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Magnetic-field-induced quantum criticality in a planar ferromagnet with single-ion anisotropy

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Abstract. We analyze the effects induced by single-ion anisotropy on quantum criticality in a d-dimensional spin-3/2 planar ferromagnet. To tackle this problem we employ the two-time Green's function method, using the Tyablikov decoupling for exchange interactions and the Anderson-Callen decoupling for single-ion anisotropy. In our analysis the role of non-thermal control parameter which drives the quantum phase transition is played by a longitudinal external magnetic field. We find that the single-ion anisotropy has substantial effects on the structure of the phase diagram close to the quantum critical point.

The study of quantum magnetic systems is a very active research subject in condensed matter physics [1, 2]. In particular, field-induced quantum phase transitions (QPTs) and anisotropy effects have been the focus of a great amount of theoretical and experimental investigations [3, 4]. Moreover, quantum critical points (QCPs) [3] are currently considered responsible for the unusual low-temperature properties observed in many low-dimensional magnetic compounds. Furthermore, there are also clear evidences that magnetic anisotropies of different nature play a fundamental role and lead to new interesting phenomena [5, 6]. Indeed, anisotropic Heisenberg models have attracted increasing attention as a basic tool to understand relevant properties of several magnetic materials. In particular, Heisenberg models with easy-plane or easy-axis anisotropies [7] have attracted a lot of interest, especially for low-dimensional ferromagnets (FM) and antiferromagnets (AFM). In this context, in constructing spin models for several anisotropic magnetic samples, one has also to face the presence of additional crystal anisotropic fields as single-ion anisotropy (SIA) [8].

Intensive studies [9, 10] have been focused on the XXZ model with exchange anisotropies and in the presence of an external longitudinal field h. In this paper we consider also the inclusion of SIA [8, 11], hence we are going to study the following Hamiltonian (in convenient units)

$$\mathcal{H} = -\frac{1}{2}\sum_{i,j=1}^{N}\Big[J_{ij}(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)\Big] - h\sum_{i=1}^{N} S_i^z - D\sum_{i=1}^{N}(S_i^z)^2. \qquad (1)$$

Here $S_i^\alpha (\alpha = x, y, z)$ denote the components of the vector spin operator $\mathbf{S}_i$ at site i of a $d-$dimensional hypercubic lattice with N sites, satisfying the commutation relations $[S_i^\alpha, S_j^\beta] = i\varepsilon_{\alpha\beta\gamma}\delta_{ij}S_i^\gamma$ ($\varepsilon_{\alpha\beta\gamma}$ is the usual Levi-Civita tensor), J_{ij} (with $J_{ii} = 0$) is the planar exchange coupling between the spins at sites i and j, Δ gives the strength of the exchange anisotropy,

h is the external longitudinal field and D is the SIA paramter. Positive or negative values of J_{ij} correspond to FM or AFM cases. Our attention is devoted to FM models and one has a uniaxial ferromagnet if $\Delta > 1$ (easy-axis exchange anisotropy), the isotropic Heisenberg model when $\Delta = 1$, and the planar ferromagnet (PFM) if $\Delta < 1$ (easy-plane exchange anisotropy). As concerning the SIA parameter D, we have that $D > 0$ gives rise to easy-axis SIA and $D < 0$ to easy-plane one. This kind of anisotropy has to be necessarily included in several situations to obtain a more accurate correspondence between theoretical finding and experimental data and occur only in system with spin $S \geq 1$. In the present paper we are going to consider the peculiar case $S = 3/2$, also because there are certain ferromagnetic compounds, containing Fe^{3+}, which assumes the unusual spin S=3/2 state and where crystal field effects are important [12].

In absence of SIA, the PFM exhibits a magnetic-field-induced QPT [4, 9, 13] with characteristics similar to those of the XY model in a transverse field [14]. The model was first introduced and studied, at a mean-field approximation (MFA) level, by Matsubara and Matsuda [15] as a pseudospin model for a gas of hard core bosons with attractive interaction to explain superfluidity in ^{4}He. On the experimental front, several magnetic compounds as spin dimer $Cs_3Cr_2Br_9$ [16], the cuprate $Sr_{14}Cu_{24}O_{41}$ [17], the crystal CaV_2O_5 with layers of coupled two-leg ladders [18], and the laddered $SrCu_2O_5$ [19], exhibit field-induced QPTs which appear to be qualitatively well described by the model (1) with easy-plane exchange anisotropy.

However, due to the complex crystalline structure of magnetic materials and, in several cases, the quantitatively unclear validity of the conventional paradigm of quantum criticality [3], it seems reasonable to explore the effects of additional anisotropic crystal fields on the ideal quantum criticality in the PFM.

We use the two-time Green function (GF) method by adopting the Tyablikov decoupling (TD) [20] for the exchange anisotropy and the Anderson-Callen decoupling (ACD) [21] for the single-ion term in the equations of motion chain. At this level of approximation one must keep in mind that the TD was shown to achieve nearly exact predictions close to the field-induced QCP of the PFM in absence of the SIA [10, 9], while the ACD is widely used for the SIA term, although this approximation may be not valid for large values of the parameter D [11, 22].

We follow the Callen approach for the isotropic Heisenberg model [23], which has been used also when anisotropies are presents [9, 24]. We introduce the retarded two-time GF for commutator (in units $\hbar = 1$)

$$G_{ij}(t-t') \;=\; \langle\langle S_i^+(t-t'); e^{aS_j^z} S_j^- \rangle\rangle = -i\theta(t-t')\langle [S_i^+(t-t'), e^{aS_j^z} S_j^-] \rangle \tag{2}$$

where $A(t) = e^{i\mathcal{H}t} A e^{-i\mathcal{H}t}$, $\theta(x)$ is the step function, $\langle \cdots \rangle$ denotes the equilibrium average with Hamiltonian (1) and $\beta = 1/T$ is the inverse temperature. The auxiliary Callen parameter a has to be set to zero at the end of calculations for extracting the physics of the model.

The equation of motion for the time Fourier transform $G_{ij}(\omega)$ is

$$\begin{aligned}(\omega - h)\langle\langle S_i^+ | e^{aS_j^z} S_j^- \rangle\rangle_\omega = \psi(a)\delta_{ij} - \sum_h \langle\langle J_{hi}(S_i^z S_h^+ \\ -\Delta S_h^z S_i^+) | e^{aS_j^z} S_j^- \rangle\rangle_\omega + D\langle\langle S_i^z S_i^+ + S_i^+ S_i^z | e^{aS_j^z} S_j^- \rangle\rangle_\omega,\end{aligned} \tag{3}$$

where

$$\psi(a) = \langle [S_i^+, e^{aS_i^z} S_i^-] \rangle, \tag{4}$$

is a site independent quantity to be determined as a function of the Callen parameter a. In particular, $\psi(0) = \langle [S_i^+, S_i^-] \rangle = 2m$, where $m = \langle S_i^z \rangle$ is the longitudinal magnetization per spin.

In order to get a closed equation of motion for $G_{ij}(\omega)$, the higher order Green's functions on the right hand side of eq. (3) has to be properly decoupled. We adopt here the TD [20], for the exchange interaction term

$$\langle\langle S_h^z S_k^+ | e^{aS_j^z} S_j^- \rangle\rangle_\omega \simeq \langle S_h^z \rangle \langle\langle S_k^+ | e^{aS_j^z} S_j^- \rangle\rangle_\omega \tag{5}$$

and the ACD [21] to decouple the single-ion term,

$$\langle\langle S_i^z S_i^+ + S_i^+ S_i^z | e^{aS_j^z} S_j^- \rangle\rangle_\omega \simeq 2C_S \langle S_i^z \rangle \langle\langle S_i^+ | e^{aS_j^z} S_j^- \rangle\rangle_\omega, \tag{6}$$

with $C_S = 1 - \frac{1}{2S^2}\left[S(S+1) - \langle (S_i^z)^2 \rangle\right]$. The term $\langle (S_i^z)^2 \rangle$ in the coefficient C_S can be expressed in terms of $\langle S_i^z \rangle$ and $\langle S_i^- S_i^+ \rangle$ using the identity $(S_i^z)^2 = S(S+1) - S_i^z - S_i^- S_i^+$. With $S = 3/2$, as in the case under study, it is $C_{3/2} = 1 - 2/9[\langle S_i^z \rangle + \langle S_i^- S_i^+ \rangle]$.

Using the above decoupling approximation, the equation of motion eq. (3) reduces to

$$\left[\omega - h - m(\Delta J(0) + 2C_{3/2}D)\right] G_{ij}(\omega) = \psi(a)\delta_{ij} - m \sum_l J_{li}^{\perp} G_{lj}(\omega) \tag{7}$$

where $J(0)$ is the zero-wave-vector Fourier component of J_{ij}.

Using Fourier transforms, with the wave-vectors $\mathbf{k}$ ranging within the first Brillouin zone (1BZ), the equation of motion is readily diagonalized providing the formal solution

$$G_{\mathbf{k}}(\omega) = \frac{\psi(a)}{\omega - \omega_{\mathbf{k}}}, \tag{8}$$

with $\omega \to \omega + i\varepsilon$ ($\varepsilon \to 0^+$) for retarded GF where

$$\omega_{\mathbf{k}} = \omega_0 + m[J(0) - J(\mathbf{k})] \tag{9}$$

is the energy spectrum of the undamped spin-excitations and

$$\omega_0 = h - m[J(0)(1-\Delta) - 2C_{3/2}D] \tag{10}$$

is the corresponding energy gap. The quantity $C_{3/2}$ can be consequently written, via the spectral theorem [20], as $C_{3/2} = 1 - \frac{2m}{9}(1 + 2\Phi)$, where

$$\Phi = \frac{1}{N}\sum_{\mathbf{k}} \frac{1}{e^{\beta\omega_{\mathbf{k}}} - 1} \overset{N\to\infty}{=} \int_{1BZ} \frac{d^d k}{(2\pi)^d} \frac{1}{e^{\beta\omega_{\mathbf{k}}} - 1} \ . \tag{11}$$

The key point in our scheme is to determine the expression of $\psi(a)$ as a function of the Callen parameter a and hence of $m = \psi(0)/2$.

Following ref.[24] we find for the magnetization

$$m = \frac{3}{2} - \Phi + \frac{4}{(1 + \Phi^{-1})^4 - 1} \equiv M(\Phi) \ . \tag{12}$$

Eqs. (9)-(12) constitute a set of self-consistent equations to determine m, $G_{\mathbf{k}}(\omega)$ (at $a = 0$), and other relevant thermodynamic quantities as functions of T and h, for fixed values of the SIA parameter D.

To study the static and dynamic properties, we need the transverse GF

$$G_\perp(\mathbf{k}, \omega) = \langle\langle S_i^+ | S_j^- \rangle\rangle_{\mathbf{k},\omega} \equiv G_{\mathbf{k}}(\omega)|_{a=0} \ , \tag{13}$$

which, at our level of approximation, is given by

$$G_\perp(\mathbf{k}, \omega) = \frac{2m}{\omega - \omega_{\mathbf{k}}}. \tag{14}$$

This leads to the dynamical transverse susceptibility

$$\chi_\perp(\mathbf{k},\omega) = -G_\perp(\mathbf{k},\omega) = \frac{2m}{\omega_\mathbf{k} - \omega}\ , \tag{15}$$

and, as a particular case, the static one

$$\chi_\perp(T,h) = \chi_\perp(\mathbf{k}=0,\omega=0) = \frac{2m}{\omega_0}\ . \tag{16}$$

The stability condition on susceptibility ($\chi_\perp > 0$) implies that $\omega_0 \geq 0$. Thus the equation $\omega_0 = 0$ ($\chi_\perp \to \infty$) defines the stability boundary and hence the possible critical points.

Hereafter we will consider only the case of easy-plane exchange anisotropy ($\Delta < 0$), in order to explore the effects of the SIA on the quantum criticality in a planar ferromagnet (related to the field-driven in-plane ordering described by the quantities ω_0 and $\chi_\perp$ as function of T and h [9, 10, 24].

In absence of SIA ($D = 0$) the systems exhibit a QPT at the value of the field $h_{0c} = \frac{3}{2}J(0)(1-\Delta)$. In the presence of the SIA, the QPT does not survive for all the value of the parameter D. Let us rewrite the expression for ω_0

$$\omega_0 = h - M(\Phi)\left[J(0)(1-\Delta) - 2D\left(1 - \frac{2M(\Phi)}{9}(1+2\Phi)\right)\right], \tag{17}$$

which is a self-consistent expression, reminding the definition of Φ (see (11)).

As first step we study the problem at zero temperature. Since as $T \to 0$ we have that $\Phi \to 0$, then from eq.(12), we get $M(0) = m(T=0) = 3/2$. Hence, for the gap equation at $T = 0$ we have

$$\omega_0 \quad = \quad h - \frac{3}{2}\left[J(0)(1-\Delta) - \frac{4}{3}D\right] = h - h_{0c} + 2D. \tag{18}$$

Thus, for $h > h_{0c} - 2D$, we have for the static transverse susceptibility

$$\chi_\perp = \frac{3}{h - h_{0c} + 2D} \equiv \frac{3}{h - h_c}. \tag{19}$$

This expression clearly shows that a QCP exist for any dimensionality of the systems and the critical magnetic field at $T = 0$ is $h_c = h_{0c} - 2D$, which depends on the strength of the SIA parameter D. Moreover we obtain for susceptibility a mean-field like behavior, with critical exponent $\gamma = 1$, hence at zero temperature the SIA produce just a shift of the QCP. However if $D = h_{0c}/2$ we have a zero critical field, i.e. a suppression of the QPT. Thus, quantum criticality is realized only if $D < \bar{D} = \frac{3}{4}J(0)(1-\Delta)$.

We now analyze the system at finite temperature. We limit ourselves to the case of short-range exchange interaction and d-dimensional hypercubic lattice, with interaction $J(\mathbf{k}) = J(0) - Jk^2$, with $J(0) = 2dJ$. From now on all quantities will be written in unity of J. The basic quantity to solve our problem is the function Φ (11). On the phase boundary, being $\omega_0 = 0$, we have

$$\Phi_c = \Phi|_{\omega_0=0} = \int_{1BZ} \frac{d^dk}{(2\pi^d)} \frac{1}{e^{\frac{3}{2}\beta M(\Phi_c)k^2} - 1}\ . \tag{20}$$

It is evident that,while a QCP exists for any d, a finite temperature critical line ending in the QCP, may take place only for dimensionalities which assure the convergency of the integral (20). Indeed, we see that only for $d > 2$ we have a finite temperature critical line, ending in the QCP

$(T = 0, h = h_c)$, consistently with the Mermin-Wagner theorem [26]. In such a case we can write a critical line equation (using eqs. (17) and (20))

$$h - M(\Phi_c)\left[\frac{2}{3}h_{0c} - 2D\left(1 - \frac{2M(\Phi_c)}{9}(1 + 2\Phi_c)\right)\right] = 0. \tag{21}$$

This self-consistent equation has been solved numerically for $d = 3$, for three different value of the exchange anisotropy parameter Δ, and changing the SIA parameter D. The results are shown in fig. 1.

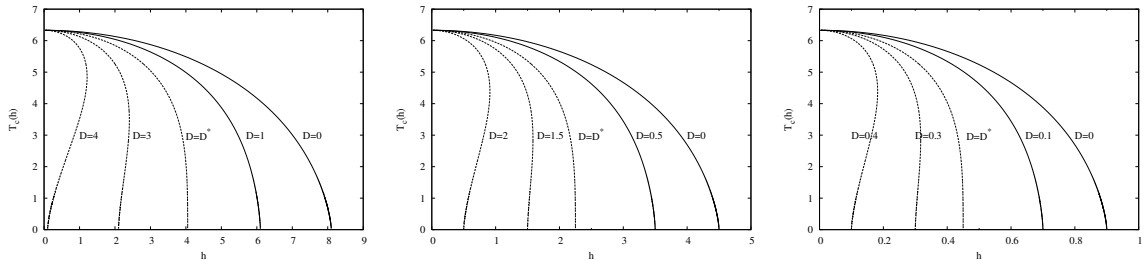

Figure 1. Critical lines for different values of single-ion anisotropy parameter D in the (h, T)-plane as obtained by solving numerically the general equation (21) at $d = 3$ for three different values of the parameter Δ: namely, from left to right, $\Delta = 0.1$, $\Delta = 0.5$ and $\Delta = 0.9$. The borderline value D^* of the single-ion anisotropy parameter D is defined in the text.

By inspection of the figure it is evident the effect played by the SIA on the phase diagram of the ideal PFM ($D = 0$). The typical behavior of the known critical line ending at the QCP of PFM with $D = 0$ breaks down when D exceeds the value D^*, which numerically for $d = 3$ is found to be $D^* = \frac{9}{4}(1 - \Delta)$. For smaller values of D the shape of the critical line remains essentially similar to that of the PFM in absence of the SIA, hence we expect that also the low-temperature behavior around the critical line should be preserved.

In the following we will consider only $D < D^*$, in order to study the effects of SIA on conventional quantum criticality of PFM.

Analytical expressions can be derived in asymptotical regimes. Hence we consider the two following cases: (i) $T \to 0$ and (ii) $h \to 0$. In the first case, i.e. close to the QCP, we have [24]

$$h_c(T) \simeq h_c - b_d(h_c - 2D)T^{d/2} \tag{22}$$

where b_d is a coefficient which depends on the spin S and on the dimensionality of the system. In the present case, with $S = 3/2$ $b_d = \zeta(d/2)/[2^{(d-2)/2}3^{(d+2)/2}\pi^{d/2}]$, in particular for $d = 3$ we have $b_3 = \zeta(1.5)/(9\sqrt{6}\pi^{3/2}) \simeq 0.0213$. By inspection of equation (22), we see that there exists a value of D above which the coefficient of $T^{d/2}$ changes sign, i.e. for $D > h_c$, and hence the low-temperature part of the critical line shows the reentrant behavior. This provides the analytical expression for the borderline value D^* of the SIA parameter D, which is $D^* = h_{0c}/4 = 3J(0)(1 - \Delta)/8$, that coincides with the numerical result for $d = 3$.

For case (ii), i.e. in the limit $h \to 0$, we have [24]

$$T_c(h) \simeq T_c(0)\left[1 - \left(12 + \frac{5d^2}{d^2 - 4}\right)\frac{3}{125}\frac{h^2}{h_c + \frac{2}{3}D}\right], \tag{23}$$

where

$$T_c(0) = \frac{5}{16\pi}\frac{d}{d-2}\left(\frac{d}{2}\Gamma(d/2)\right)^{-2/d}. \tag{24}$$

The critical temperature at zero field $T_c(0)$ does not depend on the value of the SIA parameter D.

We proceed now by studying the low-temperature properties and crossovers of our anisotropic XXZ ferromagnet in the easy-plane disordered phase close to the QCP where the energy gap ω_0 is small. In this low-T regime, being Φ very small, we can use a simplified expression for magnetization $M(\Phi) \simeq \frac{3}{2} - \Phi$. Furthermore, considering the limit $T \to 0$ in the integral (11) which defines Φ and with $\omega_{\mathbf{k}} = \omega_0 + mk^2$, we have

$$\Phi \simeq \frac{K_d}{2} \left(\frac{T}{m}\right)^{d/2} \int_0^\infty dx \frac{x^{(d-2)/2}}{e^{\beta\omega_0 + x} - 1}. \tag{25}$$

The integral which appear in the above equation is the Bose-Einstein function $F_\nu(y) = \int_0^\infty dx \frac{x^{\nu-1}}{e^{x+y}-1}$ for which the asymptotic expansions in y for different values of ν are known [9]. This provides for the magnetization

$$m \simeq \frac{3}{2} - \frac{K_d}{2} \left(\frac{2}{3}T\right)^{d/2} F_{d/2}(\omega_0/T). \tag{26}$$

We can then write a self-consistent equations for the energy gap ω_0, to leading order in T

$$\omega_0 \simeq h - h_c + \frac{K_d}{3}(h_c - 2D) \left(\frac{2}{3}T\right)^{d/2} F_{d/2}(\omega_0/T)\ . \tag{27}$$

For any d, with $h > h_c$ in the quantum regime we find for the energy gap (with $F_{d/2}(\omega_0/T) \simeq \Gamma(d/2)e^{-\omega_0/T}$)

$$\omega_0(T,h) \simeq (h - h_c) + \Gamma(d/2) C_{d,D} T^{d/2} e^{-(h-h_c)/T}\ , \tag{28}$$

which differs from the MF result $\omega_0 \simeq (h - h_c)$ for an exponentially small correction in $(h - h_c)/T \gg 1$. In eq.(28) it is $C_{d,D} = \frac{K_d}{3}(2/3)^{d/2}(h_c - 2D)$.

We now determine the asymptotic solution of (27) in the classical ($\omega_0/T \ll 1$) and quantum ($\omega_0/T \gg 1$) regimes, and related crossovers, for $d = 2$ and $d > 2$.

• $d = 2$

For this value of dimensionality we do not have a critical line, but only a QCP for $D < \bar{D}$. Close to the QCP, the energy gap equation is obtained setting $d = 2$ and $F_1(\omega_0/T) = \ln(1 - e^{-\omega_0/T})^{-1}$ in eq. (27). This equation, which is valid for arbitrary ω_0/T, cannot be solved in general but one can obtain explicit asymptotic expressions in relevant regimes.

We start with the classical regime. Simple considerations show that no physical solution occurs for $h \geq h_c$ while, for $h < h_c$ and $(h_c - h) \gg T$, one easily gets

$$\omega_0(T,h) \simeq T \exp\left[-\frac{h_c - h}{C_{2,D}T}\right]\ . \tag{29}$$

This result implies that the transverse susceptibility (and hence the correlation length $\xi_\perp \propto \chi_\perp^{1/2}$) diverges exponentially as $T \to 0$ at fixed $h < h_c$.

Within the region of the (h,T)-plane delimited by the two crossover lines $T^- = h_c - h$ and $T^+ = h - h_c$, we can obtain a consistent asymptotic solution for $|h - h_c| \ll T$ only assuming ω_0/T finite. Let us first rewrite eq.(27) for $d = 2$

$$\frac{\omega_0}{T} = \frac{h - h_c}{T} + \frac{h_c - 2D}{9\pi} \ln\left(\frac{1}{1 - e^{-\omega_0/T}}\right). \tag{30}$$

We are interested in the behavior on and around the so-called quantum critical trajectory (QCT) $h = h_c$, where we can rewrite eq. (30) in the form $\omega_0 = bT$ where b is determined by the equation $b = C_{2,D} \ln\left(\frac{1}{1-e^{-b}}\right)$. Then by expanding eq. (30) in $(h - h_c)/T$ with $\omega_0/T = b$ as zero order solution, to leading order in $h - h_c$, the energy gap reads

$$\omega_0(T, h) \simeq bT + [1 + C_{2,D}(e^b - 1)^{-1}]^{-1}(h - h_c) \, . \tag{31}$$

The phase diagram for $d = 2$ is shown in fig. 2. It is worth noting that the previous scenario breaks down as $D \to D^*$ (i.e. $C_{d,D} \to 0$), as expected.

• $d > 2$

For such dimensionalities a critical line exists ending in the field-induced QCP, and it is convenient to rewrite eq. (27) in terms of the distance $g(T) = h - h_c(T) \geq 0$ from the critical line in the disordered phase. This is simply performed by combining eqs. (22) and (27) to write

$$\omega_0 = g(T) + C_d(D) T^{d/2} G_{d/2}(\omega_0/T) \, , \tag{32}$$

where $G_{d/2}(y) = F_{d/2}(y) - F_{d/2}(0)$. The full low-temperature physics for $d > 2$ follows from this equation but we limit ourselves to $2 < d < 4$ since for $d \geq 4$ MF-like findings can be trivially obtained with logarithmic corrections at $d = 4$, consistently with the theory of critical phenomena. We first explore the classical regime $\omega_0/T \ll 1$, where $G_{d/2}(\omega_0/T) \simeq -\pi |\sin(\pi d/2)|^{-1} (\omega_0/T)^{d/2-1}$. This lead us to the equation for the ω_0

$$\omega_0 \simeq g(T) - \frac{\pi C_{3,D}}{|\sin(\pi d/2)|} T \omega_0^{d/2-1} \tag{33}$$

whose asymptotic solution are

$$\omega_0(T, h) \simeq \begin{cases} g(T) & \text{far from the critical line} \\ \left(\frac{|\sin(\pi d/2)|}{\pi C_{3,D}}\right)^{\frac{2}{d-2}} \left(\frac{g(T)}{T}\right)^{\frac{2}{d-2}} & \text{close to the critical line.} \end{cases} \tag{34}$$

For transverse susceptibility, in the region close to the critical line, we then have

$$\chi_\perp(T, h) \simeq 3 \frac{h}{h_c(T)} \left(\frac{\pi C_{3,D}}{|\sin(\pi d/2)|}\right)^{\frac{2}{d-2}} T^{\frac{2}{d-2}} \left(h - h_c(T)\right)^{-\frac{2}{d-2}} \, , \tag{35}$$

hence the susceptibility diverges with a power-law in $h - h_c(T)$, with the spherical critical exponent $\gamma = 2/(d-2)$, independently of the anisotropy parameter. Far from the critical line we have instead the mean-field critical exponent $\gamma = 1$. The crossover line which separates this two asymptotic behavior is

$$h_G(T) \simeq h_c(T) + \left[\frac{\pi C_{d,D}}{|\sin(\pi d/2)|}\right]^{\frac{2}{4-d}} T^{\frac{2}{4-d}} . \tag{36}$$

Let us finally consider the behavior along the QCT, where $h = h_c$. Here we have

$$\omega_0 = b_d(h_c - 2D) T^{d/2} - \frac{\pi C_{3,D}}{|\sin(\pi d/2)|} T \omega_0^{d/2-1} \tag{37}$$

and hence, at low temperature, we have for the susceptibility $\chi_\perp \sim T^{-d/2}$. Within a narrow region around $h = h_c$ the energy gap and all the related thermodynamic quantities behave essentially as along the QCT, except for negligible corrections in $(h - h_c)/T$.

Increasing $h - h_c > 0$ a crossover to the quantum regime $\omega_0/T \gg 1$ occurs which is characterized by the solution (28), valid for any d. In fig. 2 the phase diagram with crossover line for $d = 3$ is shown and compared with that for $d = 2$.

Previous results for $2 < d < 4$ close to the field-induced QCP in the presence of SIA with $D < D^*$ is similar to those found for conventional paradigmatic quantum models [3] by means of the RG approach. However, if $D \to D^*$ the SIA tends to destroy the conventional quantum critical scenario.

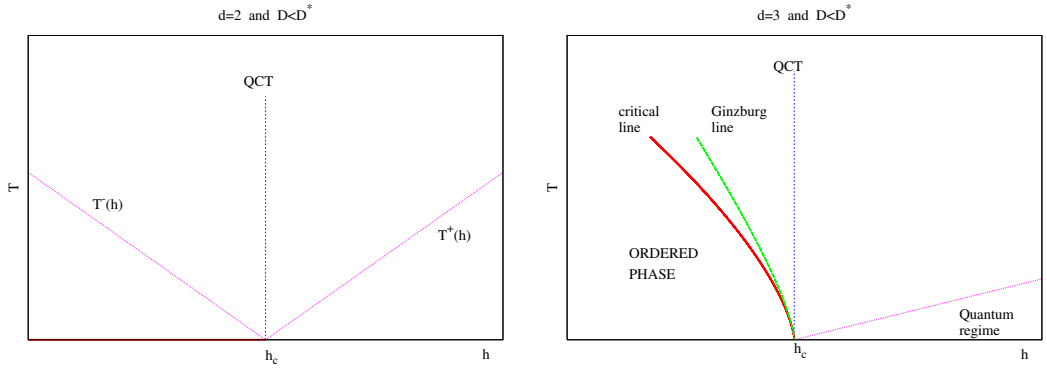

Figure 2. Phase diagram with crossover lines for dimensionalities $d = 2$ (left) and $d = 3$ (right). In both cases we consider small value of single-ion anisotropy parameter D, namely $D < D^*$. The phase diagram looks qualitatively similar to the one obtained in absence of single-ion anisotropy [9].

In conclusion, we used the two-time GFs method to explore the effects of the SIA on the magnetic-field-induced quantum criticality of a d-dimensional spin-3/2 PFM with short-range exchange couplings. The full phase diagram in the (h, T)-plane and the main physical properties at fixed single-ion parameter D have been obtained at the level of the TD and the ACD for higher order exchange interactions and SIA GFs, respectively. We stress that in our study the applied magnetic field h is assumed as the non-thermal control parameter driving quantum criticality, in contrast with recent literature where this role is played by the parameter D at fixed h (frequently assumed equal to zero). The emerging feature is that the phase boundary of the PFM is significantly influenced by the SIA, especially close to the QCP, but the quantum criticality of PFM for different dimensionalities remains essentially unchanged for values of the SIA parameter smaller than the threshold value $D^* = 3J(0)(1-\Delta)/8$. Indeed we obtain, within our approximation, mean-field-like critical exponents at the QPT and critical exponents belonging to the spherical model universality class at finite temperature. These are the same results obtained in absence of SIA [9, 10]. Hence, the inclusion of SIA does not affect the universality class of the model. For $D^* < D < \bar{D}$, where $\bar{D} = h_{0c}/2$ is the value of D at which the QCP is suppressed, our self-consistent equations are substantially modified. As a consequence, for $d > 2$, a reentrant low-temperature structure of the phase diagram appears (see fig. 1) and a modified quantum critical scenario is expected. The detailed analysis of the reentrant feature in the the phase diagram will be the subject of a future study. One may argue that the reentrant behavior may be due to the approximation used. Unfortunately, to our knowledge, no landmarks are available from literature about the SIA effects on quantum criticality controlled by an external magnetic field. Hence, at the present time, one cannot draw definitive statements about the role played by large easy-axis anisotropy in the vicinity of a magnetic-field-induced QCP. On the other hand, there is no clear reasons to consider "a priori" the ACD inadequate to provide physical findings around a QCP. Moreover, reentrant features in the phase diagram

usually arise when there are competing interactions. Furthermore, in the light of our numerical and analytical results, we speculate that the existence of D^* and of the easy-axis anisotropy regime for values of D in the limited range $D^* < D < \bar{D}$ are physically plausible. Indeed, the feature that the zero-temperature critical field h_c vanishes as $D \to \bar{D}$ suggests that increasing of the easy-axis anisotropy parameter tends, as expected, to suppress quantum critical fluctuations until, for $D \geq \bar{D}$, no field-induced QPT takes place. In this picture, D^* represents a borderline value of D between an anisotropy regime for $0 < D < D^*$, where the conventional quantum critical scenario is preserved, and that for $D^* < D < \bar{D}$, where reentrant phenomena appear in the low-temperature phase diagram close to the QCP and a non-conventional magnetic-field-induced quantum critical scenario is expected.

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