], for the magnetic field induction in the centre of superconducting magnet bore $|b_0|_{\max}=10$ [T] in contrast to 200 [T²/m], for magnetic field induction of $|b_0|_{\max}=5$ [T] in previous studies) i. e.: [8],[9],[10],[11],[12].

The goal of presented investigation was an analysis of transition from soft turbulence to hard turbulence regimes in the case of thermal and magnetic convection. This was achieved, by employing high gradient magnetic field generated by the superconducting magnet and application of Fast Fourier Transform method in phenomenon analysis.

Throughout the paper the magnetic field induction conditions were used to identify different flow regimes since it was a simple and easily controllable parameter in experimental investigations, although the magnetic field gradient was the main driving mechanism behind the magnetization force [9].

2. Experiment

2.1. Experimental setup

Experimental setup presented in Figure 1 consisted of: a superconducting magnet generating strong magnetic field, a cubical enclosure, a thermo-stating bath, a power supply and a data acquisition system. The superconducting magnet opening has a diameter of 0.1 [m] and length of 0.5 [m]. Experimental enclosure is shown in detail in Figure 2.

The central part of experimental setup was cubical enclosure of 0.032 [m] size. It was placed inside magnet opening in Rayleigh-Bénard configuration. The lower horizontal wall was electrically heated, while the upper one was cooled with thermo-stated water. Temperature of both walls was controlled with six thermocouples. Six other thermocouples were inserted into the enclosure to measure temperature of the fluid. Due to them it was possible to recognize type of the flow and identify the transition between steady and turbulent flow.

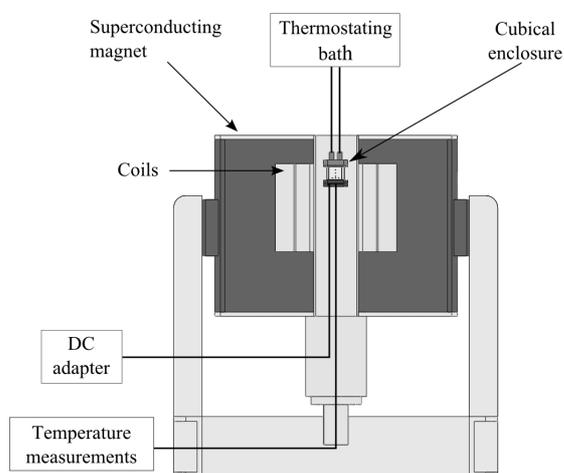


Figure 1. Experimental setup and location of the cubical enclosure with paramagnetic working fluid

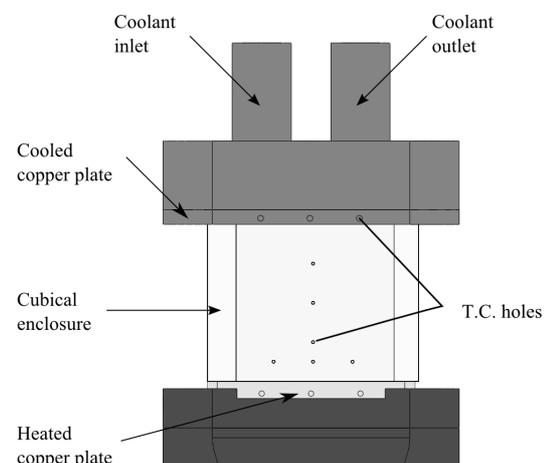


Figure 2. Cubical enclosure ($L=D=H=0.032$ [m]).

2.2. Working Fluid

As a working fluid for experimental studies 50% volume aqueous solution of glycerol with 0.8 [mol/(kg of solution)] concentration of gadolinium nitrate hexahydrate ($Gd(NO_3)_3 \cdot 6H_2O$) was chosen. All important for present studies properties of fluid were measured experimentally: the magnetic susceptibility (magnetic susceptibility balance, Evan's method), the thermal expansion coefficient

(pycnometer), the dynamic viscosity coefficient (viscosimeter, thermally controlled), the thermal conductivity (thermal conductivity meter) and the heat capacity (differential calorimeter). Due to the narrow temperature range studied, it could be assumed that almost all needed properties were constant, except viscosity. Therefore the dependence of viscosity on the temperature was checked experimentally and approximated function could be written as follows:

$$\mu_{(\theta)} = 1.057 - 6.5 \cdot 10^{-3} \theta + 1.0 \cdot 10^{-5} \theta^2 \quad (1)$$

All important for presented investigation fluid properties are listed in Table 1.

Table 1. Properties of the working fluid at 298 K.

Property	Symbol	Value	Unit
Heat capacity	C_p	$2.92 \cdot 10^3$	[J/kg·K]
Thermal diffusivity	α	$9.13 \cdot 10^{-8}$	[m ² /s]
Thermal expansion coefficient	β	$4.78 \cdot 10^{-4}$	[1/K]
Dynamic viscosity	μ	$1.30 \cdot 10^{-2}$	[kg/m·s]
Thermal conductivity	λ	0.376	[W/m·K]
Kinematic viscosity	ν	$9.25 \cdot 10^{-6}$	[m ² /s]
Density	ρ	1411	[kg/m ³]
Mass magnetic susceptibility	χ_m	$2.39 \cdot 10^{-7}$	[m ³ /kg]

2.3. Experimental course

The main aim of this paper was to identify transition to turbulent flow and then to analyze its influence on the heat transfer. Thermal measurements were used to obtain both stated aims. The experiment was divided into two steps. First step it was thermal measurements with increasing magnetic induction for temperature difference between heated and cooled walls equal to 5 and 11 [K]. Signal of thermocouples inserted into the cubic enclosure was then analyzed with FFT. The second step was connected with the energy balance and estimation of heat loss to the environment. To estimate heat loss, the conduction experiment was carried out with reversely placed thermally active horizontal walls of experimental enclosure at investigated position. At first a specific temperature difference between horizontal walls was selected and then after thermal stabilization the heating power was measured, which together with theoretical heat flux obtained from Fourier's law allow estimation of heat loss:

$$Q_{\text{loss}} = Q_{\text{cond}} - Q_{\text{theor_cond}} \quad (2)$$

$$Q_{\text{theor_cond}} = D\lambda(\theta_h - \theta_c) \quad (3)$$

where: θ_h , θ_c - are temperature of hot and cold walls respectively.

Heat loss was calculated according to eq. (2) and could be linearly approximated for various heating rates ($\Delta\theta = \theta_h - \theta_c = 2.5 \div 15$ [K], where temperature of cold wall was kept at ambient temperature inside magnet), according to:

$$Q_{\text{loss}} = 0.8\Delta\theta \quad (4)$$

2.4. Dimensionless numbers

The thermo-magnetic convection can be characterized by a group of non-dimensional parameters such as Nusselt, Prandtl, thermal and magnetic Rayleigh and magnetization numbers.

Nusselt number was calculated as a ratio between the net convective heat transfer rate and the net pure conduction contribution with applied method invented by Ozoe and Churchill [13] to estimate net heat fluxes:

$$\text{Nu} = \frac{Q_{\text{net_conv}}}{Q_{\text{net_cond}}} = \frac{Q_{\text{conv}} - Q_{\text{loss}}}{Q_{\text{cond}} - Q_{\text{loss}}} \quad (5)$$

Assuming that convection heat flux was equal to heater heat flux ($Q_{\text{net_conv}}=Q_{\text{heater}}$) and applying eq. (2) and (3) to eq. (5) it could be rewritten in following form:

$$\text{Nu} = \frac{Q_{\text{heater}} - Q_{\text{loss}}}{Q_{\text{theor_cond}}} = \frac{IU - Q_{\text{loss}}}{D\lambda(\theta_h - \theta_c)} \quad (6)$$

The Prandtl number definition was:

$$\text{Pr} = \frac{\nu}{\alpha} \quad (7)$$

The thermal Rayleigh number was defined as:

$$\text{Ra}_T = \frac{\beta \mathbf{g} \Delta \theta D^3}{\nu \alpha} \quad (8)$$

where β is the thermal expansion coefficient, \mathbf{g} is the gravitational vector, $\Delta \theta$ is the temperature difference between horizontal thermally active walls, D is the enclosure size ($D=0.032$ [m]), ν is the kinematic viscosity coefficient and α is the thermal diffusivity.

The magnetic Rayleigh number was defined as:

$$\text{Ra}_M = \left(1 + \frac{1}{\beta \theta_0}\right) \left(\frac{\gamma \beta \mathbf{g} \Delta \theta D^3}{2\nu \alpha}\right) \quad (9)$$

The magnetization number:

$$\gamma = \frac{\chi_m |b_{0\text{max}}|^2}{\mu_0 \mathbf{g} D} \quad (10)$$

where: $\mu_0=4\cdot\pi\cdot 10^{-7}$ [H/m].

The thermo-magnetic Rayleigh number was defined as:

$$\text{Ra}_{\text{TM}} = \text{Ra}_T + \text{Ra}_M \quad (11)$$

3. Results and discussion

Temperature time series of investigated fluid for $\Delta\theta=5$ [K] case at chosen $|b_{0\text{max}}$ are presented in Figure 3. The black color of signal corresponds to the position of chosen thermocouple placed at the central lowest position (0.004 [m] above bottom wall of enclosure – Figure 2). For further analysis only signal from this one thermocouple was selected to obtain detailed information on flow phenomena. The Fast Fourier Transform (FFT) analysis was conducted for all cases of chosen thermocouple signal and the results are presented as the amplitude versus frequency in Figure 3 (right). In initial state ($|b_{0\text{max}}=0$ [T]) temperature signal is oscillating. In presence of weak magnetic induction temperature signal is stabilized. Result at $|b_{0\text{max}}=4$ [T] shows temperature signal before transition to turbulence and temperature signal at $|b_{0\text{max}}=5$ [T] after that. The last case ($|b_{0\text{max}}=10$ [T]) shows temperature signal with smaller amplitude than 5 [T] of magnetic induction because it was gradually decreasing with increase in the magnetic induction.

For $\Delta\theta=5$ [K] case (Figure 3), at $|b_{0\text{max}}=0$ [T] oscillation of temperature signal has characteristic frequency equal to 0.012 [Hz]. At $|b_{0\text{max}}=4$ [T], before transition to turbulence, the characteristic

frequency of small oscillations is 0.074 [Hz]. The FFT analysis of temperature time series obtained at $|b_{0\max}|=5$ [T] shows the highest peak at 0.052 [Hz]. With further increase of magnetic induction amplitude of oscillation was decreasing and got more uniform but far more intensive than results before transition, which can be seen at $|b_{0\max}|=10$ [T] (Figure 3).

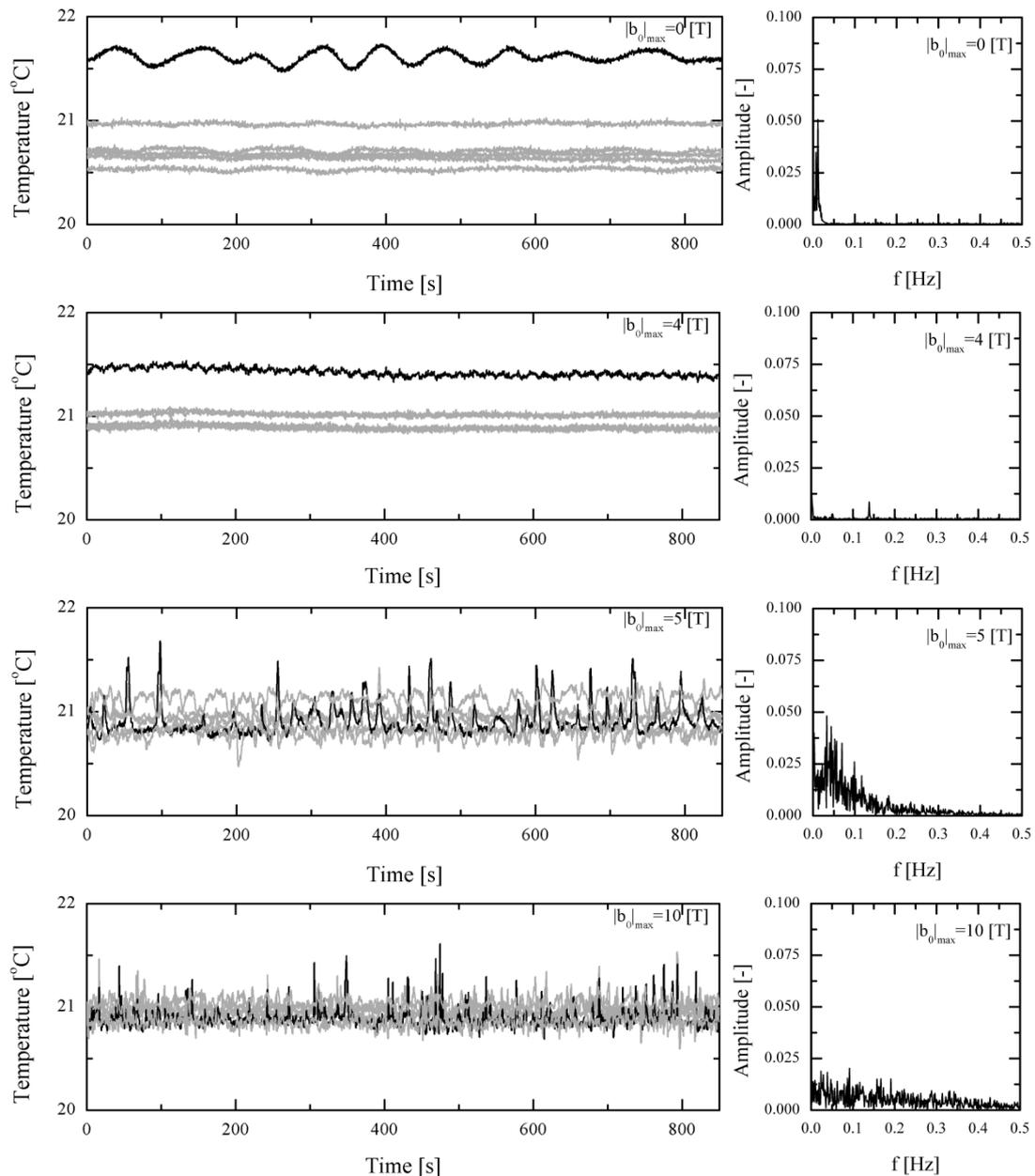


Figure 3. Temperature time series (left column) with FFT amplitude versus frequency (right column) for $\Delta\theta=5$ [K] case at $|b_{0\max}|$: 0 [T], 4 [T], 5 [T], 10 [T]

Temperature time series of investigated fluid for $\Delta\theta=11$ [K] case at chosen $|b_{0\max}|$ are presented in Figure 4. Initial state ($|b_{0\max}|=0$ [T]) is stable. Then presence of weak magnetic induction (from $|b_{0\max}|=1$ [T]) results in oscillation of temperature signal up to transition to turbulence and temperature signal at $|b_{0\max}|=3$ [T] after that. The last case ($|b_{0\max}|=10$ [T]) shows temperature signal with smaller

amplitude then 3 [T] of magnetic induction because it was gradually decreasing with increase in the magnetic induction.

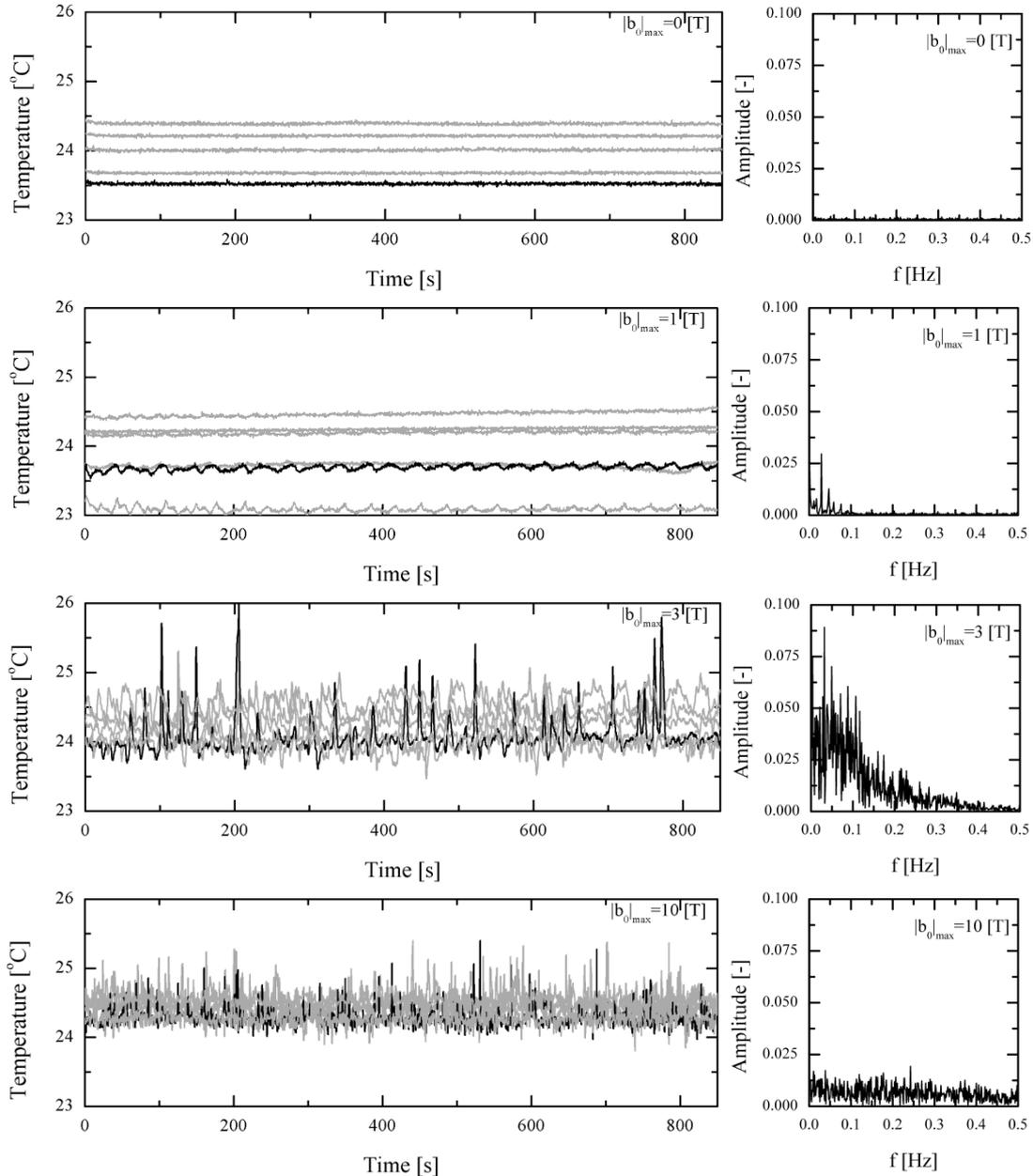


Figure 4. Temperature time series (left column) with FFT amplitude versus frequency (right column) for $\Delta\theta=11$ [K] case at $|b_{0\max}|$: 0 [T], 4 [T], 5 [T], 10 [T]

For $\Delta\theta=11$ [K] case (Figure 4), at $|b_{0\max}|=0$ [T] temperature signal is stable. At $|b_{0\max}|=1$ [T] the results indicate the most characteristic frequencies at 0.029 [Hz] which is kept up to transition. The FFT analysis of temperature time series obtained after transition at $|b_{0\max}|=3$ [T] shows the highest peak at 0.032 [Hz] (Figure 4 - right). With further increase of magnetic induction amplitude of oscillation was decreasing and got more uniform but far more intensive than first results before transition, which can be seen at $|b_{0\max}|=10$ [T] (Figure 4).

The Nusselt number plotted versus the magnetic induction in the centre of magnet ($|b_0|_{\max}$) for 5 [K] and 11 [K] of temperature difference between horizontal thermally active walls ($\Delta\theta=\theta_h-\theta_c$) are presented in Figure 5. The Prandtl number was 121 and 101 for $\Delta\theta=5$ [K] and $\Delta\theta=11$ [K] respectively. Measurements error was calculated with combined uncertainty analysis and was less than: 6.8% and 3% for $\Delta\theta=5$ [K] and $\Delta\theta=11$ [K] respectively. The heat transfer enhancement with increasing value of magnetic induction was observed for both studied temperature differences. The Nusselt number increased more than 300% at 10 [T] of magnetic induction in comparison with natural convection case.

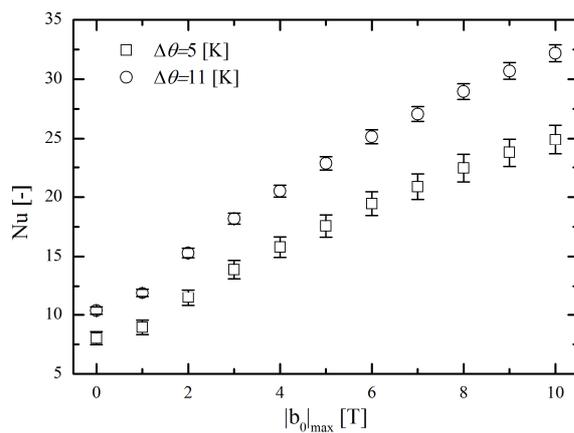


Figure 5. The Nusselt number plotted versus the magnetic induction at the centre of magnet ($|b_0|_{\max}$) for $\Delta\theta=5$ [K] and $\Delta\theta=11$ [K]

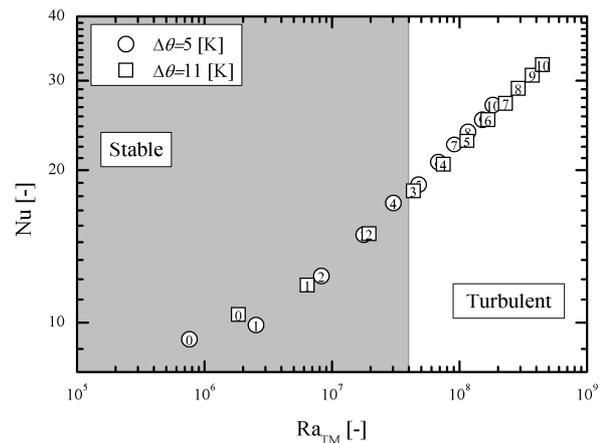


Figure 6. Nusselt number versus thermo-magnetic Rayleigh number with magnetic induction $|b_0|_{\max}$ value inside each caption

The Nusselt number versus thermo-magnetic Rayleigh number with caption consisting $|b_0|_{\max}$ value for all investigated cases is presented in Figure 8. Background color of Figure 8 represents type of fluid behavior and transition point at $Ra_{TM}=4\cdot 10^7$ which was found by Heslot [1] for natural thermal convection.

4. Conclusions

The analysis of unsteady flow in the case of thermo-magnetic convection was studied experimentally. An influence of Rayleigh number and magnetic field strength was examined. Due to the application of steady magnetic field at chosen temperature difference two flow types were presented. With an application of Fast Fourier Transform it was possible to determine the characteristic frequency for temperature time series. The FFT analysis clearly indicates the type of the flow, what was shown at first in the form of temperature time series and then amplitude vs. frequencies. Identification of characteristic frequencies highest peak in the case of 5 and 11 [K] temperature differences at 5 and 3 [T] of magnetic induction allowed calculation of Rayleigh number at transition to turbulence. It was $Ra_{TM}=4,3\cdot 10^7$ and $Ra_{TM}=4,7\cdot 10^7$ respectively, which correspond to value found by Heslot [1] and was first time found after him. In the turbulent region the heat transfer was significantly augmented. The heat exchange increased more than 300% in comparison with natural convection. The presented results showed that this experimental method can be applied in the investigations of time stable and, what is more important time and space unstable fluid flow.

5. Acknowledgments

This work was supported by the Polish National Science Centre Project No. 2012/07/B/ST8/03109.

6. References

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