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Condensation of bosons with Rashba-Dresselhaus spin-orbit coupling

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Abstract. Cold atomic Bose-Einstein systems in the presence of simulated Rashba-Dresselhaus spin-orbit coupling exhibit novel physical features. With pure in-plane Rashba coupling the system is predicted in Bogoliubov-Hartree-Fock to have a stable Bose condensate below a critical temperature, even though the effective density of states is two-dimensional. In addition the system has a normal state at all temperatures. We review here the new physics when the system has such spin-orbit coupling, and discuss the nature of the finite temperature condensation phase transition from the normal to condensed phases.

1. Introduction

The ability to simulate gauge fields in neutral ultracold atom laboratory systems, reviewed in [1], has opened a wide variety of opportunities to create and study novel physical systems. Not only can one create artificial Abelian gauge fields, which provide analogs of conventional magnetic fields, one can in principle produce non-Abelian gauge fields; the prospect of simulating lattice gauge theories, reviewed in [2], with potential applications to quantum chromodynamics, is a remarkable new direction.

Among the simplest such non-Abelian fields are artificially induced spin-orbit couplings of the linear momentum, $\mathbf{p}$, of a particle to its "spin" $\boldsymbol{\sigma}$ – which in cold atom systems is more generally a hyperfine multiplet (reviewed in [3]). Of particular interest is the in-plane Rashba-Dresselhaus interaction [4, 5] with two hyperfine states, (a, b), with the single particle Hamiltonian in the (a, b) spinor basis,

$$H_{RD} = \frac{p^2 + \kappa^2}{2m} + \frac{\kappa}{m}(\sigma_x p_x + \eta \sigma_y p_y); \tag{1}$$

here m is the atomic mass, κ is the coupling constant, σ_x and σ_y are Pauli matrices, and η between 0 and 1, measures the anisotropy of the interaction; $\eta = 1$ corresponds to a pure Rashba interaction, and $\eta = 0$ to an equal mixture of Rashba and Dresselhaus couplings. This latter situation, together with a Zeeman coupling term $\propto \sigma_z$, was first realized experimentally by Lin et al. [6] and then by Zhang et al. [7] in bosonic systems, and P. Wang et al. in fermions [8].

Here we focus on two component bosonic systems with a pure Rashba interaction, $\eta = 1$, in which the two body interactions between particles in the same hyperfine states are described by short range pseudopotentials $v_{aa}(\mathbf{r}) = g_{aa}\delta(\mathbf{r})$, and $v_{bb}(\mathbf{r}) = g_{bb}\delta(\mathbf{r})$, and the interaction between

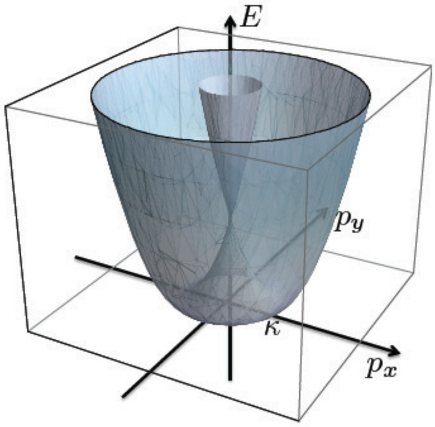

Figure 1. Spectrum of the single particle Hamiltonian with Rashba in-plane spin-orbit coupling.

different hyperfine states by $v_{ab}(\mathbf{r}) = g_{ab}\delta(\mathbf{r})$. As we shall see, this system presents intriguing questions not encountered in simple Bose-Einstein condensation. At $\eta = 1$, the single particle Hamiltonian can be written as

$$H_R = \frac{(\boldsymbol{p}_\perp + \kappa\boldsymbol{\sigma})^2 + p_z^2}{2m}. \tag{2}$$

where $\mathbf{p}_\perp \equiv (p_x, p_y)$, $\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y)$. The single particle eigenstates are naturally constructed by diagonalizing the Hamiltonian, in which basis the single particle eigenenergies become

$$\epsilon_{\mathbf{p}} = \frac{(p_\perp \pm \kappa)^2 + p_z^2}{2m}; \tag{3}$$

the ground state branch $(-)$ has a circle of minima at $p_\perp = \kappa$, while the excited branch $(+)$ is connected to the ground state branch by a Dirac point at $p_\perp = 0$. See Fig. 1.

At zero temperature the system can be condensed in any linear combination of states on the circle. The preferred condensates depend, however, on the interactions [9]. For $g_{aa} = g_{bb} > g_{ab}$, i.e., greater repulsion between like than different hyperfine states, the preferred condensate within mean-field theory is a plane wave state, e.g.,

$$\Psi_p(\mathbf{r}) = \sqrt{\frac{n}{2}} e^{i\kappa x} \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{4}$$

Here the wave vector of the condensate, $\boldsymbol{\kappa} = \kappa\hat{x}$, points along the x axis, but it could point anywhere on the circle of minima. In the opposite regime, $g_{aa} < g_{ab}$ the preferred condensate is a striped state, e.g.,

$$\Psi_s(\mathbf{r}) = \sqrt{n} \begin{pmatrix} \cos\kappa x \\ -i\sin\kappa x \end{pmatrix}, \tag{5}$$

where again the wavevectors in the condensate, $\pm\kappa\hat{x}$, are chosen arbitrarily to lie along the x axis. Condensate with a larger number of wavevectors lying on the degenerate circle have higher energy.

In the remainder of this paper we limit the discussion, for simplicity, to the plane wave condensate with isotropic interactions, $g_{aa} = g_{bb} = g_{ab}$. We first ask whether the condensate is stable against quantum fluctuations at zero temperature, and thermal fluctuations. It is not immediately obvious that the system can have a stable condensate, since the density of single particle states in the low energy limit is two dimensional in nature, $\sim m\kappa/2\pi$. Thus one could imagine that, as in two-dimensional Berezhinskii-Kostelitz-Thouless (BKT) systems [10], the condensate is unstable owing to the number of excited particles diverging. As we shall see, the condensate is in fact stabilized by interactions. A second important feature of this system is that the normal state is not kinematically forbidden, as in usual Bose-Einstein condensation below the transition temperature. We also ask for the nature of the finite temperature phase transition to the normal state.

2. Stability of the Bose-Einstein condensate

Unlike in BKT systems, condensates at zero temperature are stabilized by the interparticle interactions [11]. As we now discuss, the number fluctuations at $T = 0$ are finite and of order $\sqrt{(mg)^3 n} \sim \sqrt{na^3}$ in weak coupling, similar to usual Bose gases in the Bogoliubov approximation, where we write $g = 4\pi\hbar^2 a/m$ [owing to renormalization effects in the presence of spin-orbit interactions, the length a is not simply related to the actual scattering length [12]]. The modes of the condensate, and the corresponding condensate depletion, are readily derived within Bogoliubov mean-field from the single particle matrix Green's function with anomalous components,

$$\mathbf{G}(\mathbf{q}, t_1 - t_2) \equiv -i\langle T\left(\Psi_{\mathbf{q}}(t_1)\Psi^\dagger_{\mathbf{q}}(t_2)\right)\rangle, \tag{6}$$

where

$$\Psi_{\mathbf{q}}(t) \equiv \left(\psi_{-,\boldsymbol{\kappa}+\mathbf{q}}(t), \psi^\dagger_{-,\boldsymbol{\kappa}-\mathbf{q}}(t), \psi_{+,\boldsymbol{\kappa}+\mathbf{q}}(t), \psi^\dagger_{+,\boldsymbol{\kappa}-\mathbf{q}}(t)\right); \tag{7}$$

here $\psi_{\pm,\mathbf{p}} \equiv (\mathbf{a}_{\mathbf{p}} \pm \mathbf{b}_{\mathbf{p}})/\sqrt{2}$, with $\mathbf{a}_{\mathbf{p}}$ and $\mathbf{b}_{\mathbf{p}}$ removing particles of momentum $\mathbf{p}$ from the a and b hyperfine states, respectively. In the Bogliubov approximation, where $\psi_{-,\boldsymbol{\kappa}} \to \sqrt{N_0}$ with $N_0 = n_0 V$ the number of condensate particles, and with Hartree-Fock energies included, one finds the Fourier transform in time of the inverse Green's function,

$$\mathbf{G}^{-1}(\mathbf{q}, z) = \begin{pmatrix} z - A & -gn_0 & i(\kappa/m)q_y & 0 \\ -gn_0 & -z - A & 0 & i(\kappa/m)q_y \\ -i(\kappa/m)q_y & 0 & z - B & 0 \\ 0 & -i(\kappa/m)q_y & 0 & -z - D \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned} A(\mathbf{q}) &\equiv q^2/2m - \mu + g(2n_0 + 2n_- + n_+) \\ B(\mathbf{q}) &\equiv (2\boldsymbol{\kappa} + \mathbf{q})^2/2m - \mu + g(n_0 + n_- + 2n_+) \\ D(\mathbf{q}) &\equiv B(-\mathbf{q}).. \end{aligned} \tag{9}$$

In leading order, the chemical potential is $\mu_0 = \partial\langle\mathcal{H}\rangle/\partial N_0 = gn_0 + 2gn_- + gn_+$, where

$$n_\mp = \frac{1}{V}\sum_{\mathbf{p}\neq\boldsymbol{\kappa}} \langle\psi^\dagger_{\mp,\mathbf{p}}\psi_{\mp,\mathbf{p}}\rangle \tag{10}$$

are the number of non-condensate particles in the $(-)$ and $(+)$ states.

The modes of the condensate, found as zeroes of $\det\mathbf{G}^{-1}(\mathbf{q}, z)[= \det\mathbf{G}^{-1}(-\mathbf{q}, -z)]$ come in pairs: two of positive and two of negative frequency, for each $\mathbf{q}$. One excitation is

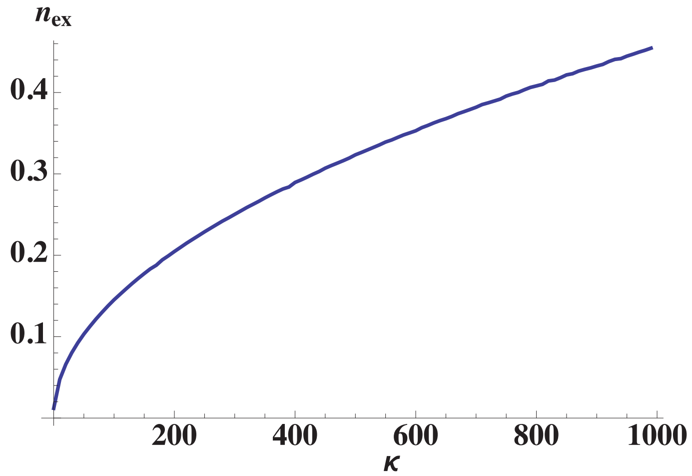

Figure 2. The number of excited particles, in units of $(2mgn_0)^{3/2}$ as a function of the spin-orbit coupling strength κ in units of $\sqrt{2mgn_0}$.

gapless as $\mathbf{q} \to 0$, and the other is gapless as $\mathbf{q} \to -2\boldsymbol{\kappa}$. With strong spin-orbit coupling, $\kappa^2/m \gg g|n_+ - n_-| \equiv g|\Delta n|$, the spectrum to leading order for $|\mathbf{q}| \ll \kappa$ is

$$\epsilon_1(\mathbf{q}) \approx \sqrt{2gn_0}\left[\frac{q_x^2+q_z^2}{2m} + \frac{q_y^2}{4\kappa^2}\left(g\Delta n + \frac{q_y^2}{2m}\right)\right]^{1/2}; \tag{11}$$

since $q_y^2/2m$ is generally larger than $g|\Delta n|$ in typical experimental setups [6], the dispersion is essentially quadratic for $q_x = q_z = 0$. Similarly, the gapless spectrum for $\mathbf{q}' \equiv \mathbf{q} + 2\boldsymbol{\kappa} \ll \kappa$ is quadratic and free-particle like,

$$\epsilon_2(\mathbf{q}') = \frac{q_x'^2 + q_z'^2}{2m} + \frac{gn_0}{\kappa^2/m + gn_0}\frac{q_y'^2}{4m}. \tag{12}$$

The condensate depletion at $T = 0$ is given by

$$n_{ex} = n_- + n_+ = i\int \frac{dzd^3q}{(2\pi)^4}\left(G_{11}(\mathbf{q}, z) + G_{33}(\mathbf{q}, z)\right), \tag{13}$$

with the z integration contour surrounding the negative poles in the positive sense; the integral converges in the ultraviolet. Figure 2 plots the depletion evaluated numerically. Generally $n_- \gg n_+$ and $n_{ex} \sim n_0\sqrt{(mg)^3 n_0} \sim n_0\sqrt{n_0 a^3}$. The leading behavior in κ is $n_{ex} \sim \sqrt{\kappa}$, as shown analytically in Ref. [13].

The shift in the ground state energy arising from fluctuations, $\Delta E \equiv E - gn^2/2$, is of order $gn(2mgn)^{3/2}$ for weak coupling [11]. The energy decreases with increasing κ, and ΔE changes from positive to negative at $\kappa \sim 0.6\sqrt{2mgn_0}$.

We next show that the condensate remains stable under thermal fluctuations. In terms of the mode energies (11) and (12), the infrared contribution to the condensate depletion is

$$n_{ex} \sim T\int \frac{d^3q}{(2\pi)^3}\left(\frac{gn_0}{\epsilon_1(\mathbf{q})^2} + \frac{1}{\epsilon_2(\mathbf{q})} + C\right), \tag{14}$$

where C is a constant as $\mathbf{q} \to 0$. The integral converges in the infrared, and condensate depletion does not destroy the Bose-Einstein condensate at finite temperature.

Unlike in usual weakly interacting Bose systems in three dimensions in the absence of spin-orbit coupling, the normal state ($n_0 = 0$) of the system is kinematically allowed at low temperatures. Is the normal state preferred over the condensate at low temperature? To answer this we evaluate the Helmholtz free energy density of the normal state,

$$\mathcal{F} = \mu n - \frac{3}{4}gn^2 + \frac{1}{\beta V}\sum_{\mathbf{p}}\left\{\ln\left(1 - e^{-\beta\xi_-(\mathbf{p})}\right) + \ln\left(1 - e^{-\beta\xi_+(\mathbf{p})}\right)\right\}, \tag{15}$$

where $\xi_\pm(\mathbf{p}) \equiv \{(p_\perp \pm \kappa)^2 + p_z^2\}/(2m) - \mu + 3gn/2$; the chemical potential is determined by the number equation $n = (1/V)\sum_{\mathbf{p}}\{f(\xi_-(\mathbf{p})) + f(\xi_+(\mathbf{p}))\}$, where $f(x) \equiv 1/(e^{\beta x} - 1)$. To a good approximation, with $\Delta\mu = \mu - 3gn/2$,

$$n(\mu) \approx -\frac{m\kappa T}{2\pi}\ln\left(-\Delta\mu/T\right), \tag{16}$$

which is essentially the mean field result for a two-dimensional BKT system [10]. Unlike for free bosons in three dimensions, there always exists a μ which satisfies the number equation, thus the non-condensed state is not kinematically forbidden at any non-zero temperature.

In the normal state, as $T \to 0$, $\mu \to 3gn/2$, and $\mathcal{F} \to 3gn^2/4$. (Note that the point $T = 0$, $\mu = 3gn/2$ is highly singular.) This energy is larger than the ground state energy in the condensed phase, $(gn^2/2)(1+\mathcal{O}(\sqrt{(2mg)^3 n}))$, indicating that at sufficiently low temperature, the condensate is energetically preferred. However, this calculation does not exclude the possibility that the system has a correlated normal state with energy lower than that of the condensate, and thus the system would not condense.

3. Phase transition between the normal and condensed states

As we see, both the normal state and condensed state at the present level of mean-field calculation are stable at zero temperature, where the condensate, of lower energy, is favored. However, at sufficiently high temperature the condensate goes away. Thus we now ask, what is the nature of the phase transition from condensate to normal; first or second order? See Fig. 3. As we find in mean-field Bogoliubov-Hartree-Fock, the transition is first order [14].

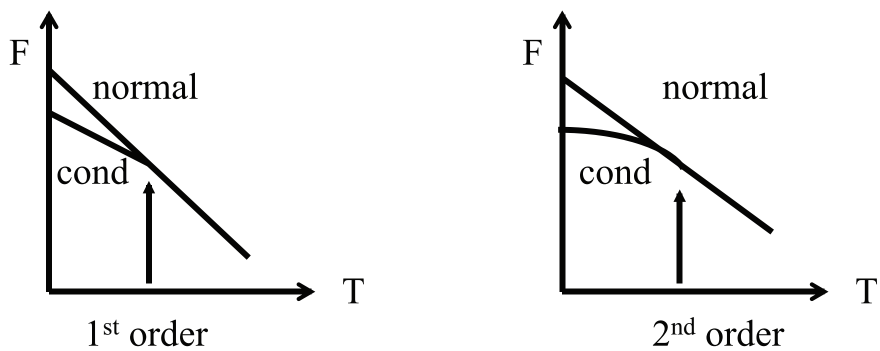

Figure 3. Schematic of the free energies of the normal and condensed states. If the free energies meet at a finite angle (left panel) the transition if first order, while if they meet at zero angle (right panel) the transition is second order.

This first order transition is distinct from the spurious first order phase transition one finds within the Bogoliubov-Hartree-Fock in usual Bose gases [15] (see also [16]). There the spurious

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transition is driven by order parameter fluctuations which lead to a density of particles excited out of the condensate near the transition temperature,

$$n_{ex}(n_0) = \int \frac{d^3q}{(2\pi)^3}\left[\left(\frac{1}{e^{\beta E_q}-1}+\frac{1}{2}\right)\frac{q^2/2m+gn_0}{E_q}-\frac{1}{2}\right] = n_{ex}(0) - \frac{1}{\lambda^2}\sqrt{mgn_0} + \mathcal{O}(n_0^{3/2}); \quad (17)$$

here $\lambda = \sqrt{2\pi/mT}$ is the thermal wavelength and $E_q \equiv \sqrt{(q^2/2m)^2 + gn_0q^2/m}$. The $\sqrt{n}_0$ term, which is non-analytic in the condensate fraction, leads to a $-n_0^{3/2}$ term in the free energy, and as a consequence the system, as cooled from above T_c^0 – the transition temperature of the ideal Bose gas – undergoes a first order transition, with a jump in n_0, at a temperature $T > T_c^0$ (cf. [16]).

This first order transition is removed when correctly determining the critical behavior at the phase transition. The issue is that the point of vanishing scattering length $a = mg/4\pi \to 0$ and deviation from the transition temperature $t = (T_c - T)/T_c \to 0$ is highly singular. The Bogoliubov-Hartree-Fock approximation is valid in the limit $a \to 0$ at non-zero t, while to study the order of the transition, one needs $t \to 0$ at non-zero a. These two limits are not equivalent. Near the transition in a weakly interacting gas, one has the scaling structure for the condensate fraction in terms of a dimensionless scaling function $h(x)$:

$$\frac{n_0}{n} = \frac{a}{\lambda}h\left(\frac{t\lambda}{a}\right), \quad (18)$$

where $h(x \to \infty) \sim x$, the non-interacting gas limit, and for $x \to 0$, as one approaches the transition at finite a, one has $h \sim x^{2\beta}$, with $\beta \approx 1/3$, the critical exponent for the order parameter.

The spurious first order transition does not occur in the presence of spin-orbit coupling. In the simple Bose gas the relevant momentum scale is $\sqrt{an_0}$. However, Rashba spin-orbit coupling introduces a second scale, κ, and as a consequece the density of excited particles is analytic in n_0 for $\kappa^2 \gg an_0$.

To determine the nature of the phase transition, we calculate the free energy $\mathcal{F}$ in terms of the Green's function using,

$$\frac{\partial\mathcal{F}}{\partial n_0} = -\mu + gn_0 - gT\sum_\nu \int \frac{d^3q}{(2\pi)^3}\left(2G_{11}(\mathbf{p},z_\nu) + G_{33}(\mathbf{p},z_\nu) + \frac{1}{2}\left(G_{21}(\mathbf{p},z_\nu) + G_{12}(\mathbf{p},z_\nu)\right)\right). \quad (19)$$

Expanding in small $gn_0 \ll \kappa^2/m$, as detailed in [14], one obtains

$$\frac{\partial\mathcal{F}}{\partial n_0} = -\mu + \frac{3}{2}gn(\mu) + Xgn_0 + Y(gn_0)^2 + \ldots, \quad (20)$$

where

$$X(\mu,T) \equiv 1 - \frac{4m^2gT}{\kappa}\alpha\left(\frac{\Delta\mu}{\epsilon_\kappa}\right), \quad Y(\mu,T) \equiv \frac{4m^2gT}{\kappa\epsilon_\kappa}\beta\left(\frac{\Delta\mu}{\epsilon_\kappa}\right), \quad (21)$$

$\epsilon_\kappa \equiv \kappa^2/2m$, and $\alpha(x)$ and $\beta(x)$ are dimensionless functions of x, with the asymptotic forms as $x \to 0^-$, $\alpha(x) \simeq -19/32\pi x$ and $\beta(x) \sim 0.16/x^2$, and approaching 0 as $x \to -\infty$.

Integrating with respect to n_0 we have

$$\mathcal{F}(n_0) = \mathcal{F}_{\rm n} - \Delta\mu\, n_0 + X\frac{g}{2}n_0^2 + Y\frac{g^2}{3}n_0^3, \quad (22)$$

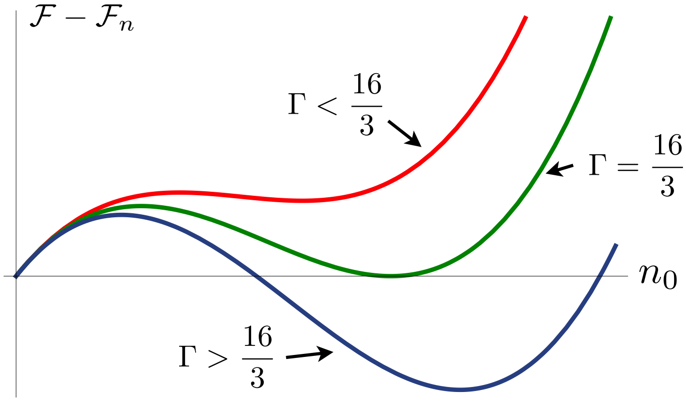

Figure 4. The free energy difference of the normal and condensed phases, showing the first order phase transition from the normal to condensed phase at $\Gamma = 16/3$. The top line is for $T > T_c$; the middle line is at T_c; and the bottom line is for $T < T_c$.

where $\mathcal{F}_\mathrm{n} = \mathcal{F}(n_0 = 0)$ is the free energy in the normal phase. The coefficients of n_0 and n_0^3 are both positive; however,for given $\mu > 0$, X goes from positive at high T to negative at low T, and decreases continuously with decreasing temperature.

This change in the sign of X drives a first order phase transition, since at sufficiently small T, the two conditions for the transition $\mathcal{F}(n_0) = \mathcal{F}_\mathrm{n}$ and $\partial\mathcal{F}(n_0)/\partial n_0 = 0$ become simultaneously satisfied at T_c where $X^2 = -16Y\Delta\mu/3$. At this temperature the system undergoes a transition to the condensed phase, as illustrated schematically in Fig. 4. The combination $X^2/(-Y\Delta\mu) \equiv \Gamma$ monotonically decreases with T, as long as $X < 0$. At the transition, n_0 jumps from zero to $|3X/4Yg| > 0$ on the condensate side.

As derived in [14], the transition temperature is given by

$$T_c \approx \frac{2\pi n(\mu)}{m\kappa}\frac{1}{|\ln(2m\kappa g)| + C}, \tag{23}$$

where $C \sim 3.4$. The transition temperature depends linearly on the density, and as expected approaches zero as $g \to 0$. In addition, the jump in the condensate density at the transition is $n_0/n(\mu) \sim 0.32/(|\ln(2m\kappa g)| + C)$, approaching zero as $g \to 0$, and increasing with increasing g.

A deep issue is whether the present transition remains first order at a higher level of approximation. On the one hand, this system is similar to other bosonic systems with continuously degenerate single-particle minima, such as a weak-crystallization model [17] and magnon systems [18], in which condensation transitions are predicted to be first order.

On the other hand, the single particle density of states closely resembles that in a two-dimensional BKT system hinting at a second order transition. In finite geometry the condensate fraction is discontinuous at the BKT transition, but in a macroscopic system correlation corrections change the transition from first order to continuous, as one sees from scaling arguments [10]. To fully address this issue requires scaling or renormalization group analyses; Monte-Carlo calculations of the transition would also be useful.

Several additional factors need to be taken into account in the analysis. The first is the dependence of the order of the transition on the geometry of the order parameter, plane wave, stripes, or with higher symmetry, e.g., triangular. A second is the effect of the renormalization of the interparticle interactions in the presence of spin-orbit coupling [12]

Finally we ask whether one can construct normal (non-condensed) states of lower free energy than the optimal condensed phase. A key feature of the Rashba spin-orbit coupling considered here is the circle of degenerate single particle states, which the normal state does not take particular advantage of. For example. rapidly rotating Bose condensates, develop a set of nearly degenerate (lowest Landau level) single particle excitations; this degeneracy allows the formation of highly correlated normal, or totally fragmented condensates, with low occupation per level, at sufficiently rapid rotation. Whether a similar highly correlated state is better than a Bose-Einstein condensate remains an open question (see, e.g, [19]).

Acknowledgments

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