

The proton-proton weak capture reaction within chiral effective field theory

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Abstract. We review the results of the most recent calculation for the astrophysical S -factor of the weak proton-proton capture reaction, over a range for the center-of-mass relative energy of 0–100 keV. The so-called chiral effective field theory approach is used, where the chiral two-nucleon potential is derived up to next-to-next-to-next-to leading order and is augmented by the full electromagnetic interaction. The low-energy constants (LEC’s) entering the weak current operators are fixed so as to reproduce the $A = 3$ binding energies and magnetic moments, and the Gamow-Teller matrix element in tritium β -decay. Contributions from S and P partial waves in the incoming two-proton channel are retained. The S -factor at zero energy is found to be $\sim 1\%$ larger than the value reported in the literature, mostly due to the P -waves contributions.

1. Introduction

The proton weak capture on protons, i.e., the reaction ${}^1\text{H}(p, e^+\nu_e){}^2\text{H}$ (hereafter labelled pp), is the most fundamental process in stellar nucleosynthesis: it is the first reaction in the pp chain, which converts hydrogen into helium in main sequence stars like the Sun. Its reaction rate is expressed in terms of the astrophysical S -factor, $S(E)$, where E is the two-proton center-of-mass (c.m.) energy, by the relation

$$S(E) = E \exp(2\pi\eta) \sigma(E) , \quad (1)$$

where $\eta = \alpha/v_{\text{rel}}$, α being the fine structure constant and v_{rel} the pp relative velocity, and $\sigma(E)$ is the pp weak capture cross section. The energy-dependence of $S(E)$ is often parametrized as [1]

$$S(E) = S(0) + S'(0)E + S''(0)E^2/2 + \dots , \quad (2)$$

where $S(0)$, $S'(0)$ and $S''(0)$ are the zero-energy value of the S -factor, its first and second derivatives, both evaluated at $E = 0$. At the center of light stars like the Sun, with temperature of the order of 1.5×10^7 K, the Gamow peak is at $E \simeq 6$ keV, while in larger-mass stars, whose central temperature becomes of the order of 5×10^7 K, the Gamow peak turns out to be $E \sim 15$ keV. At these energies, the reaction cross section cannot be measured in terrestrial laboratories, and it is necessary to rely on theoretical predictions. The many studies on $S(0)$, and the few for $S'(0)$ and $S''(0)$, have been extensively reviewed in Ref. [1]. The currently recommended value for $S(0)$, $(4.01 \pm 0.01) \times 10^{-23}$ MeV fm² [1], is the average of values obtained within three different approaches, the “potential model” approach (PMA), “hybrid chiral effective field theory” (χ EFT*) and “pionless effective field theory” (π EFT). The first one uses phenomenological realistic models for the nuclear potential, fitted to reproduce the



large body of two-nucleon (NN) bound and scattering state data with a $\chi^2/\text{datum} \sim 1$. The axial current operator includes both one- and two-body terms, these latest derived from meson-exchange mechanisms and the excitation of Δ -isobar resonances. The axial coupling constant for the N -to- Δ transition (g_A^*) is constrained to reproduce the experimental value of the Gamow-Teller (GT) matrix element of tritium β -decay. In the χEFT^* approach, transition operators derived in χEFT are sandwiched between initial and final wave functions generated by potential models. The only unknown low-energy constant (LEC), which parametrizes the strength of a contact-type four-nucleon coupling to the axial current, is determined by fitting the experimental GT matrix element. Finally, λEFT is an approach applicable to low-energy processes—such as the pp reaction under consideration here—, where pions are integrated out and the NN interaction and weak currents are described by classes of point-like contact interactions, each class corresponding to given order in a systematic expansion in powers of the characteristic momentum over the pion mass.

For $S'(0)$ and $S''(0)$ the situation is much less clear than for $S(0)$. The adopted value for $S'(0)$ in Ref. [1] is $S'(0)/S(0) = (11.2 \pm 0.1) \text{ MeV}^{-1}$, as obtained in Ref. [2] and later confirmed in Ref. [3] in a PMA. No value is reported for $S''(0)$ in Ref. [1]. In Ref. [2] it was estimated by dimensional considerations that the contribution of $S''(0)$ to the pp rate would be at the level of 1% for temperatures characteristic of the solar interior. Only very recently, $S'(0)$ and $S''(0)$ have been studied in λEFT [4] to the third-order in the power expansion, with the results $S'(0)/S(0) = (11.3 \pm 0.1) \text{ MeV}^{-1}$ and $S''(0)/S(0) = (170 \pm 2) \text{ MeV}^{-2}$.

In parallel with the λEFT study of Ref. [4], a systematic calculation of $S(E)$ in (pionfull) χEFT has been reported in Ref. [5]. This latest work will be reviewed in this contribution.

2. Formalism

The pp weak capture cross section $\sigma(E)$, from which the S -factor is obtained (see equation 1), is written in the c.m. frame as

$$\begin{aligned} \sigma(E) &= \int 2\pi\delta(\Delta m + E - \frac{q^2}{2m_d} - E_e - E_\nu) \frac{1}{v_{\text{rel}}} F(Z, E_e) \\ &\times \frac{1}{4} \sum_{s_e s_\nu} \sum_{s_1 s_2 s_d} |\langle f | H_W | i \rangle|^2 \frac{d\mathbf{p}_e}{(2\pi)^3} \frac{d\mathbf{p}_\nu}{(2\pi)^3}, \end{aligned} \quad (3)$$

where $\Delta m = 2m_p - m_d$ (m_p and m_d are the proton and deuteron masses, respectively), $\mathbf{p}_e$ ($\mathbf{p}_\nu$) and E_e (E_ν) are the electron (neutrino) momentum and energy, $\mathbf{q} = \mathbf{p}_e + \mathbf{p}_\nu$ is the momentum transfer, and $F(Z, E_e)$ is the Fermi function (with $Z = 1$), which accounts for the Coulomb distortion of the final positron wave function, including radiative corrections [6]. The transition amplitude $\langle f | H_W | i \rangle$ is given by

$$\langle f | H_W | i \rangle = \frac{G_V}{\sqrt{2}} l^\sigma \langle -\mathbf{q}; d | j_\sigma^\dagger | \mathbf{p}; pp \rangle, \quad (4)$$

where G_V is the Fermi constant, $|-\mathbf{q}; d\rangle$ and $|\mathbf{p}; pp\rangle$ represent, respectively, the deuteron bound state with recoiling momentum $-\mathbf{q}$ and the pp scattering state with relative momentum $\mathbf{p}$, and l_σ and $j^\sigma(\mathbf{q})$ are the leptonic and nuclear weak currents, respectively. A standard multipole decomposition of the nuclear weak current operator leads to [5]

$$\frac{1}{4} \sum_{s_e s_\nu} \sum_{s_1 s_2 s_d} |\langle f | H_W | i \rangle|^2 = (2\pi)^2 G_V^2 L_{\sigma\tau} N^{\sigma\tau}, \quad (5)$$

where $L^{\sigma\tau}$ and $N^{\sigma\tau}$ are the lepton and nuclear tensors, respectively. The latter can be expressed in terms of the reduced matrix elements for the Coulomb, longitudinal, transverse magnetic

and transverse electric multipole operators between the initial pp state with orbital angular momentum L , channel spin S ($S = 0, 1$), total angular momentum J , and the final deuteron state with total angular momentum $J_d = 1$. The integrations over $\mathbf{p}_e$ and $\mathbf{p}_\nu$ are performed by Gaussian quadratures [7], and a moderate number of Gauss points (~ 10 – 20 for each integration) suffices to achieve convergence to within better than 1 part in 10^3 .

The two-body wave functions have been obtained using the NN potential derived in χ EFT up to next-to-next-to-next-to leading order (N3LO) in the chiral expansion by Entem and Machleidt [8, 9]. The two-body Schrödinger equation has been solved variationally with the technique described in Refs. [10, 5]. To be noticed that, in the pp sector, the nuclear NN potential has been augmented by the Coulomb interaction and the higher-order electromagnetic (EM) terms, due to two-photon exchange and vacuum polarization. These higher-order terms are the same as those of the Argonne v_{18} (AV18) NN potential [11], and therefore also retain short-range corrections associated with the finite size of the proton charge distribution. The additional distortion of the pp wave function, induced primarily by vacuum polarization, has been shown to reduce $S(0)$ by $\sim 1\%$ in Ref. [3]. The technique has been tested by verifying that, in the case of the AV18, the 1S_0 phase shifts are in agreement with those reported for the AV18 in Ref. [11]. Of course, the N3LO phase shifts listed in Ref. [9], obtained by including only the Coulomb potential, have also been reproduced. Note that the N3LO NN potential of Ref. [8] has been used in conjunction with the three-nucleon interaction (TNI) obtained up to next-to-next-to leading order (N2LO), in the version of Ref. [12], when reactions involving $A > 2$ nuclear systems are considered.

The charge-changing weak current has been first derived in χ EFT by Park and collaborators in the late nineties, using the so-called heavy-baryon chiral perturbation theory (HB χ PT) approach, where the baryons are treated as heavy static sources, and the perturbative expansion is performed in terms of the involved momenta over the baryon mass (see Ref. [13] and references therein). These are the same currents used to study the pp reaction in χ EFT*. In particular, the polar-vector part is related, via the conserved-vector-current constraint, to the (isovector) EM current, and at N3LO includes, apart from one- (OPE) and two-pion-exchange (TPE) terms, two contact terms—one isoscalar and the other isovector—whose strengths are parametrized by the LEC's g_{4S} and g_{4V} . To be noticed that few years ago, the problem of deriving the electromagnetic current and charge operators in χ EFT has been revisited by Pastore *et al.* [14] and, in parallel, by Kölling *et al.* [15]. Pastore *et al.* have used time-ordered perturbation theory (TOPT) to calculate the EM transition amplitudes, which allows for an easier treatment of the so-called reducible diagrams than the HB χ PT approach. On the other hand, Kölling *et al.* have used the method of unitary transformation, the same one used to derive the chiral potentials mentioned above. The EM operators of Pastore *et al.* have been found significantly different from those of Park *et al.*, especially for the structure of the contact terms, while they are in good agreement with those of Kölling *et al.*. However, the TOPT approach has not yet been used to derive the axial component of the weak current operator, and work along this line is currently underway.

The two-body axial-vector current are diagrammatically represented in figure 1, where they are listed according to their scaling in Q , the pions' and nucleons' momenta. The leading-order (LO) contribution consists of the well known single-nucleon axial current, and is of order Q^{-3} . At order Q^{-2} it turns out that there are no contributions, and therefore the next-to-leading (NLO) contribution is of order Q^{-1} , and arises from the $(Q/m)^2$ relativistic corrections to the LO contribution (m is the nucleon mass). The next-to-next-to leading order (N2LO) currents consist of the OPE term and a contact term with one LEC, denoted with d_R . Note that N3LO contributions arise from loop and TPE terms, and they have not been calculated yet.

All the chiral potentials and currents presented above have power-law behavior in momentum space, and must be regularized before they can be used in practical calculations. This is

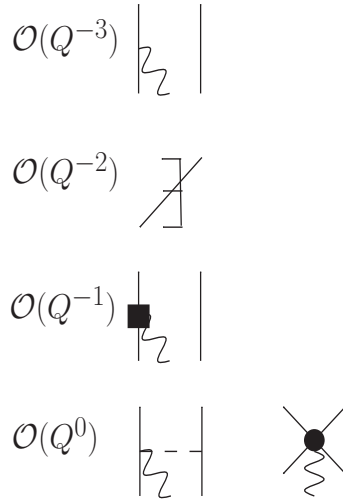


Figure 1. Diagrams illustrating one- and two-body χ EFT axial currents entering at LO (Q^{-3}), NLO (Q^{-1}), and N2LO (Q^0). Nucleons, pions, and weak probes are denoted by solid, dashed, and wavy lines, respectively. The solid square represents the relativistic corrections to the one-body current, while the solid circles represent the contact terms. Only the relevant topologies are indicated. No contribution of order Q^{-2} exists.

accomplished by multiplying them by a momentum-cutoff function, whose cutoff Λ is taken to be in the range (500–600) MeV.

Some remarks on the fitting procedure of the LEC d_R are in order. As first shown in Ref. [16], d_R can be related to the LEC c_D entering one of the two contact terms present in the TNI at N2LO, via the relation

$$d_R = \frac{m}{\Lambda_\chi g_A} c_D + \frac{1}{3} m (c_3 + 2c_4) + \frac{1}{6}, \quad (6)$$

where g_A is the single-nucleon axial coupling constant, c_3 and c_4 are LEC's of the πN Lagrangian, already part of the chiral NN potential at NLO, and $\Lambda_\chi = 700$ MeV is the the chiral-symmetry-breaking scale. Therefore, it has become common practice to fit c_D (and c_E — the other LEC entering the N2LO TNI) to the triton binding energy and the Gamow-Teller matrix element in tritium β -decay. The values obtained in this way for c_D and c_E are listed in Ref. [17], where they have been used to study the muon capture on deuteron and ^3He , leading to predictions (with an estimated theory uncertainty of about 1%) in excellent agreement with the experimental data. Note that the first studies of $A = 3$ and 4 elastic scattering observables, as cross sections and analyzing powers, with these values for c_E and c_D have been reported in Ref. [18]. In particular, in table 1 we report the central values of c_D and c_E obtained with the N3LO NN potential and N2LO TNI mentioned above, for two different values of the cutoff Λ , 500 and 600 MeV, and the corresponding values for the ^4He binding energy and the $n - d$ doublet scattering length. The values of c_D and c_E obtained within this fitting procedure for two other values of the cutoff Λ , 414 and 450 MeV, have been used also to study infinite nuclear matter [19].

3. Results

In the present section we summarize the results for the pp astrophysical S -factor, as function of the c.m. pp energy E , obtained in Ref. [5]. We recall that (i) $S(E)$ is studied in the wide

Table 1. Values for the LEC's c_D and c_E obtained with the N3LO NN potential and N2LO TNI, for two different values of the cutoff Λ , 500 and 600 MeV. The corresponding values for the ^4He binding energy (in MeV) and the $n-d$ doublet scattering length (in fm) are also given. The row labelled “N3LO/N2LO*” indicates the results obtained with the original N2LO TNI of Ref. [12].

Model	Λ	c_D	c_E	$B(^4\text{He})$	$^2a_{nd}$
N3LO/N2LO*	500	1.0	-0.029	28.36	0.675
N3LO/N2LO	500	-0.12	-0.196	28.49	0.666
N3LO/N2LO	600	-0.26	-0.846	28.64	0.696

energy range 2 – 100 keV. The solar Gamow peak is at $\simeq 6$ keV, but in larger-mass stars the Gamow peak becomes 15 keV. (ii) The χEFT N3LO NN potential is augmented not only of the Coulomb interaction but also of the higher-order EM interaction contributions, like those due to two-photon exchange, Darwin-Foldy and vacuum polarization. (iii) All the $L \leq 1$ pp partial waves are considered. To our knowledge, this is the first time that the P -wave contributions are retained.

The results for the zero-energy S -factor $S(0)$ obtained retaining only the 1S_0 pp partial wave are given in table 2, where the one-body and the full one- plus two-body current contributions are listed. We also show the results obtained neglecting the higher-order EM contributions (i.e., with only the Coulomb interaction), and, for comparison, those obtained using the PMA. By inspection of the table we can conclude that (i) the value for $S(0)$ is in agreement with the recommended one of Ref. [1]; (ii) the higher-order EM contributions are of the order of 1% or less; (iii) the χEFT and PMA results are in excellent agreement; (iv) the cutoff dependence is extremely small ($\leq 1\%$). Note that the theoretical uncertainty on the results of table 2 induced by the adopted fitting procedure of the LEC d_R (axial N -to- Δ coupling constant g_A^*) in the χEFT approach (PMA) are not reported, since they are affecting the third decimal digits.

The cumulative contributions to $S(0)$ due to S - and P -waves are listed in table 3. As it can be seen by inspection of the table, the P -wave contributions sum up to a $\sim 1\%$ of the total value, and the theoretical uncertainty due to the fitting procedure as well as the cutoff dependence is negligible. Again, the χEFT and PMA results are in excellent agreement.

Table 2. Zero-energy S -factor (in 10^{-23} MeV fm²) obtained retaining only the 1S_0 pp partial wave. Two different values for the cutoff Λ are used, 500 and 600 MeV. The results obtained including the full EM (only the Coulomb) interaction are listed in the columns labelled “ $V_{NN} + V_{EM}$ ” (“ $V_{NN} + V_{Coul}$ ”). The labels IA and FULL are used when only the one-body or the full one- and two-body current contributions are retained. The PMA results are also given.

	$V_{NN} + V_{Coul}$	$V_{NN} + V_{EM}$
$\chi\text{EFT}(500)\text{-IA}$	3.96	3.94
$\chi\text{EFT}(500)\text{-FULL}$	4.03	4.01
$\chi\text{EFT}(600)\text{-IA}$	3.94	3.93
$\chi\text{EFT}(600)\text{-FULL}$	4.01	4.01
PMA-IA	3.99	3.96
PMA-FULL	4.03	4.00

Table 3. Cumulative S - and P -wave contributions to $S(0)$ in units of 10^{-23} MeV fm². The results labelled “ χ EFT(500)” and “ χ EFT(600)” have been obtained within the χ EFT approach with two different cutoff values, 500 and 600 MeV. The results obtained within the PMA are also shown. The theoretical uncertainties are given in parentheses and are due to the fitting procedure adopted for the LEC’s (or g_A^* within the PMA) in the weak current.

	1S_0	$\dots + ^3P_0$	$\dots + ^3P_1$	$\dots + ^3P_2$
χ EFT(500)	4.008(5)	4.011(5)	4.020(5)	4.030(5)
χ EFT(600)	4.007(5)	4.010(5)	4.019(5)	4.029(5)
PMA	4.000(3)	4.003(3)	4.015(3)	4.033(3)

In conclusion, the χ EFT results of table 3 can be summarized in the conservative range $S(0) = (4.030 \pm 0.006) \times 10^{-23}$ MeV fm², with a P -wave contribution of $\simeq 0.2 \times 10^{-23}$ MeV fm².

Finally, we show in figure 2 the energy dependence of $S(E)$ in the energy range 2 – 100 keV, as obtained within the χ EFT approach. The S - and $(S + P)$ -wave contributions are displayed separately, and the theoretical uncertainty is included—the curves are in fact very narrow bands. As expected, the P -wave contributions become significant at higher values of E . From these results, a least-squares polynomial fit to $S(E)$ has been performed up to order $O(E^2)$, i.e., by

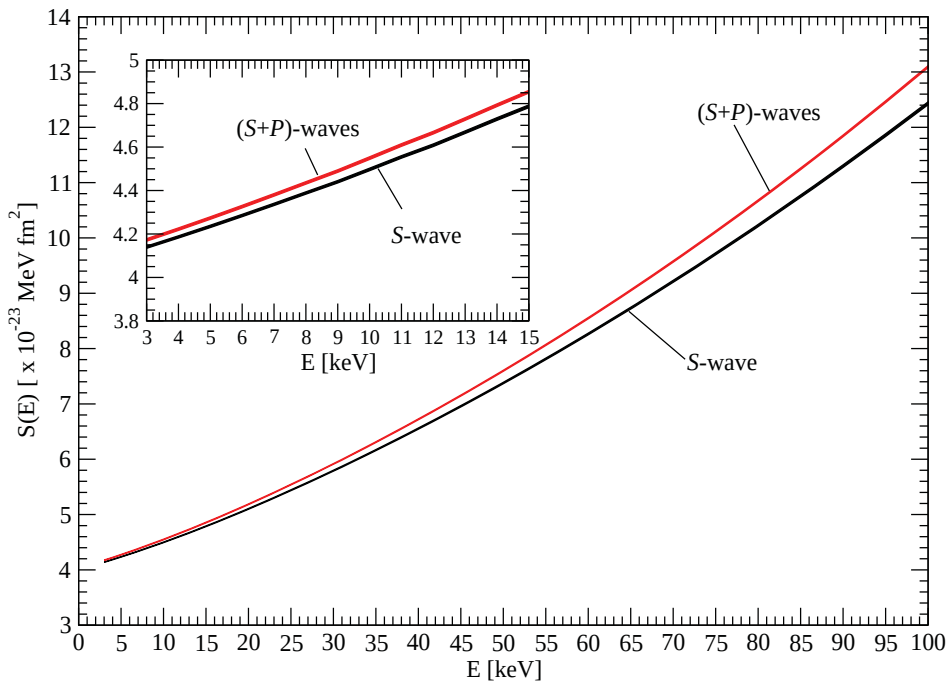


Figure 2. (Color online) Energy dependence of $S(E)$ in the range 2 – 100 keV. The S - and $(S + P)$ -wave contributions are displayed separately. In the inset, $S(E)$ is shown in the range 3–15 keV.

Table 4. Values for the first $[S'(0)/S(0)]$, second $[S''(0)/S(0)]$ and third $[S'''(0)/S(0)]$ derivatives of $S(E)$ at zero energy, in units of MeV^{-1} , MeV^{-2} , MeV^{-3} , respectively, as obtained with a polynomial fit of $S(E)$ up to order $O(E^2)$ (Fit 1) and $O(E^3)$ (Fit 2), retaining all $(S + P)$ -waves or only the 1S_0 initial partial wave. The numbers given in parentheses are the theoretical uncertainties, which account for the cutoff sensitivity and the error due to the LEC's fitting procedure. The χ^2 value is also shown, as defined in the text.

	$S'(0)/S(0)$	$S''(0)/S(0)$	$S'''(0)/S(0)$	χ^2
$S + P$ - Fit 1	12.59(1)	199.3(1)		8.8×10^{-4}
$S + P$ - Fit 2	11.94(1)	248.8(2)	-1183(8)	1.9×10^{-4}
1S_0 - Fit 1	12.23(1)	178.4(3)		1.2×10^{-3}
1S_0 - Fit 2	11.42(1)	239.6(5)	-1464(5)	1.9×10^{-4}

using equation (2) itself, and up to order $O(E^3)$, by adding the $S'''(0)E^3/3!$ term ($S'''(0)$ is the third derivative of $S(E)$ evaluated at $E = 0$). The values for $S'(0)/S(0)$, $S''(0)/S(0)$, and $S'''(0)/S(0)$ for the two fits (up to $O(E^2)$ - Fit 1, and $O(E^3)$ - Fit 2) are listed in table 4, along with the χ^2 value, here defined as $\chi^2 = \sum_i (1 - f_i^{fit}/f_i^{calc})^2$, with f_i^{calc} (f_i^{fit}) being the calculated (fitted) $S(E)$ results. By inspection of the table we conclude that the values for the derivatives of $S(E)$ are strongly dependent on the order of the polynomial function used in the fit. However, an accurate description of the data can be obtained with a desired degree of accuracy by increasing the number of polynomial terms. Finally, we would like to remark that the values for $S'(0)/S(0)$ and $S''(0)/S(0)$ obtained in Ref. [4] are similar to those listed in the table only in the case of Fit 1, with only the 1S_0 contribution. The χ^2 in this case, though, assumes the highest value.

4. Conclusions

We have presented the most recent calculation of the pp astrophysical S -factor within the χ EFT approach in a wide energy range, up to 100 keV, retaining all the S - and P -waves in the initial wave function, and including, beyond the Coulomb interaction, also higher order EM contributions, like two-photon exchange, Darwin-Foldy and vacuum polarization. The value at zero energy, $S(0)$, has been calculated also within the PMA, finding a remarkable agreement between the two different approaches. The theoretical uncertainty has been reduced below the 1% level.

As a final remark, we notice that the present χ EFT model for the nuclear current operator is that of Park *et al.*, up to N2LO. Recently, the model for the EM current and charge operators of Pastore *et al.*, developed up to N3LO, has been used to study the EM structure of $A = 2, 3$ nuclei [20], finding a good agreement between theory and experiment. It has also been shown that the TOPT approach used by Pastore *et al.* is more suitable than the HB χ PT one of Park *et al.* to treat reducible diagrams and to derive the contact terms. Therefore, it is highly desirable to derive the axial currents up to N3LO in TOPT, following the footsteps of Pastore *et al.* for the EM operators, and to repeat this study for the pp reaction. Finally, it should be noticed that the radiative proton-deuteron and the weak proton- ^3He captures, also relevant in astrophysics, are at reach within the present framework. Work along these lines is currently underway.

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