

# On the Goos-Hänchen effect in neutron optics

**A I Frank**

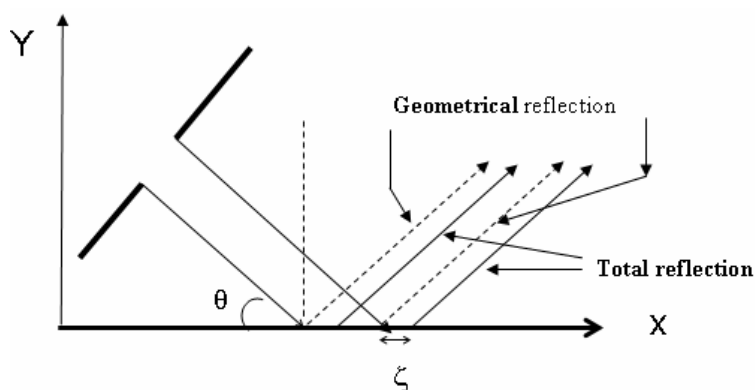
Frank Laboratory of Neutron Physics, Joint Institute for Nuclear Research,  
141980 Moscow region, Dubna, Russia.

frank@nf.jinr.ru

**Abstract.** The Goos-Hänchen effect, a longitudinal shift of a wave beam at total inner reflection, is a well-known optical phenomenon. It was discovered in 1947 and have been observed many times at reflection of micro- and ultrasonic waves since then. Here we present an elementary theory of the GH. effect to describe reflection of a massive particle and demonstrate how the shift is related to delay time at reflection. It is shown that giant positive or negative longitudinal shifts of the neutron beam may occur at neutron reflection from some specially manufactured planar system. The possibility of delay time measurement at neutron reflection and direct experimental observation of the GH shift are also discussed

## 1. Introduction

The discovery by Goos and Hänchen [1,2] of the wave beam longitudinal shift at total inner reflection of light called the Goos - Hänchen effect has had remarkable influence on the progress in understanding of reflection. It is recognized that the phenomenon is a very general one and its investigation goes beyond the scope of conventional light optics. The GH shift was observed for ultrasonic waves [3-5] and later, for microwaves [6,7] and X-rays [8]. Obviously it was quite natural to suppose that massive particles must also demonstrate the GH. shift at their reflection from the region of the potential.



**Figure 1.** The Goos-Hänchen effect. The longitudinal shift at total reflection

As far as we know the corresponding quantum problem was first analyzed by Hora [9]. Later such approach has become quite common [10,11] especially since the connection of the GH. effect with the quantum problem of reflection time was realized.

Till recently the problem of GH. shift at reflection of a massive particle remained purely theoretical in spite of its more than half-century history. Best conditions for detection of the phenomenon seem to exist in neutron optics. To our knowledge, A.A. Seregin was first to raise the question of the GH. effect at neutron reflection [13]. Later the theory of the effect and possibilities of its observation in neutron experiments were discussed in [14, 15]. The recent work [16] reported that the GH. effect was observed for neutrons for the first time. This gave rise to discussion and made the problem even more actual.

Next section presents main relations concerning the shift of a neutron beam at reflection from matter border. Also, the relation of the shift with the group delay time (GDT) at reflection is discussed. In the third section some examples of possible giant and negative GH. shifts are given. In the final section possibilities of experiments to measure the GDT and the neutron beam shift at reflection are briefly discussed.

## 2. The Goos-Hänchen shift and neutron reflection time from matter

Below our attention is focused on reflection of neutron waves. The peculiarity of neutron optics is that in most cases interaction of neutrons with matter may be described by introducing the effective potential

$$U = \frac{2\pi\hbar^2}{m}\rho b, \quad (1)$$

where  $\rho$  is the volume density of atoms and  $b$  is the volume average of the coherent scattering length. To illustrate how the longitudinal shift of the wave beam occurs at reflection of neutron waves from the border of matter we address here to theoretical approach of Artmann [18]. Artmann's method is widely known and with some modification it was multiply used in a number of subsequent papers (see for example [4, 19, 20,14]).

Let a medium with the effective potential  $U$  be bordered by the plane  $y = 0$ . Considering the reflection of the wave beam from the border and following [21] we accept for simplicity that the beam is formed by just two waves whose wave vectors slightly differ in their direction. For each of the waves the condition of total reflection  $E_n < U$  holds. Here

$$E_n = \frac{\hbar^2 k_y^2}{2m}, \quad (2)$$

and  $k_y$  is the component of the wave vector normal to the surface.

On the surface of matter  $x$ -components of incoming waves are  $\exp(ik_x x)$  and  $\exp[i(k_x + \Delta k_x)x]$ . The corresponding reflected waves differ from the incoming ones by phase and their  $x$ -components on the surface are  $\exp(ik_x x + \varphi)$  and  $\exp[i(k_x + \Delta k_x)x + (\varphi + \Delta\varphi)]$ , respectively. The phase shift  $\Delta\varphi$  may be determined in a conventional way from continuity conditions at the border. Such waves interfere and for the  $x$ -component of the superposition of the reflected waves we have

$$\Psi_x = \exp(ik_x x + \varphi) \left\{ 1 + \exp[i(\Delta k_x x + \Delta\varphi)] \right\}. \quad (3)$$

The condition of maximum intensity or of constructive interference is

$$\Delta k_x x + \Delta\varphi = 2\pi v, \quad (4)$$

where the integer number  $v$  is the order of interference.

In the absence of total reflection, in particular for neutrons with the energy above the barrier  $E_n > U$  the phase shift is absent and both  $\varphi$  and  $\Delta\varphi$  are zero. In this case the condition of maximum

intensity is  $\Delta k_x x_0 = 2\pi v$ . Consequently, the phase shift associated with total reflection leads to the beam shift at  $\varsigma = x - x_0 = -\frac{\Delta\phi}{\Delta k_x}$ . In the limit of  $\Delta k_x$  going to zero we come to famous Artmann's formula for the longitudinal Goos-Hänchen shift.

$$\varsigma = -\frac{d\phi}{dk_x}. \quad (5)$$

Above it was believed that the wave number  $k_x$  varies only due to variation of the angle of incidence. Taking into account that the total k-number  $k_0^2 = k_x^2 + k_z^2$  is constant we write the displacement  $\varsigma$  in equation (5) in terms of the phase derivative with respect to  $E_n$

$$\varsigma = \frac{\hbar^2 k_x}{m} \frac{d\phi}{dE_n}, \quad (6)$$

Finding the relation for the phase shift of the reflected wave and substituting it into (6) we obtain

$$\varsigma = \frac{2k_x}{k_y \sqrt{k_b^2 - k_y^2}}, \quad (k_y < k_b) \quad (7)$$

in full agreement with the results [11]. Here  $k_b = \sqrt{2mU}/\hbar$  is the border wave number.

Note the following remarkable feature of equation (6). The value

$$\tau = \hbar \frac{d\phi}{dE_n}, \quad (8)$$

which enters in this equation is the well-known group delay time (GDT) introduced by Eisenbud, Bohm and Wigner [22-24] as a measure of interaction time in quantum mechanics. Consequently, equation (6) can be written in the form

$$\varsigma = \tau v_x. \quad (9)$$

With some reservations the GDT may be interpreted as neutron reflection time. The relation between the reflection time and the longitudinal GH shift was first noted by Agudin [12]. Note that in the case under consideration reflection from the potential barrier GDT is

$$\tau = \frac{\hbar}{\sqrt{E_n (U - E_n)}}. \quad (10)$$

Alternative approach to the theory of GH effect based on the fluxes balance in condition of total reflection was developed by Renard [10]. We present here only some based principle of this theory limiting our self only by the case of the total reflection of neutron by matter.

Since at the condition of total reflection a wave penetrates into the region of matter, attenuating there exponentially, a finite particle density and correspondent flux directed parallel to the surface are in this classical forbidden region. This extra flux  $J_t$  caused by the partial penetrating of neutrons through interface must be compensated by decreasing of the flux in the region correspondent to the geometrical reflection of the initial beam. Simple calculations lead to the result

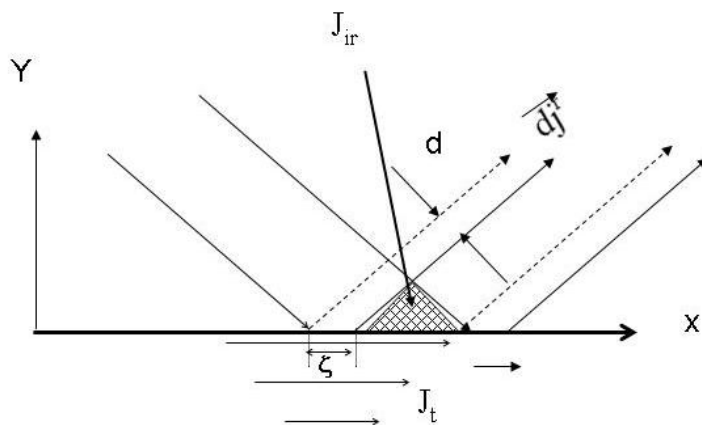
$$J_t = \frac{\hbar}{2m} \frac{k_x}{\sqrt{k_b^2 - k_y^2}} \frac{4k_y^2}{k_b^2}, \quad (11)$$

from which it is easy to define the transversal displacement  $d$  of the reflected beam and correspondent longitudinal shift  $\varsigma$  along the interface

$$\varsigma = \frac{k_x}{k_b^2} \frac{2k_y}{\sqrt{k_b^2 - k_y^2}}. \quad (12)$$

This result differs from the Artmann - Carter – Hora formula (7) and consequently contradicts to the hypothesis of relation between GH shift and GDT (9). Nevertheless, the equation (12) was used as a basis for calculation of the results proposed [14] and realized [16] in neutron optical experiments.

For the long time the origin of the discrepancy between the results which follow from these two physically approaches was not properly understood. Only two decades after the publication of Renard's the paradox was resolved by Yasimoto, Oishi and Fedoseev [25, 26]. The point is that it was supposed by Renard, that total reflection differs from ideal-geometrical one only by the presence of the flux of evanescent waves inside the matter. It was not taken into account that phase difference between incoming and reflected waves leads to distortion of the flux in the region of crossing of two beams. Whereas as we could see from Artmann's approach just this phase shift at total reflection is the origin of the GH effect.



**Figure.2.** Illustration of the idea of flux balance. The sum of the fluxes  $J_t$  and  $J_{ir}$  must be compared with the disappearing flux in the region  $d$  [25].

Consequently, an extra flux that rises at total reflection is the sum of the flux of evanescent waves (11) inside matter and the additional flux  $J_{ir}$  arising due to interference of incoming and outgoing beams (see figure 2). The calculation that takes into account the above circumstance leads to equation (7). Thus, both approaches, namely, the method of Artmann and the one based on flux balance are equivalent. This means that the longitudinal shift  $\xi$  of the beam may be represented as a product of longitudinal velocity and GDT.

Taking into account the fact that the effective potential  $U$  is typically of the order of  $10^{-7}$  eV and that the condition of total reflection is  $E_n < U$ , from equation (10) we obtain that with the exception of two narrow regions, i.e., that close to zero energy and the one close to the barrier, the group delay time is  $\tau_r \approx 4\div 6$  ns

### 3. Giant and negative longitudinal displacements of reflected neutron beam

Direct observation of neutron beam shift at total reflection is quite a difficult task. Smallness of the effect is not the only problem. In the pioneering works of Goos and Hänchen [1,2] the position of the totally reflected wave beam is compared with that of the beam reflected from a metal film because that is very small [27]. In the neutron experiment no reference position which the shift could be counted from was found. In fact, in [13,14] it is suggested that the position of the reflected beam should be compared with the results of calculation.

In addition, [14] suggests measuring the phase of the reflected wave using an interferometer. However, though phase shift at reflection and longitudinal shift of the reflected wave are closely related phenomena, they are different from the physics viewpoint. Therefore, it is not quite correct to interpret rather impressive manifestation of the phase shift effects detected in [16] as direct observation of the GH shift (see discussion in [17]).

The solution to the problem of observation of the GH effect in a neutron experiment will probably be found if one turns to the idea of resonant amplification of the beam shift which may occur at reflection from multilayer structures as proposed by Tamir and Bertony [28]. Later the problem was theoretically investigated [29,30] and a giant shift at light reflection from the waveguide structure was observed [31]. With respect to neutrons a similar assumption was discussed from a slightly different standpoint in [15].

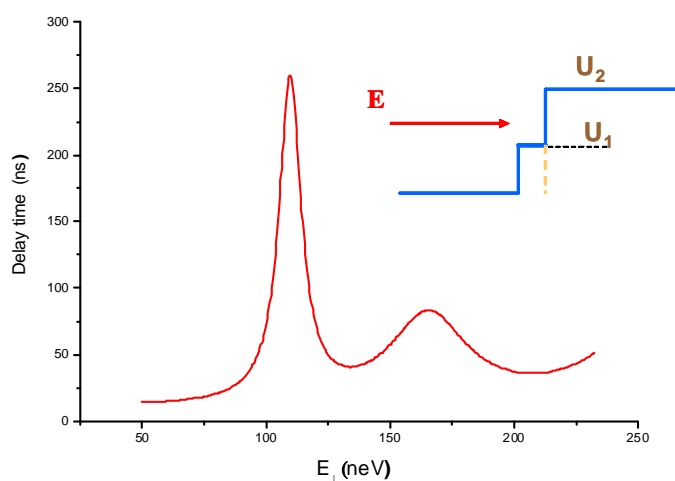
The conditions for the waveguide-type propagation of the neutron flux along the surface of matter may be satisfied for quite various types of planar structures. The simplest one is a homogeneous film with the effective potential  $U_1$  on the substrate with the potential  $U_2 > U_1$ . It is obvious that neutrons with  $E_n < U_2$  will be totally reflected from such a structure. In the case of normal incidence the general expression for the amplitude of the reflected wave is

$$r(k_0) = \frac{(k_1 + k_2)(k_0 - k_1) + (k_1 - k_2)(k_0 + k_1)e^{2ik_1\xi}}{(k_1 + k_2)(k_0 + k_1) + (k_1 - k_2)(k_0 - k_1)e^{2ik_1\xi}}, \quad (13)$$

where  $\xi$  is the film thickness,  $k_0$  is the wave number of the initial wave and  $k_1 = \sqrt{k_0^2 - 4\pi\rho b_1}$ , and  $k_2 = \sqrt{k_0^2 - 4\pi\rho b_2}$  are the wave numbers in the matter of the film and of the substrate, respectively. In the considered case of total reflection from nonabsorbing matter  $r = \exp(i\phi)$ ,  $k_2$  is imaginary and  $k_1$  may be either real or imaginary depending on neutron energy.

Having obtained from eq. (13) the phase of the reflected wave  $\phi(k_0) = \arctg(\text{Im}(r)/\text{Re}(r))$  it is easy to find the derivative  $d\phi/d(k_0^2)$  and GDT (8) which is directly related to the longitudinal shift (9). In figure 3 the result of such calculation for the film with the thickness 90nm and the effective potential  $U_1 = 91\text{neV}$  (Ag) on the substrate with  $U_2 = 245\text{ neV}$  (Ni) is shown.

As is clearly seen in the figure directly in the resonance the GDT is as large as 260ns. Note that in accordance with (10) in the case of neutron reflection from a pure substrate the GDT is only 5.4 ns. As it follows from equation (9), the longitudinal shift of the reflected beam experiences resonant amplification equal to that of the delay time.



**Figure 3.** The group delay time at neutron reflection from a thin film on a substrate. In the insert there is shown the potential structure of such a combination. See the text for details.

At neutron reflection from multilayers another remarkable phenomenon may also arise. Under some conditions the group time and the longitudinal shift of the reflected beam may be negative. The possibility of a negative GH shift was first pointed to in paper [28] cited above. Later, the problem of negative beam shift and negative GDT was the subject of many theoretical [32-35] and experimental

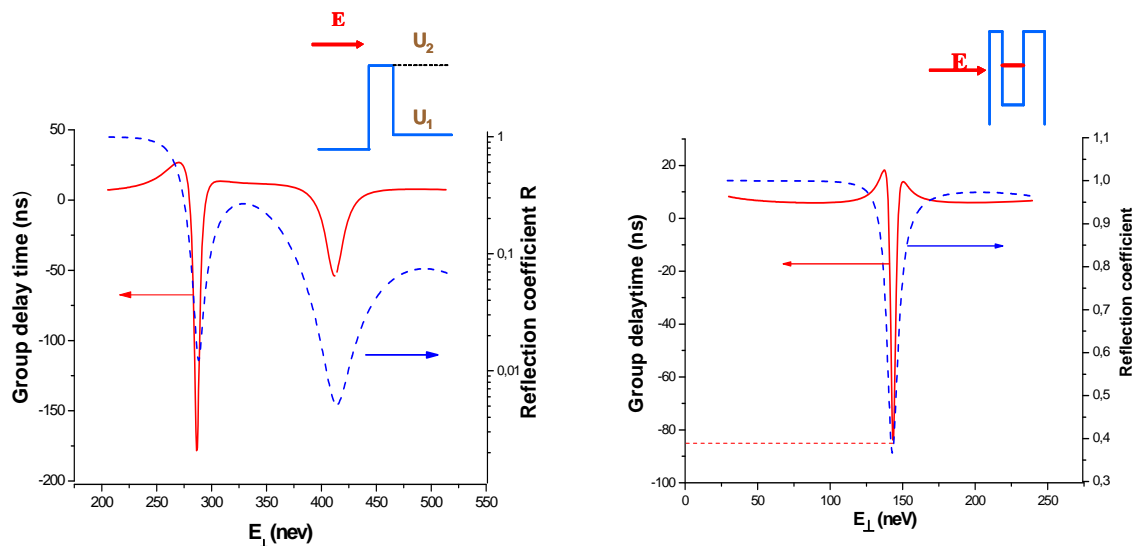
[36-39] investigations. There exist a large number of various objects and media reflection from which is accompanied by negative beam shift. It is beyond the scope of the given work to present any detailed review of related papers. We will only dwell on the noted above case of neutron reflection from a simple planar structure and turn back to the simplest one, namely, to a homogeneous film on a substrate.

Differently from the case considered above the film potential,  $U_1$ , is now larger than that of the substrate,  $U_2$ . The effective potential of such a structure looks like an asymmetric potential barrier. The reflection coefficient of the barrier is then  $R(k_0) = |r(k_0)|^2$  and it can be easily found from equation (13). Above the barrier the coefficient  $R$  oscillates and decreases rather rapidly with increasing  $k_0$ . The GDT from that asymmetric potential barrier was found in [41] where it is also shown that in the minima of the reflection curve the GDT is negative. In accordance with (9) the reflected beam has to undergo negative shift in such conditions.

On the left figure 4 shows the result of calculation of the GDT and the neutron reflection coefficient from a nickel film 70nm thick deposited on a quartz substrate. It is seen that in the minima of the reflection curve the GDT reaches rather large negative values. In spite of that the reflection coefficient is relatively small in this case one may hope that the negative GDT and the negative longitudinal shift will be measurable.

The variety of multilayers characterized by negative GDT is though quite wide. This permits choosing an optimal relation between an absolute GDT value and the reflected beam intensity.

As an example the results of calculation for a three-layer structure are given. Its effective potential is two barriers not equal in width separated by a well. The data used in the calculation correspond to Ni-Ti-Ni films with the thickness 23, 13, 33 nm, respectively. In this case the reflection coefficient is about 0.4 in the resonance and the negative GDT has a twice smaller absolute value than in the previous case.



**Figure 4.** The group delay time (solid red) and the reflection coefficient (dash blue) for two multilayers. On the left hand side – a film on a substrate, on the right hand side – three films. The corresponding potential form is shown in the upper right hand corner of each figure. See the text for details.

## 4. Possibilities of experimental observation

### 4.1. Measurement of the group delay time

The possibility of direct measurement of the GDT at neutron reflection was experimentally demonstrated many years ago. The measurement is based on the so-called Larmor clock. The idea is to use the Larmor frequency of neutron spin precession in the magnetic field as a standard of frequency. Baz' [41] was the first to propose using such a clock as an original theoretical method to calculate the time of interaction of a particle with a three dimensional potential. Then Rybachenko [42] used the concept to calculate the particle tunneling time through a potential barrier. The Larmor time appears in many theoretical works dedicated to the problem of interaction time (see, for example [43,44]). It is closely related with the group delay time of Bohm-Wigner. Indeed, the additional angle of Larmor precession arising due to interaction with any object may be identified with the phase difference  $\Delta\phi$  between two spin components of the resulting wave function whose wave numbers are different.

$$k_{\pm} = k_0(1 \mp \mu B/E)^{1/2}, \quad E = \hbar^2 k_0^2 / 2m, \quad (14)$$

where  $k_0$  is the neutron wave number in the absence of the magnetic field,  $\mu$  is the neutron magnetic moment and  $B$  is the magnetic induction. Defining after Baz' the time delay due to interaction as

$$\Delta t_L = \frac{\Delta\phi}{\omega_L}, \quad \omega_L = \frac{2\mu B}{\hbar}, \quad (15)$$

where  $\omega_L$  is the Larmor frequency and taking into account that  $2\mu B = (\hbar^2/2m)(k_+^2 - k_-^2) = \Delta E$  we come to the relation  $\Delta t_L = \hbar(\Delta\phi/\Delta E)$  coincident with (8) in the limit  $B \rightarrow 0$ .

The Larmor clock method was successfully used in neutron experiments for measuring the delay time at refraction, Bragg reflection from a multilayer mirror and also, at tunneling in the resonance of a quasi-bound state [45-48]. Note that the measured time error was about 0.5ns which is less than GDT at total reflection by an order of magnitude.

### 4.2. Direct measurement of the Goos – Hänchen shift.

As is shown above the GDT from multilayer structures may reach in the resonance a positive or negative value of an order of 100 - 200 ns. Standard practice of the GH type experiment is that instead of the longitudinal shift  $\zeta = \tau_r v_x$ , the transverse beam shift  $d = (v_y/v_0)\zeta$  is measured which is proportional to  $\zeta$  (see figure 2). By substituting eq. (9) and the relation for the total velocity  $v_0$  we obtain

$$d = \frac{v_y v_x}{\sqrt{v_y^2 + v_x^2}} \tau \approx v_y \tau \quad (v_y \ll v_x) \quad (16)$$

As soon as we discuss either total reflection or the case when the energy just slightly exceeds the barrier, the velocity component  $v_y$  normal to the matter interface is on the order of some meters per second. Therefore, the inequality in (16) is valid for all neutrons except the slowest neutrons and UCNs. The transverse shift  $d$  depends on the normal velocity alone. For the total reflection from homogeneous matter the beam shift is  $d \cong v_y \tau_r \approx 10\text{nm}$  and is hardly measurable.

Use of multilayer structures makes possible resonant enhancement of the GDT and of the longitudinal beam shift which is proportional to the former. The transverse shift is then of an order of a micrometer, which provides hope of performing appropriate experiments. The resonant behavior of the effect allows us to solve another serious problem. Because outside the very narrow resonance region the beam shift is small, the position of the beam beyond its limits may be accepted as a reference position with respect to which a relatively large resonant shift can be measured.

It is natural to think that the experiment can be performed with a neutron reflectometer operating in the time-of-flight mode. A narrow neutron beam will then fall on the sample at a fixed incidence angle

$\theta$  and the normal to surface component of the wave vector  $k_y(t) = k_x(t)\theta$  will depend on the time of flight. Thus reflected beam will only shift from its reference position over a short and well-known time interval. The experiment can be performed either with a position-sensitive detector or by using partial screening of the reflected beam with a nontransparent screen with a sharp edge. The beam shift will then result at variation of the count rate.

## 5. Conclusion

In the paper the well-known Goos – Hänchen effect was analyzed for the case of neutron reflection from matter. It is shown that the longitudinal shift of the reflected beam is always determined by the product of the longitudinal neutron velocity and the group delay time. In the case of total reflection of neutrons from matter the shift is very small and is unlikely to be measurable. However, in the case of neutron reflection from planar multilayer structures resonant enhancement of the effect may take place and in some conditions both the longitudinal shift and the group delay time may be negative. Some possibilities of experimental observation of the group delay time and of the Goos – Hänchen shift in a neutron experiment were discussed.

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