

Turbulence over/inside porous surfaces and challenges to its modelling

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Abstract. To understand turbulence over porous media, a series of PIV measurements were carried out in porous-wall channel flows. The porous walls were made of three types of foamed ceramics which had the same porosity but different permeability. For turbulence inside porous media, LES studies of fully developed flows in several model porous media were performed. Referring to these databases, a multi-scale $k-\varepsilon$ four equation eddy viscosity model for turbulence around/inside porous media was developed. The proposed model was validated with satisfactory accuracy.

1. Introduction

Turbulent flows over/inside highly porous materials are often seen in various engineering devices due to the advantages for their performance of heat and mass transfer by their large contact area [1,2]. It is readily understood that to provide gas and fuel through the porous media for chemical reaction, mass transfer enhancement over the porous wall is crucially important. Thus, understanding and predicting flows and mass transfer around/inside porous media are very important in industry. However, turbulent flow physics in such flows is not fully understood though many studies have been historically performed experimentally and numerically (e.g. Refs. [3-5]). Moreover, numerical schemes (e.g. Refs. [6,7]) for those flows are still immature compared to for single phase turbulence [8].

Therefore, the present authors' group have been performing PIV experiments [9-11] and high resolution LES studies [12,13] to understand turbulence over/inside highly porous media. Based on those studies and the knowledge in the literature, the present authors have proposed new engineering turbulence models [14,15]. In this paper, the major outcomes of those studies are summarized for serving a part of the state of the art in the studies on turbulence over/inside porous media.

2. PIV measurements of turbulence over porous walls

2.1. Experimental procedures

Turbulent flows in a channel whose bottom wall was made of a porous medium were measured by the PIV. Two flow geometries: porous wall channels [9,10] and rib-mounted porous wall channels [11], were considered. Figure 1 shows those flow geometries. The experimental setup of the flow facility is illustrated in figure 2. Tap water was used as the working fluid. The flow was fully developed in a driver channel before the test section channel. Both the channels consisted of solid smooth acrylic top and side walls and a porous bottom wall. The porous media applied were three types of foamed ceramics shown



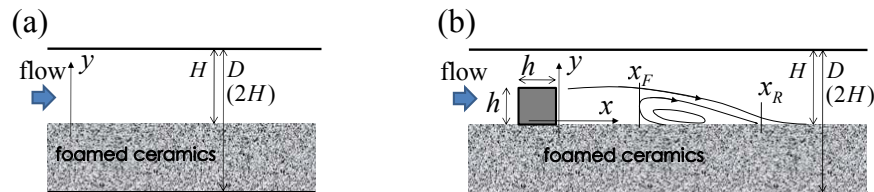


Figure 1. Flow geometries; (a) porous wall channel, (b) rib-mounted porous wall channel.

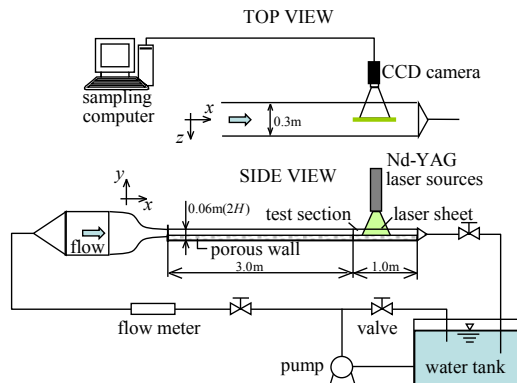


Figure 2. Experimental setup.

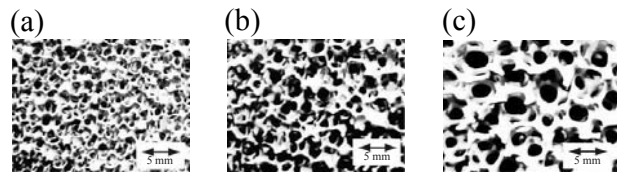


Figure 3. Surface photographs of the foamed ceramics; (a) #20, (b) #13, (c) #06.

in figure 3. Their properties are listed in table 1. The thickness of the porous wall was 0.03 m which was set to be the same as the channel height H for the clear fluid region (figure 1(a)) whose width was 0.3 m. In cases of rib-mounted channels, at 0.2 m from the beginning of the test section, a square sectioned rib whose height is $h=0.015$ m was mounted as shown in figure 1(b). It had been confirmed that the flow was nominally two-dimensional near the symmetry plane. The measured range of the bulk Reynolds number $Re_b (\rho U_b H / \mu)$ was 1000-10000 where the fluid viscosity μ and density ρ were obtained from their relations to the water temperature. The bulk mean velocity U_b was obtained by integrating the measured sectional mean velocity profile. For the rib-mounted channel, it was performed at $x=-5h$ where the flow was confirmed to be unaffected by the rib.

The two-component PIV system consisted of a double-pulse Nd-YAG laser with 120 mJ/pulse at a frequency of 532 nm, a CCD camera of 30 fps, a 60 mm camera lens of f/2.8 and a computer for data processing. The laser beam was formed into a sheet of approximately 1.0 mm thickness. The sheet illuminated the symmetry plane of the channel where the instantaneous images were recorded by the CCD camera. A single recorded frame covered a zone of $30 \times 30 \text{ mm}^2$ with 2048×2048 pixels. For the tracer particles, acrylic colloid particles were used. The mean diameter, specific gravity and refractive index of the particles are respectively $3.1 \text{ } \mu\text{m}$, 1.19 and 1.50. The seeding density was adjusted to obtain 10-15 particle-image pairs in each interrogation window whose size was set to 64×16 or 32×32 pixels for the channel or the rib-mounted channels, respectively. Thus the measurement sampling volume was $0.8(x) \times 0.2(y) \times 1.0(z)$ or $0.4(x) \times 0.4(y) \times 1.0(z) \text{ mm}^3$. The image sampling rate was 4 Hz. The average particle displacement was set to be about 25 % length (6-8 pixels) of the interrogation window. To obtain the statistical data, in each zone, 3200-4800 image pairs were processed. The recorded data were

Table 1. Characteristics of the foamed ceramics.

Porous med. No.	Porosity ϕ	Mean pore diameter D_p [mm]	Permeability K [mm^2]	Forchheimer coef. c_F
#20	0.82	1.7	0.020	0.17
#13	0.81	2.8	0.033	0.10
#06	0.80	3.8	0.087	0.095

processed by commercial software (Dynamics Studio 2.0, DANTEC DYNAMICS) with the fast Fourier transform cross-correlation technique. The removed error vectors were less than 3% of the total data processed. The average number of pixels for a particle image captured by the CCD camera was counted to be about 3.1 pixels. This indicated that the particle images were well resolved in the experiments and the estimated error of the instantaneous velocity magnitude was less than 4% of the maximum velocity.

2.2. Channel flow experimental results [9,10]

Figure 4(a) shows examples of the mean velocity distributions in the porous wall channel. The position at $y = 0$ corresponds to the surface of the porous medium whilst the $y = H$ represents the top wall surface. Due to the Reynolds shear stress distribution shown later in figure 6(c), the velocity distributions look asymmetric and deviate from the parabolic profile of the laminar case ($Re_b=900$) as Re_b increases. This clearly indicates that the flow tends to be turbulent even at $Re_b=1300$. Although it is not shown here, this tendency was enhanced in the higher permeability cases. Obviously, the flow became turbulent at a lower Re_b compared with a solid wall channel flow. This suggests that porous surface enhances the onset of turbulence. At $y = 0$, finite velocities exist and their magnitude increases as Re_b increases. This tendency of the slip velocity U_w is shown in figure 4(b). The characteristic profiles of the slip velocities look fairly generalized against the permeability Reynolds number $Re_K (=u^p_\tau K^{1/2}/\nu)$. This implies that Re_K can be a good measure for permeable turbulent boundary layers. Indeed, in the range of $0 < Re_K < 3$ a sharp increase of U_w can be seen irrespective of the porous media. It suggests that there is transition to full porous wall turbulence in that range. The friction velocity here u^p_τ was calculated from the shear stress on the porous wall obtained by extrapolating the measured Reynolds shear stress distribution to the porous wall. For turbulent flows above rough beds or submerged canopies, the log law form:

$$U^+ = \frac{1}{\kappa} \ln \left(\frac{y+d}{h_r} \right)^+, \quad (1)$$

has been often applied, where d is the zero-plane displacement and h_r is the equivalent roughness height. In this log law, d and h_r are empirical parameters. Note that as shown in figure 4(c), the parameter d is the depth of the zero-plane from the surface. From the semi-log velocity distribution as shown in figure 4(d), κ , d^{p+} and h_r^{t+} were obtained. Note that values with the superscripts ' $p+$ ' and ' $t+$ ' denote normalized values by u^p_τ and u^t_τ which are the friction velocities on the porous and top solid walls, respectively. It was found that they had close relationships with Re_K as shown in figure 5(a)-(c).

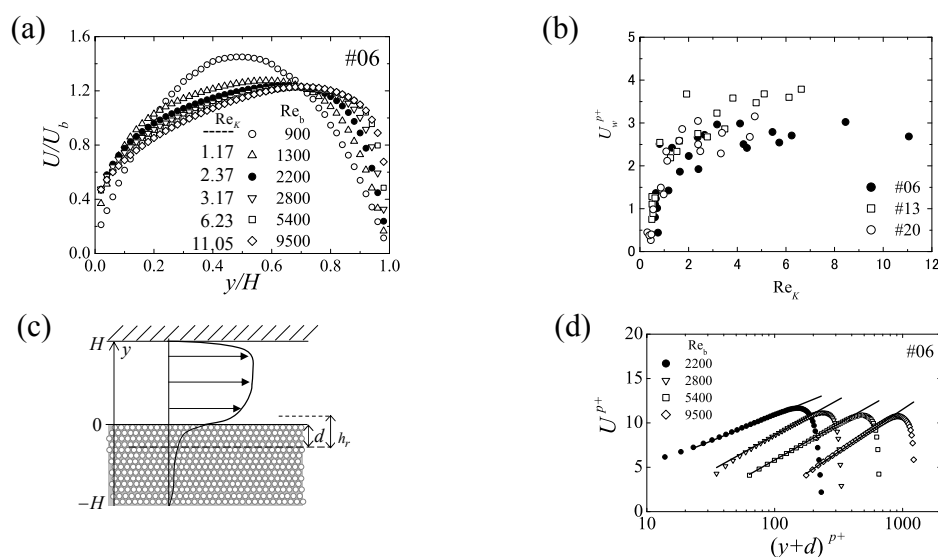


Figure 4. Time mean velocities; (a) mean velocity profiles of case #06, (b) slip velocities, (c) zero-plane displacement d and equivalent roughness h_r , (d) mean velocities of case #06 in semi-log scale.

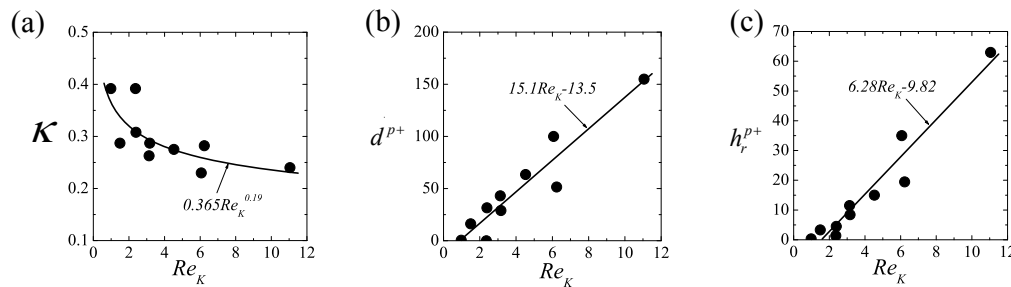


Figure 5. Distribution against the permeability Reynolds number; (a) von Kármán constant, (b) zero-plane displacement, (c) equivalent roughness height.

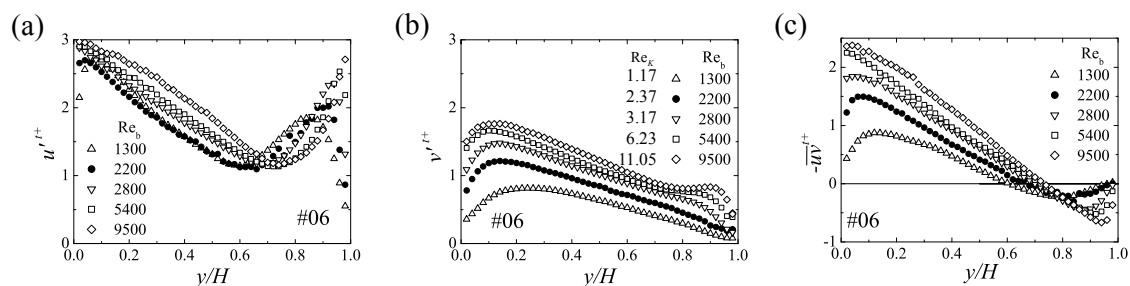


Figure 6. Turbulent quantities of case #06; (a) streamwise turbulent intensity, (b) wall normal turbulent intensity, (c) Reynolds shear stress.

To emphasise and to see the effects of the wall permeability further, u_{τ}^t was applied as the reference velocity for normalisation. The distribution profiles of the turbulent intensities u'^{+} and v'^{+} of case #06 are shown in figure 6. Although near the porous walls the peak of u'^{+} changes a little with Re_b , v'^{+} increases drastically as Re_b increases. It suggests that the effects of the porous wall appear in the wall normal component rather than in the streamwise component. Since wall blocking effects are weakened more due to the higher wall permeability, the wall normal fluctuation is kept at a certain level even near the wall. In other words, the wall normal fluctuation is enhanced near the porous wall due to the permeability. This tendency becomes stronger as the Reynolds number and thus Re_K increases. Figure 6(c) shows the Reynolds shear stress $-\overline{uv}^{+}$ distribution. Due to the large contribution of v'^{+} , $-\overline{uv}^{+}$ monotonically increases with Re_b (and Re_K) near the porous wall. Consequently, it suggests that the higher wall permeability weakens the wall blocking effects more and the turbulent wall normal fluctuation is kept at a higher level (or enhanced) leading to the intensified turbulent shear stress near the porous wall.

Figure 7 shows the distribution of the joint probability density function of the fluctuating velocities $p(u', v')$ in the buffer regions at the position of $y^+ = 15$. Figure 7(a) corresponds to the distribution near the solid top wall whereas figure 7(b)-(d) corresponds to those near the porous walls at different conditions of $Re_K = 1.1-11.1$. It is obvious that the distribution shape (called distribution “oval” hereafter) changes as Re_K increases. (Note that at the solid wall, Re_K is essentially zero since the wall does not have permeability.) As illustrated in figure 8(a), when Re_K becomes larger, the distribution oval becomes rounder and the angle between its major axis and the u' -coordinate becomes larger up to a certain level. Indeed, figure 7(c) looks very similar to figure 7(d) though their permeability Reynolds numbers are different: $Re_K = 4.5$ and 11.1 , respectively. Since the rounder distribution oval indicates that the turbulence is less anisotropic, figure 7 indicates that the turbulence field tends to be less anisotropic up to a certain level as Re_K increases. In order to quantitatively discuss on the shape of the distribution oval, figure 8(b) indicates the ratio of the mean deviations from the minor and major axes of the distribution oval of fluctuating velocities. Figure 8(c) indicates the difference between the major axis angles of the

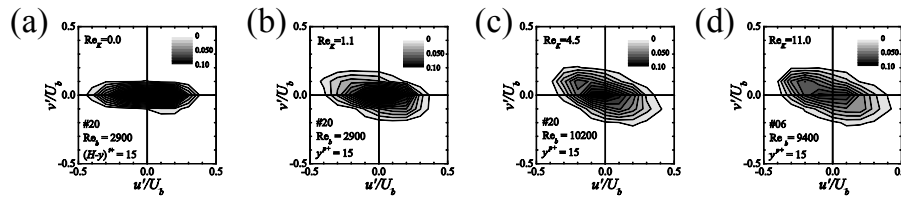


Figure 7. Joint probability density function $p(u', v')$ of fluctuating velocity at $y^+ = 15$; (a) near the solid wall side of case #20 at $Re_b = 2900$ ($Re_K = 0.0$), (b) near the porous wall side of case #20 at $Re_b = 2900$ ($Re_K = 1.1$), (c) near the porous wall side of case #20 at $Re_b = 10200$ ($Re_K = 4.5$), (d) near the porous wall side of case #06 at $Re_b = 9400$ ($Re_K = 11.1$).

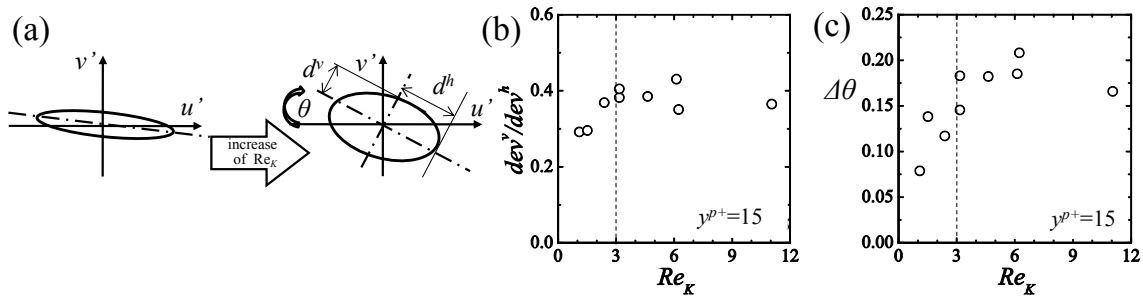


Figure 8. Measure of the distribution-oval shape; (a) distribution oval of fluctuating velocities, (b) ratio of the deviations from the minor and major axes of the distribution oval of fluctuating velocities, (c) difference between the angles of major axes.

distribution ovals near the porous and solid walls: $\Delta\theta = \theta^p - \theta^s$. The centre point, the major and minor axes of the oval were determined by averaging the distributed point positions and their angles from the u' - and v' -coordinates, respectively. Here, the mean deviations were defined as $dev^h = [\Sigma(d^h)^2 / N]^{1/2}$ and $dev^v = [\Sigma(d^v)^2 / N]^{1/2}$, where N , d^h and d^v were, respectively, the sampling number, the deviations from the minor and major axes of the distribution oval as shown in figure 8(a). When the ratio increases toward unity, the turbulence becomes less anisotropic in the u' - v' plane. As shown in figure 7(b), the ratio dev^v/dev^h of the distribution oval shows less anisotropic characteristics as Re_K increases. In the region of $Re_K \leq 3$, dev^v/dev^h tends to be smaller as Re_K reduces whilst it, however, tends to be saturated in the higher Re_K region. The distribution of $\Delta\theta$ shown in figure 8(c) also indicates the same tendency. These suggest that the turbulence structure changes depending on Re_K and thus the influence of the wall permeability on the vortex structure near permeable walls is significant.

2.3. Rib-mounted channel flow experimental results [11]

The above measurements suggested that due to the wall permeability, turbulent vortex motions over porous walls were not damped so much as near solid walls resulting in strong near-wall shear stress production. It is then understandable that if the flow includes separation near a porous wall, the formation of a separation bubble is strongly affected by the wall permeability. Thus, discussions on the turbulence separation over porous media were carried out by the PIV measurements.

Figure 9 compares streamlines obtained from the mean velocities in the solid rib-mounted channel with different wall materials: low permeability #20 and high permeability # 06 porous media, respectively. In case #20, with the increase of Re_b , it is recognisable that the reverse flow region in the clear channel tends to vanish: figure 9(a) and (b). This is considered to result from the fact that a higher proportion of the flow upstream the rib goes inside the bottom wall layer and bleeds out behind the rib as the Reynolds number increases. Also, as shown in figure 9(c), in the high permeable porous bottom wall case: #06, due to the increase of the wall permeability, the recirculation bubble behind the rib in the clear channel region looks almost extinguished. Indeed, as shown in table 1, #06 is about 4.5 times

more permeable than #20. It is considered that due to the increase of the wall permeability, more amount of flow upstream the rib goes inside the bottom porous layer and bleeds out behind the rib to supply the fluid to the entrainment of a developing shear layer.

When the rib has permeability, since the flow also goes through the rib, the flow pattern and turbulence quantities change. Figure 10 compares streamlines of low and high permeability cases #20 and #06 at $Re_b=3500$. Due to the flow going through the rib, as shown in figure 10(a), clearly the recirculation zone behind the rib shifts downstream compared with the solid rib flow of figure 9. Also, at the high permeability case of figure 10(b), there is no recirculation zone observed and only the wake flow behind the rib can be seen.

In summary, the measurement results indicated that a turbulent recirculation or wake flow region was formed behind the rib in all the flow cases at $Re_b=10^3-10^4$. The turbulence levels of those flows were much higher in the region around and behind the rib compared with those in the corresponding fully developed porous channel flows. However, in the solid rib-mounted channel flows, since a part of entraining fluid was supplied through the permeable bottom wall from the region upstream the rib, the recirculation behind the rib in the clear channel became smaller and eventually vanished as the wall permeability increased. The separation bubble in the solid rib flow was smaller and its reattachment length became shorter as the increase of the wall permeability and/or Re_b . Because of the reduction of the magnitude and the size of the reverse flow region in the clear channel, turbulence became weaker than those of the rib-mounted solid-wall channel flow though the turbulence levels around and behind the rib were much higher than those of fully developed porous channel flows. Due to the entrainment flow to the shear layer through the porous wall, the near wall turbulent intensities, in particular, the wall normal component tended to be large as the wall permeability increased. In the porous-rib-mounted channel flows (the rib material was the same as that of the porous wall), because of the fluid passing through the rib, the recirculation was more significantly weakened and shifted downstream. The

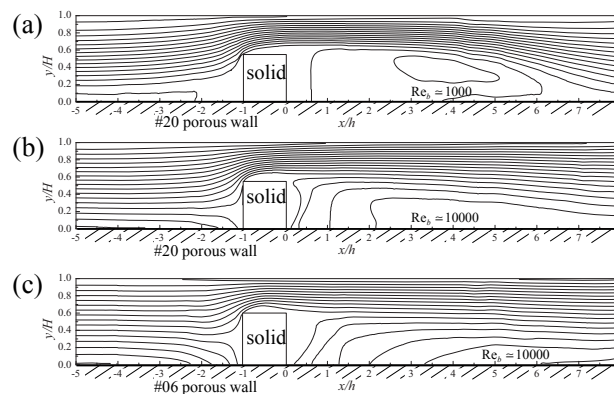


Figure 9. Comparison of streamlines over a solid rib-mounted porous wall channel; (a) case #20 at $Re_b=1000$, (b) case #20 at $Re_b=10000$, (c) case #06 at $Re_b=10000$.

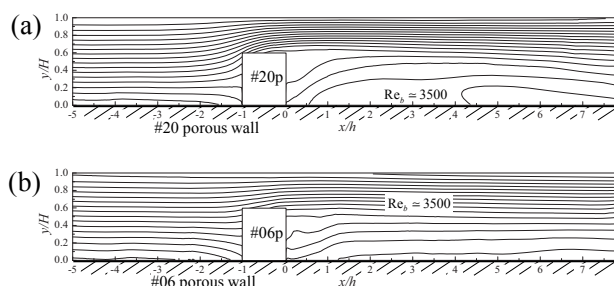


Figure 10. Comparison of streamlines over a porous rib-mounted porous wall channel; (a) case #20 at $Re_b=3500$, (b) case #06 at $Re_b=3500$.

upstream edge of the recirculation shifted further downstream as Re_b increased in the low permeability case whereas no recirculation region was observed in the clear channel at the higher wall permeability regardless of Re_b . Because of the flow through the porous rib, turbulent intensities and the Reynolds shear stress became smaller than those of the solid-rib flows.

3. Double averaged turbulence equations

So far, turbulent flows over porous media are discussed. However, to treat statistical values of turbulent flows inside porous media, so called the *double averaging* needs applying to the transport equations [16]. The double averaging consists of the conventional Reynolds and volume averaging operators. The volume averaged values are called the *superficial* and *intrinsic* (fluid phase) averaged values and defined as

$$\langle \phi \rangle = \frac{1}{\Delta V} \int_{\Delta V} \phi dv, \quad \langle \phi \rangle^f = \frac{1}{\Delta V_f} \int_{\Delta V_f} \phi dv, \quad (2)$$

where ΔV and ΔV_f are the REV (representative elementary volume) and the volume of the fluid phase contained within ΔV , respectively. Between them, the relation: $\langle \phi \rangle = \phi \langle \phi \rangle^f$ exists with the porosity of the porous medium: $\phi = \Delta V_f / \Delta V$. The volume averaging of derivatives is expressed as

$$\left\langle \frac{\partial \phi}{\partial x_k} \right\rangle = \frac{\partial \langle \phi \rangle}{\partial x_k} + \frac{1}{\Delta V} \int_A n_k ds, \quad (3)$$

where A , n_k are the superficial area of the solid phase and its unit normal vector pointing outward from the fluid to the solid phase, respectively. The dispersion is $\tilde{\phi} = \phi - \langle \phi \rangle^f$ and the interchangeable rules [17] between the Reynolds and volume averaged values are $\overline{\langle \phi \rangle^f} = \langle \tilde{\phi} \rangle^f$, $\langle \phi' \rangle^f = \langle \phi \rangle^{f'}$, $\tilde{\phi}' = (\tilde{\phi})'$ and $\tilde{\tilde{\phi}} = \tilde{\phi}$. Then the double (volume and Reynolds) averaged momentum equation is

$$\frac{\partial \langle \bar{u}_i \rangle^f}{\partial t} + \langle \bar{u}_j \rangle^f \frac{\partial \langle \bar{u}_i \rangle^f}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \langle \bar{p} \rangle^f}{\partial x_i} + \nu \frac{\partial^2 \langle \bar{u}_i \rangle^f}{\partial x_j^2} - \bar{f}_i + \bar{g}_i^{\phi} - \frac{\partial \left(\langle \tilde{u}_i \tilde{u}_j \rangle^f + \langle u'_i u'_j \rangle^f \right)}{\partial x_j}, \quad (4)$$

where f_i and g_i^{ϕ} are the drag force and hetero porous terms, respectively. The terms $\mathcal{T}_{ij} = \langle \tilde{u}_i \tilde{u}_j \rangle^f$ and $R_{ij}^A = \langle u'_i u'_j \rangle^f$ are the dispersive covariance and the volume averaged (total) Reynolds stress. The latter can be further decomposed as

$$R_{ij}^A = \underbrace{\langle u'_i \rangle^f \langle u'_j \rangle^f}_{R_{ij}} + \underbrace{\langle \tilde{u}'_i \tilde{u}'_j \rangle^f}_{r_{ij}}, \quad (5)$$

where R_{ij} and r_{ij} are namely the macro-scale and micro-scale Reynolds stresses, respectively. The transport equations of those second moments are

$$\frac{D \mathcal{T}_{ij}}{Dt} = \mathcal{D}_{ij} + \Psi_{ij} + \mathcal{D}_{ij}^p - P_{ij}^d + P_{ij}^t + \mathcal{E}_{ij}^{\phi} + \bar{f}_i \langle \hat{u}_j \rangle^f + \bar{f}_j \langle \hat{u}_i \rangle^f + \mathcal{E}_{ij}^{\phi}, \quad (6)$$

$$\frac{D R_{ij}}{Dt} = D_{ij} + \phi_{ij}^M + P_{ij} - P_{ij}^t + C_{ij}^t + G_{ij}^{\phi} - F_{ij} - \varepsilon_{ij}^M, \quad \frac{D r_{ij}}{Dt} = D_{ij}^m + \phi_{ij}^m + P_{ij}^m + P_{ij}^d - C_{ij}^t + G_{ij}^{\phi m} + F_{ij} - \varepsilon_{ij}^m, \quad (7)$$

where $\langle \hat{u}_i \rangle^f$ is the relative velocity to the solid phase, $\mathcal{D}_{ij}, \Psi_{ij}, \mathcal{D}_{ij}^p, \mathcal{E}_{ij}^{\phi}$ and $\mathcal{E}_{ij}^{\phi}$ are the diffusion, pressure-dispersive-velocity-strain, mean shear production, hetero-porous and dissipation terms of $\mathcal{T}_{ij}$, respectively. The terms $D_{ij}, \phi_{ij}^M, P_{ij}, G_{ij}^{\phi}$ and ε_{ij}^M are the diffusion, pressure-strain, mean shear production, hetero-porous and dissipation terms of R_{ij} while $D_{ij}^m, \phi_{ij}^m, P_{ij}^m, G_{ij}^{\phi m}$ and ε_{ij}^m are those of r_{ij} , respectively.

The mutual communication terms: $P_{ij}^d, P_{ij}^t, C_{ij}^t$ and F_{ij} , are the mean dispersive shear production, turbulent shear production, macro-micro turbulence cascade and turbulent drag force terms, respectively.

4. LES of turbulence inside porous media

Due to the difficulty to access the flow fields inside porous media experimentally, numerical simulation is a better choice to investigate turbulence inside porous media. Therefore, high resolution LES studies were carried out for turbulence inside several model porous media.

4.1. Numerical methods

The microscopic LES computations [12,13] were carried out by the WALE (wall-adaptive local eddy-viscosity) model [18] using an in-house code of the D3Q27 (3 dimensional 27 discrete velocity) MRT (multiple relaxation time) LBM (lattice Boltzmann method) [19]. The lattice Boltzmann equation can be obtained by discretizing the velocity space of the Boltzmann equation into a finite number of discrete velocities ξ_α $\{\alpha = 0, \dots, Q-1\}$. The MRT LBM transforms the distribution function $\mathbf{f}$ in the velocity space to the moment space by the transformation matrix $\mathbf{M}$. Since the moments of the distribution function correspond directly to flow quantities, the moment representation allows us to perform the relaxation processes with different relaxation-times according to different time-scales of various physical processes. The evolution equation is written as

$$|\mathbf{f}(\mathbf{x} + \xi_\alpha \delta t, t + \delta t)\rangle - |\mathbf{f}(\mathbf{x}, t)\rangle = -\mathbf{M}^{-1} \hat{\mathbf{S}} \left[|\mathbf{m}(\mathbf{x}, t)\rangle - |\mathbf{m}^{eq}(\mathbf{x}, t)\rangle \right] + \mathbf{M}^{-1} \left(\mathbf{I} - \frac{\hat{\mathbf{S}}}{2} \right) \mathbf{M} |\mathbf{F}\rangle \delta t, \quad (8)$$

where the notations such as $|\mathbf{f}\rangle$ is $|\mathbf{f}\rangle = (f_0, f_1, \dots, f_{Q-1})^T$, $\mathbf{I}$ is the identity matrix and δt is the time step. The term $\mathbf{F}$ is an external body force. The matrix $\mathbf{M}$ is a $Q \times Q$ matrix which linearly transforms the distribution function to the moment: $|\mathbf{m}\rangle = \mathbf{M} \cdot |\mathbf{f}\rangle$ and its collision matrix $\hat{\mathbf{S}}$ is diagonal:

$$\hat{\mathbf{S}} \equiv \text{diag}(0, 0, 0, 0, s_4, s_5, s_5, s_7, s_7, s_7, s_7, s_{10}, s_{10}, s_{10}, s_{10}, s_{13}, s_{13}, s_{13}, s_{13}, s_{16}, s_{17}, s_{18}, s_{18}, s_{20}, s_{20}, s_{20}, s_{20}, s_{23}, s_{23}, s_{23}, s_{26}), \quad (9)$$

for the D3Q27 discrete velocity model. The relaxation parameters used were

$$s_4 = 1.54, s_7 = s_5, s_{10} = 1.5, s_{13} = 1.83, s_{16} = 1.4, s_{17} = 1.61, s_{18} = s_{20} = 1.98, s_{23} = s_{26} = 1.74, \quad (10)$$

which were from Ref.[20]. The relaxation parameter s_5 is related to the fluid viscosity, and thus for the LES, it is $\nu + \nu_{SGS} = c_s^2 (1/s_5 - 1/2) \delta t$ where $c_s = 1/\sqrt{3}$ is the sound speed in the D3Q27 LBM.

In the WALE model [18], the eddy viscosity is described as

$$\nu_{SGS} = (C_w \Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(S_{ij}^d S_{ij}^d)^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}}, \quad (11)$$

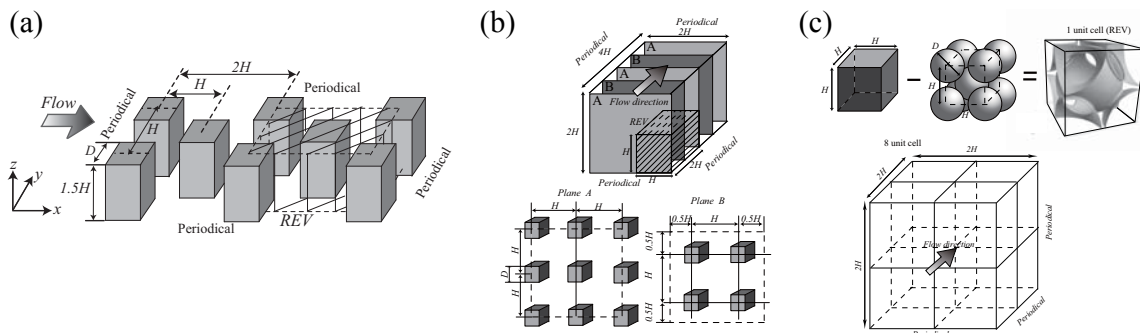


Figure 11. Model geometries and computational domains of model porous media; (a) square rod arrays, (b) staggered cube arrays, (c) BCC foam.

where $S_{ij} = (\partial \hat{u}_i / \partial x_j + \partial \hat{u}_j / \partial x_i) / 2$, $S_{ij}^d = \{(\partial \hat{u}_i / \partial x_j)^2 + (\partial \hat{u}_j / \partial x_i)^2\} / 2 - (\partial \hat{u}_k / \partial x_k)^2 \delta_{ij} / 2$ and the value $\hat{u}_i$ is the filtered velocity. The eddy viscosity coefficient applied was $C_w = 0.1$.

The computational domain considered were $4H(x) \times H(y) \times 1.5H(z)$, $4H \times 2H \times 2H$ and $2H \times 2H \times 2H$, respectively in square rod arrays, staggered cube arrays and BCC (body centered cube) foam as shown in figure 11. After performing grid sensitivity tests, uniform computational grids of $481 \times 121 \times 180$, $321 \times 161 \times 161$ and $256 \times 256 \times 256$ were used respectively for those geometries. Periodic boundary conditions were applied for each domain surfaces though a pressure difference was imposed in the streamwise direction. To describe the wall surfaces, the interpolated bounce-back boundary method was applied for the distribution functions. The computed flows were in the range of 1000-3500 of the pore Reynolds number.

4.2. Budget term analysis by the LES results [11,12]

By analyzing the simulation data, the behavior of the budget terms against the porosity was investigated. In fully developed flows inside homogeneous porous media, equations (6) and (7) reduce to

$$\frac{D\mathcal{T}_{ij}}{Dt} = \Psi_{ij} - P_{ij}^d + P_{ij}^t + \bar{f}_i \langle \hat{u}_j \rangle^f + \bar{f}_j \langle \hat{u}_i \rangle^f + \mathcal{E}_{ij} = 0, \quad (12)$$

$$\frac{DR_{ij}}{Dt} = \phi_{ij}^M - P_{ij}^t + C_{ij}^t - F_{ij} - \varepsilon_{ij}^M = 0, \quad \frac{Dr_{ij}}{Dt} = \phi_{ij}^m + P_{ij}^d - C_{ij}^t + F_{ij} - \varepsilon_{ij}^m = 0. \quad (13)$$

Since the macro-scale Reynolds stress is virtually zero in the fully developed porous medium flows, the budget terms of the micro-scale Reynolds stress were focused on. (Note that in the square rod array flows, it can be detected that a very small level of the cross-beamwise macro-scale Reynolds stress component is produced by the resonance of vortex shedding [12].) Also, it was found that the turbulent shear production P_{ij}^t became negligibly small in all the cases, and hence figure 12 only shows the behaviors of the dispersive shear production P_{ij}^d . In the square rod arrays case, P_{33}^d is zero in figure 12(a) because of zero spanwise gradient of the dispersive velocity. At $\phi = 0.5$, P_{11}^d and P_{22}^d are nearly

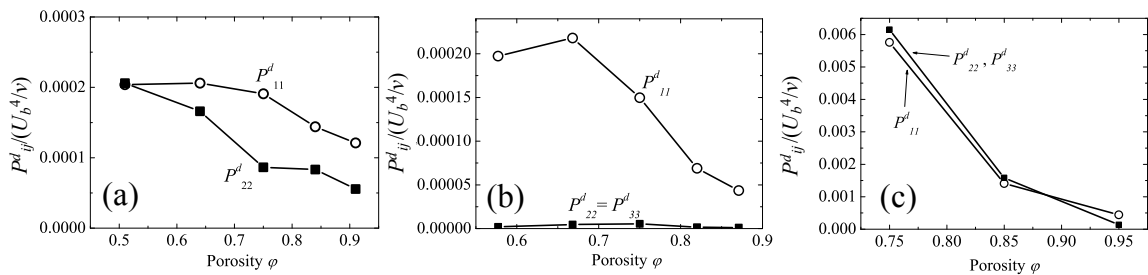


Figure 12. Dispersive shear production terms in the porous media; (a) square rod arrays, (b) staggered cube arrays, (c) BCC foam.

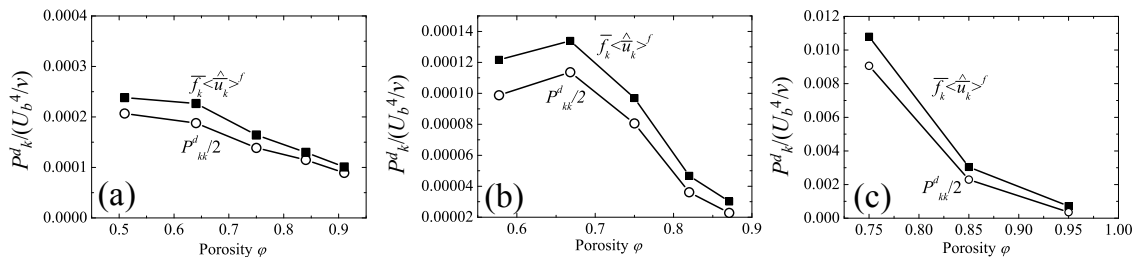


Figure 13. Production and drag terms in the porous media; (a) square rod arrays, (b) staggered cube arrays, (c) BCC foam.

the same and as the porosity increases, they tend to decrease while at $\phi > 0.75$ the decreasing rate of P_{22}^d becomes lower. In the cube array flows of figure 12(b), P_{11}^d is always dominant while the other components are marginally small. In the BCC foam flows, P_{ij}^d is nearly isotropic as shown in figure 12(c). It is thus understood that the behavior of P_{ij}^d , which is an important term to be modeled, is so diverse depending on the structure of the porous media.

In the fully developed porous media, the following relations come out from the transport equations of the dispersive and micro-scale turbulent energy which are derived from equations (12) and (13):

$$\frac{D\mathcal{T}_{kk}/2}{Dt} \approx -P_{kk}^d/2 + \bar{f}_k \langle \hat{u}_k \rangle^f - \mathcal{E}_{kk}^e/2 = 0, \quad \frac{Dr_{kk}/2}{Dt} \approx P_{kk}^d/2 - \varepsilon_{kk}^m/2 = 0, \quad (14)$$

since the macro-scale turbulence and P_{kk}^t are negligible. The drag force term $\bar{f}_k \langle \hat{u}_k \rangle^f$ in equation (14) balances with the sum of the dispersive shear production P_{kk}^d and the dissipation $\mathcal{E}_{kk}^e$ terms. The behaviours of the drag force and the dispersive shear production terms are indicated in figure 13. Interestingly, the profiles of the dispersive production and the drag force terms look synchronized in each case. This fact indicates that the majority of the dispersive kinetic energy, which is proportional to the drag force, transfers to the micro-scale turbulence energy via the dispersive shear production term P_{kk}^d . Consequently, with the relation: $P_{kk}^d \approx \varepsilon_{kk}^m$, the transferred energy dissipates in the micro-scale turbulence. For the modeling issue, it suggests that the dispersive shear production term can be scaled by the drag force term $\bar{f}_k \langle \hat{u}_k \rangle^f$ which is usually modeled using the Darcy-Forchheimer model.

5. Turbulence modelling

The present authors have been developing turbulence models considering the analysis of the LES data [14,15]. For validation of the developed models, comparison with the PIV data has been carried out. In the present paper, the development of a multi-scale four equation k - ε eddy viscosity (4 eqn. k - ε) model [15] is shown below.

5.1. Multi-scale four equation k - ε eddy viscosity model [15]

In the model, the eddy viscosity models are applied to R_{ij}^A and r_{ij} as

$$R_{ij}^A = \frac{2}{3} k^A \delta_{ij} - \nu_t^A S_{ij}, \quad \nu_t^A = c_\mu f_\mu^A \frac{(k^A)^2}{\tilde{\varepsilon}^A}, \quad r_{ij} = \frac{2}{3} k_m \delta_{ij} - \nu_t^m S_{ij}, \quad \nu_t^m = c_\mu f_\mu^m \frac{k_m^2}{\tilde{\varepsilon}_m}, \quad (15)$$

where S_{ij} is the double averaged strain tensor: $S_{ij} = \partial \langle \bar{u}_i \rangle^f / \partial x_j + \partial \langle \bar{u}_j \rangle^f / \partial x_i - (2/3) \delta_{ij} \partial \langle \bar{u}_k \rangle^f / \partial x_k$.

The (total) turbulent energy $k^A (= R_{kk}^A/2)$, the isotropic part of its dissipation rate $\tilde{\varepsilon}^A$, the micro-scale turbulent energy $k_m (= r_{kk}/2)$ and the isotropic part of its dissipation rate $\tilde{\varepsilon}_m$ are obtained respectively by solving their transport equations. Note that the isotropic dissipation rate is defined as $\tilde{\varepsilon} = \varepsilon - 2\nu(\partial\sqrt{k}/\partial x_j)^2$. The coefficient c_μ and the damping functions f_μ^A and f_μ^m are from the Launder-Sharma k - ε model [21]: $c_\mu=0.09$, $f_\mu^A = \exp[-3.4/(1+R_t^A/50)^2]$, $f_\mu^m = \exp[-3.4/(1+R_t^m/50)^2]$, $R_t^A = (k_m^A)^2/(\nu\tilde{\varepsilon}^A)$ and $R_t^m = k_m^2/(\nu\tilde{\varepsilon}_m)$.

The drag force term is modelled following the Darcy-Forchheimer term as

$$\bar{f}_i = \phi \nu K^{ij} \langle \hat{u}_j \rangle^f + \phi^2 C_{ik}^F \sqrt{K^{kj}} \langle \hat{u}_j \rangle^f \sqrt{\langle \hat{u}_l \rangle^f}, \quad (16)$$

where $\hat{u}_i$ is the relative velocity to the solid phase, K^{ij} and C_{ij}^F are the inverse matrix of the permeability tensor K_{ij} and the Forchheimer tensor, respectively. Since $\bar{f}_i$ balances with the pressure gradient of the momentum equation (4) in fully developed homogeneous porous medium flows, K^{ij} and C_{ij}^F can be obtained by fitting with the pressure gradients in several different flow rate conditions for each porous medium.

5.1.1. Modelling the total turbulence. The modelled transport equations of k^A and $\tilde{\varepsilon}^A$ are

$$\frac{Dk^A}{Dt} = \frac{\partial}{\partial x_k} \left[\left(v + \frac{v_t^A}{\sigma_k} \right) \frac{\partial k^A}{\partial x_k} \right] - R_{ij}^A \frac{\partial \langle \bar{u}_i \rangle^f}{\partial x_j} + P_{k_m}^d - P_k^t + G_{k^A}^\phi - \varepsilon^A, \quad (17)$$

$$\frac{D\tilde{\varepsilon}^A}{Dt} = \frac{\partial}{\partial x_k} \left[\left(v + \frac{v_t^A}{\sigma_\varepsilon} \right) \frac{\partial \tilde{\varepsilon}^A}{\partial x_k} \right] + F_\varepsilon + 2\nu v_t^A \left(\frac{\partial^2 \langle \bar{u}_i \rangle^f}{\partial x_j \partial x_k} \right)^2 + \frac{c_{\varepsilon 1}}{\tau^A} \left(-R_{ij}^A \frac{\partial \langle \bar{u}_i \rangle^f}{\partial x_j} + G_{k^A}^\phi \right) + \frac{c_{\varepsilon 2} f_{\varepsilon 2}}{\tau^A} \left[\frac{\tau^A}{\tau_s} (P_{k_m}^d - P_k^t) - \tilde{\varepsilon}^A \right], \quad (18)$$

where $\sigma_k = 0.5$, $\sigma_\varepsilon = 0.65$, $c_{\varepsilon 1} = 1.44$, $c_{\varepsilon 2} = 1.92$ and $f_{\varepsilon 2} = 1 - 0.3 \exp(-(R_t^A)^2)$. The time scales are $\tau^A = k^A / \tilde{\varepsilon}^A$ and $\tau_s = \ell_s / k_m^{1/2}$. The porous structural length scale is modelled as $\ell_s = (6.8 - 5\phi) \sqrt{K_{kk}^*}$ where $K_{ij}^* = (K_{ik} L_{kj} + K_{jk} L_{ki}) / 2$, $L_{ij} = \ell_i \ell_j$, $\ell_i = \langle \hat{u}_i \rangle^f / (\langle \hat{u}_l \rangle^f \langle \hat{u}_l \rangle^f)^{1/2}$. The hetero porous term $G_{k^A}^\phi$ is modelled as

$$G_{k^A}^\phi = \frac{\nu}{\phi} \left(2 \frac{\partial k^A}{\partial x_k} \frac{\partial \phi}{\partial x_k} + k^A \frac{\partial^2 \phi}{\partial x_k^2} \right) + \frac{v_t^A}{\sigma_{k^A}} \frac{\partial k^A}{\partial x_k} \frac{\partial \phi}{\partial x_k}. \quad (19)$$

For including the effects of the permeability, the term F_ε is introduced using the drag term F_k for the micro-scale turbulent kinetic energy as $F_\varepsilon = F_k c_{\varepsilon 3} (\tau_m^{-1} - \tau_M^{-1})$ where the time scales are $\tau_m = k_m / \tilde{\varepsilon}_m$ and $\tau_M = k_M / \tilde{\varepsilon}_M$ with $k_M = k^A - k_m$ and $\tilde{\varepsilon}_M = \tilde{\varepsilon}^A - \tilde{\varepsilon}_m$. The model coefficient $c_{\varepsilon 3}$ is $c_{\varepsilon 3} = 6.5 [1 - \exp\{-(R_t^A / 25)^2\}] \times [1 - \exp\{-(R_{K^*} / 9)^2\}]$ with the effective permeability Reynolds number $R_{K^*} = \sqrt{K_{kk}^*} \sqrt{k_M} / \nu$. With the assumption that the mean shear is assumed to be negligible in the REV, $P_{k_m}^d - P_k^t = (P_{ij}^d - P_{ij}^t) / 2$ is estimated as $P_{k_m}^d - P_k^t \approx \bar{f}_j \langle \hat{u}_j \rangle^f - \varepsilon^A$. The model of the dissipation ε^A of the dispersive kinetic energy $\mathcal{K}$ is shown later.

5.1.2. Modelling the micro-scale turbulence. The modelled transport equations of k_m and $\tilde{\varepsilon}_m$ are

$$\frac{Dk_m}{Dt} = \frac{\partial}{\partial x_k} \left[\left(v + \frac{v_t^m}{\sigma_k} \right) \frac{\partial k_m}{\partial x_k} \right] - r_{ij} \frac{\partial \langle \bar{u}_i \rangle^f}{\partial x_j} + P_{k_m}^d - C_k^t + F_k + G_{k_m}^\phi - \varepsilon_m, \quad (20)$$

$$\frac{D\tilde{\varepsilon}_m}{Dt} = \frac{\partial}{\partial x_k} \left[\left(v + \frac{v_t^m}{\sigma_\varepsilon} \right) \frac{\partial \tilde{\varepsilon}_m}{\partial x_k} \right] + 2\nu v_t^m \left(\frac{\partial^2 \langle \bar{u}_i \rangle^f}{\partial x_j \partial x_k} \right)^2 + \frac{c_{\varepsilon 1}}{\tau_m} \left(-r_{ij} \frac{\partial \langle \bar{u}_i \rangle^f}{\partial x_j} + G_{k_m}^\phi + \frac{c'_{\varepsilon 1} \tau_m F_k}{\tau_M} \right) + \frac{c_{\varepsilon 2} f_{\varepsilon 2}^m}{\tau_m} \left[\frac{\tau_m}{\tau_s} (P_{k_m}^d - C_k^t) - \tilde{\varepsilon}_m \right], \quad (21)$$

where $c'_{\varepsilon 1} = 0.694$ and $f_{\varepsilon 2}^m = 1 - 0.3 \exp(-(R_t^m)^2)$. The macro-micro turbulence cascade term and the mean dispersive production term are jointly modelled as

$$P_{k_m}^d - C_k^t = \bar{f}_j \langle \hat{u}_j \rangle^f - \varepsilon^A - [1 - \exp(-R_t^M / 100)] \left\{ 0.2(\varepsilon^A + \varepsilon_m) - 0.2(F_k + \bar{f}_j \langle \hat{u}_j \rangle^f) - (\mathcal{T}_{ik} + r_{ik}) \frac{\partial \langle \bar{u}_i \rangle^f}{\partial x_k} \right\}, \quad (22)$$

where $R_t^M = k_M^2 / (\nu \tilde{\varepsilon}_M)$. The models for the drag force term F_k and the hetero porous term $G_{k_m}^\phi$ are

$$F_k = \varphi \nu K^{ij} R_{ij} + \varphi^2 C_{ij}^F \sqrt{K^{jk}} R_{ki} \sqrt{\left(\langle \bar{u}_l \rangle^f\right)^2}, \quad G_{k_m}^\varphi = \frac{\nu}{\varphi} \frac{\partial(k^A + k_m)}{\partial x_k} \frac{\partial \varphi}{\partial x_k} + \frac{\nu}{\varphi} (2k_m - k^A) \frac{\partial^2 \varphi}{\partial x_k^2} + \frac{\nu_l^m}{\sigma_k} \frac{\partial k_m}{\partial x_k} \frac{\partial \varphi}{\varphi \partial x_k}. \quad (23)$$

5.1.3. Modelling the dispersive covariance. The dispersive covariance $\mathcal{T}_{ij}$ is the volume averaged second moment of the Reynolds averaged velocity dispersions and appears in the double averaged momentum equation irrespective of laminar and turbulent flows. However, this second moment has been often ignored and attributed to be included in the modelled Darcy-Forchheimer term, or it has been treated together with the turbulent kinetic energy. The present authors modelled the dispersive covariance through phenomenological considerations on the LES results [12,13] as follows.

In the local equilibrium condition in the REV of homogeneous porous media, the mean shear may be assumed to be ignored. Indeed, such a condition can be usually found deep inside a porous medium. Thus, the transport equations of k^A and the dispersive kinetic energy $\mathcal{K} = \mathcal{T}_{kk} / 2$ reduce to

$$\frac{Dk^A}{Dt} = P_{k_m}^d - P_k^t - \varepsilon^A = 0, \quad \frac{D\mathcal{K}}{Dt} = \bar{f}_j \langle \hat{u}_j \rangle^f - (P_{k_m}^d - P_k^t) - \mathcal{E} = 0. \quad (24)$$

In such a flow condition, the LES results confirmed that the micro-scale turbulence was often dominant compared to the macro-scale turbulence. Hence, $\varepsilon^A \approx \varepsilon_m$ and from equation (24), $\mathcal{E} + \varepsilon_m \approx \bar{f}_j \langle \hat{u}_j \rangle^f$.

This combined dissipation of the dispersion energy is scaled by the porous structural length scale ℓ_s as $\mathcal{E} + \varepsilon_m \approx \mathcal{K}^{1.5} / \ell_s$ and then the dispersive kinetic energy is obtained. As for the dissipation term $\mathcal{E}$, since it does not include velocity fluctuations and is considered to be a balancing term for the viscous drag force. Hence,

$$\mathcal{K} = \left(\ell_s \bar{f}_j \langle \hat{u}_j \rangle^f \right)^{2/3}, \quad \mathcal{E} = \varphi \nu K^{ij} \langle \hat{u}_i \rangle^f \langle \hat{u}_j \rangle^f. \quad (25)$$

Since in those flow conditions, the transport equation (12) of the dispersive covariance indicates that the main production of the dispersive covariance is by the drag term $\mathcal{T}_{ij} = \bar{f}_i \langle \hat{u}_j \rangle^f + \bar{f}_j \langle \hat{u}_i \rangle^f$ while $(P_{ij}^d - P_{ij}^t)$ transfers a part of energy to turbulent Reynolds stress while the dissipation term $\mathcal{E}_{ij}$ does not include turbulence effects. The dispersive covariance is thus phenomenologically modelled as the sum of two parts which include the effects of the drag force and structural contributions, respectively. They are thus modelled as

$$\mathcal{T}_{ij} = \mathcal{T}_{ij,1} + \mathcal{T}_{ij,2}, \quad \mathcal{T}_{ij,1} / \mathcal{K} = 2\beta \mathcal{T}_{ij} / \mathcal{T}_{kk}, \quad \mathcal{T}_{ij,2} / \mathcal{K} = 2(1-\beta) C_{ij}^{Fs} / C_{kk}^{Fs}. \quad (26)$$

The modified structural part of the Forchheimer tensor is $C_{ij}^{Fs} = C_{ij}^{Fl} - C_{ij}^{Fl*}$ and $C_{ij}^{Fl*} = (C_{ik}^{Fl} L_{jk} + C_{jk}^{Fl} L_{ik}) / 2$ where C_{ij}^{Fl} is the molecular part of the Forchheimer tensor obtained only in laminar flows. The bridge function is modelled as $\beta = [1 + 2 \exp(-6C_{kk}^{Fl*})] / 3$.

5.1.4. For interface regions. In the interface regions between porous media and clear fluids, the porosity distribution is modelled as $\varphi = \varphi_\infty + (1 - \varphi_\infty) \exp(-N_\varphi y' / D_p)$, where φ_∞ is the porosity of porous media, and y' is the normal distance from the porous surface. The coefficient N_φ is arranged so that the porosity varies within the distance of the mean pore diameter D_p . Hence, $N_\varphi = 4$ is applied. In the clear flow region, the permeability tensor and the Forchheimer tensor become infinite and zero, respectively. To impose those conditions, the permeability tensor and the Forchheimer tensor are respectively modified as $K_{ij} = K_{ij}^\infty / f_k$, $C_{ij}^F = f_{C_F} C_{ij}^{F\infty}$ where K_{ij}^∞ , $C_{ij}^{F\infty}$ are the permeability and Forchheimer tensors of the homogeneous porous media. The model functions are designed to vary between 0 and 1 within the distance of the mean pore diameter D_p :

$$f_k = 1 - \exp[-0.08(y' / \sqrt{K_{ij}^*})^2], \quad f_{c_F} = 1 - \exp[-1.41(y' / \sqrt{K_{ij}^*})^{1/2}]. \quad (27)$$

5.2. Validation results

5.2.1. Homogeneous porous media. Figure 14 shows comparison between the LES and the 4 eqn. k - ε model prediction results of the volume averaged turbulent energy and its dissipation rate in cases SQR, CUB and BCC for various porosity conditions. It is clearly seen that the results of the 4 eqn. k - ε model well accord with the microscopic LES in all the test flow conditions. Although the model of Ref.[7] (NK08) predicts general tendency reasonably, it overpredicts turbulent energy at lower porosity conditions in cases SQR and BCC. Figure 15 compares the dispersive covariance. Note that in the test flow field geometries, the dispersive covariance tensor is diagonal. Hence, off-diagonal components do not exist. It is clear that the magnitudes of the dispersive covariance are comparable to those of the turbulent energy. The dispersive covariance is thus never negligible in any cases whilst it has not been carefully treated in many other model proposals. As can be seen, the form applied in the 4 eqn. k - ε model for the dispersive covariance performs satisfactorily, though the agreement is not perfect. It is clear that the dispersive covariance does not become isotropic even in the isotropic porous media of cases CUB and BCC.

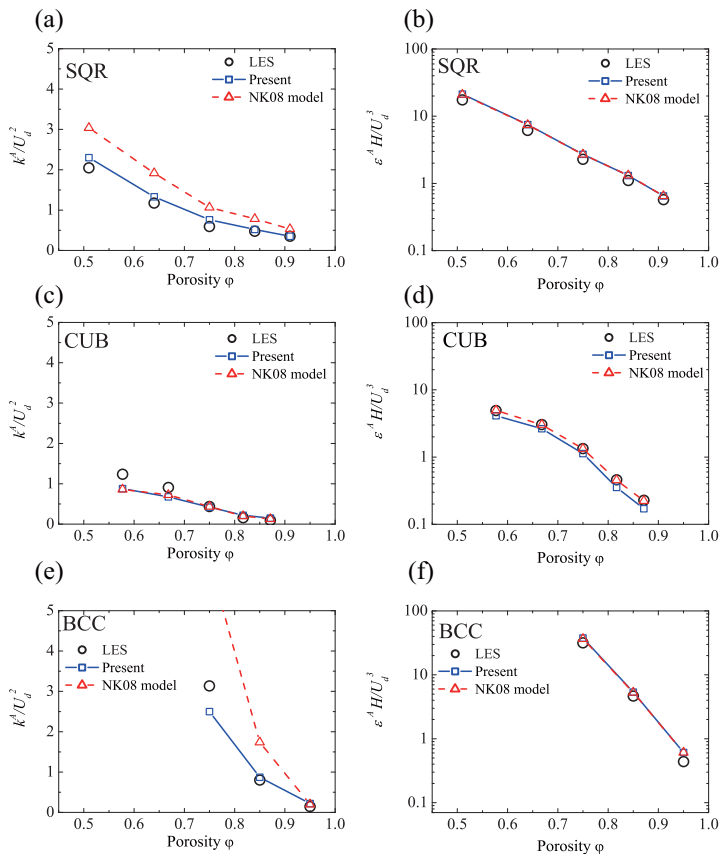


Figure 14. Comparison of the volume averaged turbulent energy and its dissipation rate between the macroscopic and LES computations; (a,b) square rod arrays (SQR) at $Re_h=3500$, (c,d) cube arrays (CUB) at $Re_h=3000$, (e,f) BCC foam (BCC) at $Re_D=1000$.

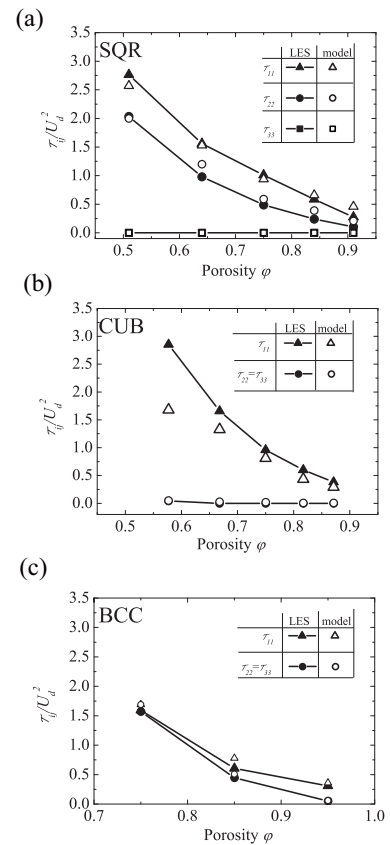


Figure 15. Comparison of the dispersive covariance between the model predictions and the LES; (a) case SQR, (b) case CUB, (c) case BCC.

5.2.2. Porous channel flows. Figures 16 and 17 compare the mean velocity and turbulent quantity profiles of the 4 eqn. $k-\varepsilon$ model with those of the porous channel experiments. In those flows over porous media, flow characteristics are characterised by the permeability Reynolds number Re_K . The experiments showed that porous wall flows had two distinctive flow regimes at $Re_K < 3$ and $Re_K \gg 3$. As shown in figure 16, the overall agreement in the mean velocities of the present results and the data is reasonable. In cases #20, #13 and #06, the permeability Reynolds numbers are respectively $Re_K = 1.1$, 3.1 and 11 which are from the transitional regime to the full porous wall turbulence regime. As Re_K increases, the mean velocity and the turbulent energy profiles tend to penetrate into the porous media and a larger slip velocity can be seen. The agreement in turbulence quantities shown in figure 17 is also acceptable though the peak values of the turbulent energy near the surfaces are not well predicted. Since the Launder-Sharma model, which is the mother model of the 4 eqn. $k-\varepsilon$ model, also has similar performance, it is considered that its defect carries over to the proposed model.

5.2.3. Rib-mounted porous channel flows. Figure 18 (a)-(c) compares the streamwise mean velocity, turbulent energy and Reynolds shear stress profiles of the porous-rib-mounted porous channel flow. The predicted profiles by the 4 eqn. $k-\varepsilon$ model well agree with those of the experiments. It is considered that the flow rate going through the rib increases as the increase of the permeability. This tendency was well captured by the 4 eqn. $k-\varepsilon$ model since the agreement in the velocity distributions just behind the rib is satisfactory although different wall permeability cases are not shown here. The experiments showed that the turbulent energy and the Reynolds shear stress in the downstream region became smaller in the higher permeability case. The 4 eqn. $k-\varepsilon$ model predicts this tendency well.

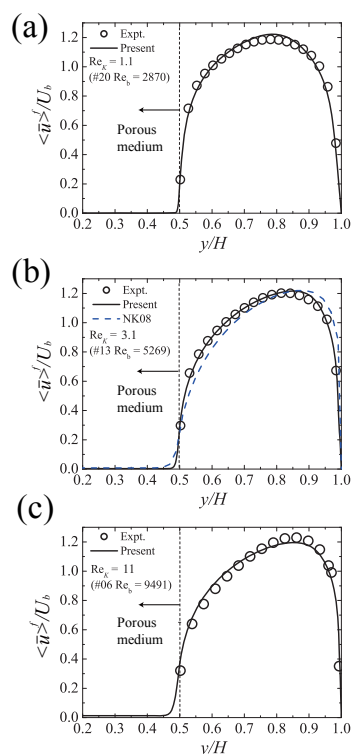


Figure 16. Comparison of streamwise mean velocity profiles between the prediction and the experiments; (a) case #20, (b) case #13, (c) case #06.

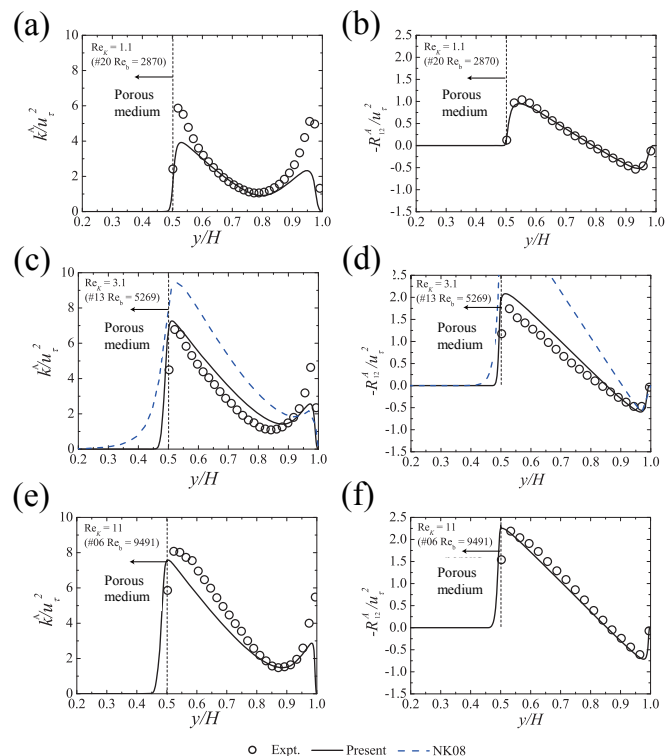


Figure 17. Comparison of turbulent energy and Reynolds shear stress between the prediction and the experiments; (a,b) case #20, (c,d) case #13, (e,f) case #06.

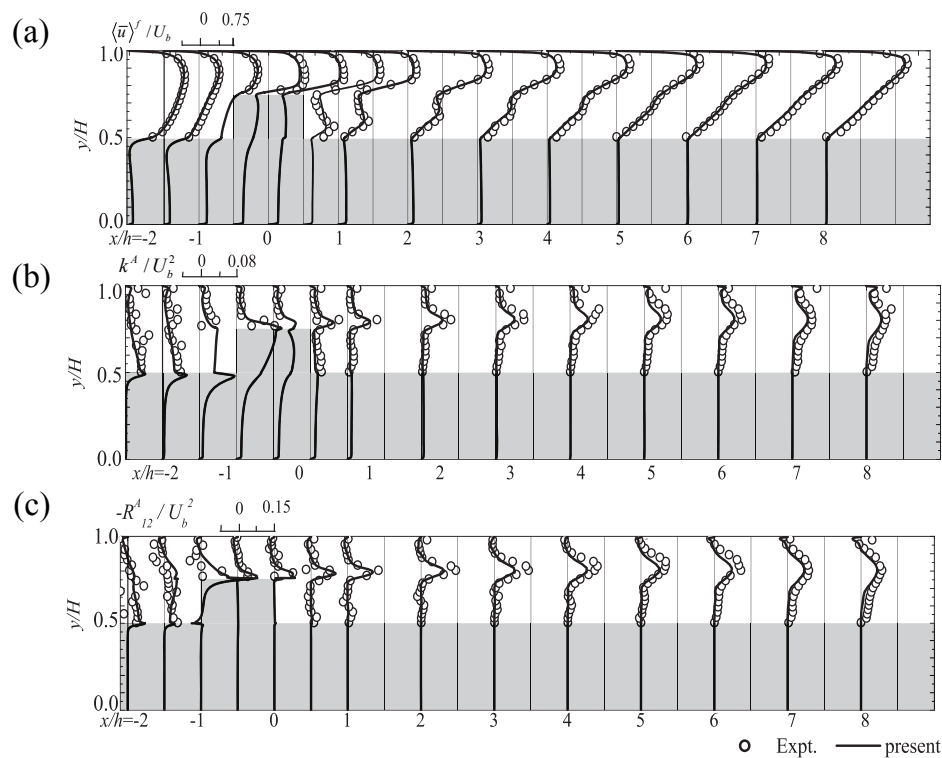


Figure 18. Comparison of the mean velocity and turbulence profiles with the rib-mounted porous channel experiments in case #06 at $Re_b=10600$; (a) streamwise mean velocity, (b) turbulent energy, (c) Reynolds shear stress.

6. Conclusions

By the PIV experiments, the following points were confirmed. Above a permeable wall, transition to turbulence occurs at progressively lower Reynolds numbers as the wall permeability increases. The magnitude of the slip velocity on the porous wall depends on Re_K . It drastically increases in a narrow range of $Re_K < 3$ corresponding to laminar to turbulent transition. The flows are not in the condition comparable to the full roughness regime due to the effects of the wall permeability superimposed on the roughness effects. The wall normal component of the velocity fluctuation near a porous wall tends to be higher as the increase of the wall permeability and the Reynolds number though the streamwise component does not have such an obvious tendency. Since blocking effects of a porous wall on vortex motions are weakened more as Re_K increases, the wall normal fluctuation is kept at a higher level. This contributes to a higher level of the turbulent wall shear stress at a porous wall.

The measurement results of the rib-mounted porous channel flows confirmed the following points. In the solid rib-mounted porous channel flows, since a part of entraining fluid is supplied through the permeable bottom wall from the region upstream the rib, the recirculation behind the rib in the clear channel becomes smaller and eventually vanishes as the wall permeability increases. Because of the reduction of the magnitude and the size of the reverse flow region in the clear channel, turbulence becomes smaller. In the porous-rib-mounted porous channel flows, because of the fluid passing through the rib, the recirculation is more significantly weakened and shifts downstream. Because of the flow through the porous rib, turbulence becomes smaller than those of the solid-rib flows.

From the LES for porous medium flows, the following points were obtained. The profiles of the dispersive production and the drag force terms of the dispersive kinetic energy are synchronized. This indicates that the majority of the dispersive kinetic energy, which is proportional to the drag force,

transfers to the micro-scale turbulence energy via the dispersive shear production term. Consequently, the transferred energy dissipates in the micro-scale turbulence. For the modelling issue, it suggests that the dispersive shear production term may be scaled by the drag force term.

To predict turbulence around and inside porous media, three kinds of second moments: the dispersive covariance, the volume averaged (total) Reynolds stress and the micro-scale Reynolds stress were individually modelled in the four equation k - ε model. In the proposed model, two sets of the modelled k and ε transport equations are solved for the eddy viscosity models of the total and the micro-scale Reynolds stresses. For the dispersive covariance, an algebraic form is applied after the discussions using the LES results. The validation in fully developed flows in homogeneous porous media, porous channel flows and porous rib mounted channel flows confirms that the proposed method is promising. Indeed, the model predicts the turbulent and dispersive kinetic energy profiles well in three kinds of porous media in a wide range of porosity. The results of the porous channel flows show that the prediction accuracy is satisfactory. The overall agreement between the prediction and the experiments is also satisfactory in the porous-rib-mounted porous channel flows.

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