

Thermodynamic instabilities in dense asymmetric nuclear matter and in compact stars

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Abstract. We investigate the presence of thermodynamic instabilities in compressed asymmetric baryonic matter, reachable in high energy heavy ion collisions, and in the cold β -stable compact stars. To this end we study the relativistic nuclear equation of state with the inclusion of Δ -isobars and require the global conservation of baryon and electric charge numbers. Similarly to the low density nuclear liquid-gas phase transition, we show that a phase transition can occur in dense asymmetric nuclear matter and it is characterized by both mechanical instability (fluctuations on the baryon density) and by chemical-diffusive instability (fluctuations on the electric charge concentration). Such thermodynamic instabilities can imply a very different electric charge fraction Z/A in the coexisting phases during the phase transition and favoring an early formation of Δ^- particles with relevant phenomenological consequences in the physics of the protoneutron stars and compact stars. Finally, we discuss the possible co-existence of very compact and very massive compact stars in terms of two separate families: compact hadronic stars and very massive quark stars.

1. Introduction

One of the very interesting aspects in nuclear astrophysics and in the heavy-ion collisions experiments is a detailed study of the thermodynamical properties of strongly interacting nuclear matter away from the nuclear ground state. In this direction, many efforts have been focused on searching for possible phase transitions in the β -stability conditions and in regime of finite temperature and density.

The new accumulating data from x-ray satellites provide important informations on the structure and formation of compact stellar objects. The recent discovery of Compact Stars (CSs) having a mass of the order of $2 M_{\odot}$ [1, 2] puts rather severe constraints on the equation of state (EOS) of matter at large densities. It is clear that matter inside a compact star, i.e. β -stable and charge neutral matter, has to be stiff to allow such massive configurations. On the other hand, we know that by increasing the density new degrees of freedom come into the game, for instance hyperons and maybe deconfined quarks. These new ingredients soften the EOS close to their production threshold, but by introducing repulsive interactions the EOS can be stiff enough at large densities to support a $2 M_{\odot}$ configuration [3, 4, 5].

On the other hand, the information coming from experiments with heavy ions in intermediate- and high-energy collisions is that the EOS depends on the beam energy but also sensibly on the electric charge fraction Z/A of the colliding nuclei, especially at not too high temperature



[6, 7]. Moreover, the study of nuclear matter with arbitrary electric charge fraction results to be important in radioactive beam experiments and in the physics of compact stars.

The extraction of information about the equation of state at different densities and temperatures by means of intermediate and high energy heavy ion collisions is a very difficult task and can be realized only indirectly by comparing the experimental data with different theoretical models, such as, for example, fluid-dynamical models. The EOS at density below the saturation density of nuclear matter ($\rho_0 \approx 0.16 \text{ fm}^{-3}$) is relatively well known due to the large amount of experimental nuclear data available. At larger density there are many uncertainties; the strong repulsion at short distances of nuclear force makes, in fact, the compression of nuclear matter quite difficult. However, in relativistic heavy ion collisions the baryon density can reach values of a few times the saturation nuclear density and/or high temperatures. The future CBM (Compressed Baryonic Matter) experiment of FAIR (Facility of Antiproton and Ion Research) project at GSI Darmstadt, will make it possible to create compressed baryonic matter with a high net baryon density [8].

In relativistic heavy ion collisions, where finite values of density and temperature can be reached, a state of high density resonance matter may be formed and the $\Delta(1232)$ -isobar degrees of freedom are expected to play a central role [9, 10, 11]. Moreover, within the non-linear Walecka model, it has been predicted that a phase transition from nucleonic matter to Δ -excited nuclear matter can take place but the occurrence of this transition sensibly depends on the Δ -meson coupling constants [12, 13, 14]. In this context, it is important to observe that also in CSs, where high baryon densities are reached, $\Delta(1232)$ -resonances can be also produced in principle and it has been pointed out that the existence of Δ s can be very relevant also in the core of neutron stars [15, 16, 17, 18]. The production of these particles softens the EOS and allows very compact configurations. On the other hand, this same softening forbids this hadronic family of CSs to reach very large masses [19, 20].

The purpose of this paper is twofold: first, to show that, for asymmetric warm and dense nuclear medium, the possible Δ -matter phase transition is characterized by mechanical and chemical-diffusive instabilities. Similarly to the liquid-gas phase transition [21], chemical instabilities play a crucial role in the characterization of the phase transition and can imply a very different electric charge fraction Z/A in the coexisting phases during the phase transition. Second, to investigate the relevance of Δ -isobar degrees of freedom in the bulk properties of compact stars and the possible presence of thermodynamic instability in the β -stable and charge neutral hadronic EOS. Finally, we will discuss a possible interpretation of the co-existence of compact and very massive CSs in terms of two separate families: hadronic stars, whose equation of state is soft, can be very compact, while quark stars, whose equation of state is stiff, can be very massive. In this respect an early appearance of Δ resonances is crucial to guarantee the stability of the branch of hadronic stars.

2. Nuclear Equation of State

Concerning the hadronic EOS, we use the relativistic mean field model with the inclusion of the octet of lightest baryons (nucleons and hyperons) in the framework of the GM3 non-linear Walecka type model of Glendenning-Moszkowsky [22]. In this framework the Lagrangian density \mathcal{L}_{NH} for nucleons and hyperons can be written as

$$\begin{aligned} \mathcal{L}_{\text{NH}} = & \sum_k \bar{\psi}_k [i \gamma_\mu \partial^\mu - (M_k - g_{\sigma k} \sigma) - g_{\omega k} \gamma_\mu \omega^\mu - g_{\rho k} \gamma_\mu \vec{t} \cdot \vec{\rho}^\mu] \psi_k \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c (g_{\omega N}^2 \omega_\mu \omega^\mu)^2 \\ & + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu}, \end{aligned} \quad (1)$$

where the sum runs over the full octet of baryons, M_k is the vacuum baryon mass of index k , the quantity \vec{t} denotes the isospin operator that acts on the baryon and the field strength tensors for the vector mesons are given by the usual expressions $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$. The $U(\sigma)$ is a nonlinear self-interaction potential of σ meson

$$U(\sigma) = \frac{1}{3}a (g_{\sigma N} \sigma)^3 + \frac{1}{4}b (g_{\sigma N} \sigma^4). \quad (2)$$

The implementation of hyperon degrees of freedom comes from determination of the corresponding meson-hyperon coupling constants that have been fitted to hypernuclear properties. Following Refs. [23, 24], the SU(6) simple quark model can be used to obtain the relations

$$\frac{1}{3}g_{\omega N} = \frac{1}{2}g_{\omega \Lambda} = \frac{1}{2}g_{\omega \Sigma} = g_{\omega \Xi}, \quad g_{\rho N} = \frac{1}{2}g_{\rho \Sigma} = g_{\rho \Xi}, \quad g_{\rho \Lambda} = 0. \quad (3)$$

In addition, we can fix the scalar σ meson-hyperon ($g_{\sigma Y}$) coupling constants to the potential depth of the corresponding hyperon in normal dense matter taking into account the following recent results [25, 26]

$$U_{\Lambda}^N = -28 \text{ MeV}, \quad U_{\Sigma}^N = +30 \text{ MeV}, \quad U_{\Xi}^N = -18 \text{ MeV}. \quad (4)$$

To incorporate Δ -isobars in the framework of effective hadron field theories, a formalism was developed to treat Δ particles analogously to the nucleons, taking only the on-shell Δ s into account and the mass of the Δ s are substituted by the effective one in the mean field approximation [27, 28]. The Lagrangian density of the Δ -isobars can then be expressed as [12, 13, 27]

$$\mathcal{L}_{\Delta} = \bar{\psi}_{\Delta \nu} [i\gamma_{\mu} \partial^{\mu} - (M_{\Delta} - g_{\sigma \Delta} \sigma) - g_{\omega \Delta} \gamma_{\mu} \omega^{\mu}] \psi_{\Delta}^{\nu}, \quad (5)$$

where ψ_{Δ}^{ν} is the Rarita-Schwinger spinor for the Δ -baryon.

Due to the uncertainty on the meson- Δ coupling constants, we limit ourselves to considering only the couplings with σ and ω meson fields, which are explored in the literature [12, 13, 29]. If the SU(6) symmetry were exact, one might adopt the universal couplings $x_{\sigma \Delta} = g_{\sigma \Delta}/g_{\sigma N} = 1$ and $x_{\omega \Delta} = g_{\omega \Delta}/g_{\omega N} = 1$. However, the SU(6) symmetry is not exactly fulfilled and one may assume the scalar coupling ratio $x_{\sigma \Delta} > 1$ with a value close to the mass ratio of the Δ and the nucleon [13]. On the other hand, QCD finite-density sum rule results show that the Lorentz vector self-energy for the Δ is significantly smaller than the nucleon vector self-energy implying therefore $x_{\omega \Delta} < 1$ [29]. In this paper we adopt different choices for the Δ -meson couplings which are consistent with the experimental flow data of heavy-ion collisions at intermediate energies [30, 10].

Because we are going to describe a dense asymmetric nuclear matter, we have to require the conservation of two charges: baryon number (B) and electric charge (C)¹. As a consequence, the system is described by two independent chemical potentials: μ_B and μ_C , the baryon and the electric charge chemical potential, respectively. Therefore, the chemical potential of particle of index i can be written as

$$\mu_i = b_i \mu_B + c_i \mu_C, \quad (6)$$

¹ In heavy ion collisions strangeness can be produced only by strong-interaction via associated production and we neglect the contribution of strange hadrons because a tiny amount of strangeness can be produced in the range of temperature and density explored in this study. On the other hand, in the β -stability condition, realized in CSs, strangeness is mainly produced by weak interaction and is not conserved.

where b_i and c_i are, respectively, the baryon and the electric charge quantum numbers of the i th hadron.

The thermodynamical quantities can be obtained from the baryon grand potential Ω_B in the standard way. More explicitly, the baryon pressure $P_B = -\Omega_B/V$ and the energy density can be written as

$$P_B = \frac{1}{3} \sum_i \gamma_i \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{E_i^*(k)} [n_i(k) + \bar{n}_i(k)] - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{4} c (g_{\omega N} \omega)^4 + \frac{1}{2} m_\rho^2 \rho^2, \quad (7)$$

$$\epsilon_B = \sum_i \gamma_i \int \frac{d^3k}{(2\pi)^3} E_i^*(k) [n_i(k) + \bar{n}_i(k)] + \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{1}{2} m_\omega^2 \omega^2 + \frac{3}{4} c (g_{\omega N} \omega)^4 + \frac{1}{2} m_\rho^2 \rho^2. \quad (8)$$

3. Stability conditions and phase transition in dense asymmetric nuclear matter

As previously discussed, we are dealing with the study of a multi-component system with two conserved charges: baryon number and electric charge. For such a system, the Helmholtz free energy density F can be written as [21]

$$F(T, \rho_B, \rho_C) = -P(T, \mu_B, \mu_C) + \mu_B \rho_B + \mu_C \rho_C, \quad (9)$$

with

$$\mu_B = \left(\frac{\partial F}{\partial \rho_B} \right)_{T, \rho_C}, \quad \mu_C = \left(\frac{\partial F}{\partial \rho_C} \right)_{T, \rho_B}. \quad (10)$$

In a system with N different particle species, the particle chemical potentials are expressed as the linear combination of the two independent chemical potentials μ_B and μ_C and, as a consequence, $\sum_{i=1}^N \mu_i \rho_i = \mu_B \rho_B + \mu_C \rho_C$.

Assuming the presence of two phases (denoted as I and II , respectively), the system is stable against the separation in two phases if the free energy of a single phase is lower than the free energy in all two phases configuration. The phase coexistence is given by the Gibbs conditions

$$\mu_B^I = \mu_B^{II}, \quad \mu_C^I = \mu_C^{II}, \quad P^I(T, \mu_B, \mu_C) = P^{II}(T, \mu_B, \mu_C). \quad (11)$$

Therefore, at a given baryon density ρ_B and at a given net electric charge density $\rho_C = y \rho_B$ (with $y = Z/A$), the chemical potentials μ_B and μ_C are univocally determined. An important feature of this conditions is that, unlike the case of a single conserved charge, the pressure in the mixed phase is not constant and, although the total ρ_B and ρ_C are fixed, baryon and charge densities can be different in the two phases. For such a system in thermal equilibrium, the possible phase transition can be characterized by mechanical (fluctuations in the baryon density) and chemical instabilities (fluctuations in the electric charge density). As usual the condition of the mechanical stability implies

$$\rho_B \left(\frac{\partial P}{\partial \rho_B} \right)_{T, \rho_C} > 0. \quad (12)$$

By introducing the notation $\mu_{i,j} = (\partial \mu_i / \partial \rho_j)_{T,P}$ (with $i, j = B, C$), the chemical stability for a process at constant P and T can be expressed with the following conditions [14]

$$\rho_B \mu_{B,B} + \rho_C \mu_{C,B} = 0, \quad (13)$$

$$\rho_B \mu_{B,C} + \rho_C \mu_{C,C} = 0. \quad (14)$$

Whenever the above stability conditions are not respected, the system becomes unstable and the phase transition take place. The coexistence line of a system with one conserved charge becomes in this case a two dimensional surface in (T, P, y) space, enclosing the region where mechanical and diffusive instabilities occur.

The chemical stability condition is satisfied if [14]

$$\left(\frac{\partial \mu_C}{\partial y}\right)_{T,P} > 0 \quad \text{or} \quad \begin{cases} \left(\frac{\partial \mu_B}{\partial y}\right)_{T,P} < 0, & \text{if } y > 0, \\ \left(\frac{\partial \mu_B}{\partial y}\right)_{T,P} > 0, & \text{if } y < 0. \end{cases} \quad (15)$$

In the above condition it has been considered possible negative values of $y = Z/A$ because of, as already observed, during a phase transition with two conserved charges, the electric charge fraction $y = \rho_C/\rho_B$ is not locally conserved in the single phase but only globally conserved. Therefore, during the compression of the system, the appearance of particles with negative electric charge (such as Δ^-) could, in principle, shift the diffusive instability region to negative values of y , even if the system is prepared with a positive y . Such a feature has no counterpart in the liquid-gas phase transition and, as we will see, it turns out to be very relevant in order to properly determine the instability region through the binodal phase diagram.

In the figure 1, we report the baryon and electric charge chemical potential isobars as a function of y , at fixed temperature $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$ in the GM3 parameters set [22]. It is relevant to observe that for the value $x_{\sigma\Delta} = 1$, we do not find any mechanical or diffusive instability. Contrariwise, by increasing the $x_{\sigma\Delta}$ coupling ratio, mechanical and chemical instabilities take place. In particular, in the range $1 < x_{\sigma\Delta} \leq 1.1$, instabilities are restricted to very low values of temperature and electric charge fraction, but for $x_{\sigma\Delta} > 1.1$, such instabilities start to be much more relevant and extend to higher values of T and y .

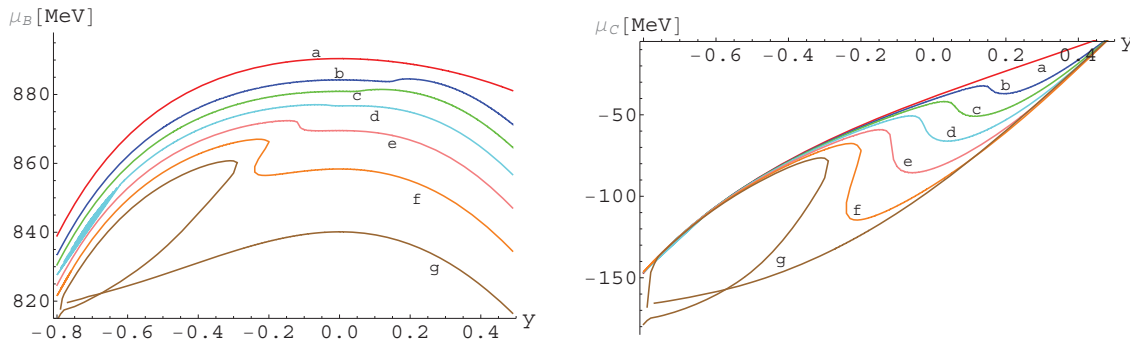


Figure 1. (Color online) Baryon (right panel) and electric charge (left panel) chemical potential isobars as a function of y at $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$. The curves labeled a through g have pressure $P=9,7,6,5,4,3,2$ MeV/fm³, respectively.

From the analysis of the above chemical potential isobars, we are able to construct the binodal surface relative to the nucleon- Δ matter phase transition and in figure 2, we show the relative binodal section with the two phases at different density and electric charge fraction y . The right branch (at lower density) corresponds to the initial phase (I), where the dominant component of the system is given by nucleons. The left branch (II) is related to the final phase at higher densities, where the system is composed primarily by Δ -isobar degrees of freedom (Δ -dominant phase). In presence of Δ -isobars the phase coexistence region results very different

from what obtained in the liquid-gas case, in particular it extends up to regions of negative electric charge fraction and the mixed phase region ends in a point of maximum asymmetry with $y = -1$ (corresponding to a system with almost all Δ^- -particles, being antiparticles and pions contribution not relevant in this regime).

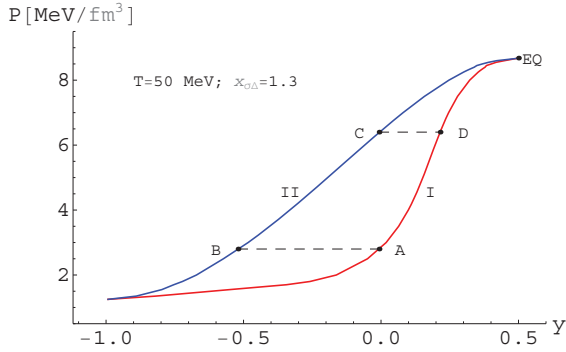


Figure 2. (Color online) Binodal section at $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$.

Let us analyze the phase evolution of the system during the isothermal compression from an arbitrary initial point A , indicated in figure 2. In this point the system becomes unstable and starts to be energetically favorable the separation into two phases, therefore an infinitesimal Δ -dominant phase appears in B , at the same temperature and pressure. Let us observe that, although in B the electric charge fraction is substantially negative, the relative Δ^- abundance must be weighed on the low volume fraction occupied by the phase II near the point B . During the phase transition, each phase evolves towards a configuration with increasing y , in contrast to the liquid-gas case, where each phase evolves through a configuration with a decreasing value of y (with the exception of the gas phase after the maximum asymmetry point).

In figure 3, we report the phase diagram with in evidence the coexistence regions of the liquid-gas and the nucleon- Δ matter phase transition for $y = 0.3$ and 0.5 ($x_{\sigma\Delta} = 1.3$). The two coexistence regions are well separated and the features of the two phase transitions differ significantly. In fact, for the liquid-gas transition, asymmetric nuclear matter implies a reduction of the second critical density and of the critical temperature T_c . Contrariwise, for the Δ -dominant phase transition, we have a slight increase of the critical temperature and a significant reduction of the first critical density. In particular at moderate temperatures ($T \approx 30 \div 40$ MeV), the system begins the mixed phase at a baryon density of the order of ρ_0 . This behavior could be phenomenologically relevant in order to identify such a phase transition in heavy ion collision experiments and in the bulk properties of protoneutron stars [31].

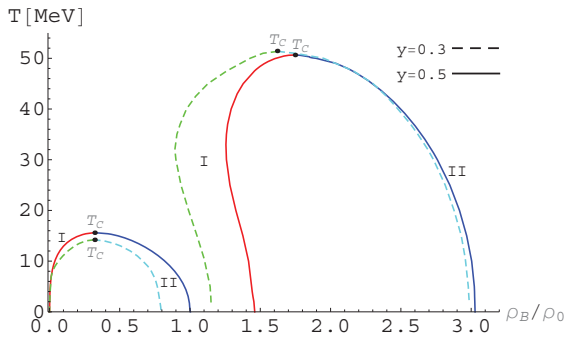


Figure 3. (Color online) Phase diagram of the liquid-gas and the nucleon- Δ matter phase transition for $y = 0.3$ (dashed curves) and $y = 0.5$ (continuous curves). The lines labeled with I and II, delimitate the first and second critical densities of the coexistence regions, respectively.

4. Mechanical and chemical instability in β -stable compact stars

In this section we are going to investigate the possible presence of thermodynamic instability in regime of β -stable hadronic matter at zero temperature. Differently by the previous section, we consider the hadronic EOS by including hyperons and electrons. Also in this case we are in presence of two conserved charge, the baryon number and the total electric charge density (fixed to $\rho_C = 0$). Such a system has the same thermodynamic stability conditions reported in the equations (12) and (15).

In figure 4 we report the pressure as a function of the baryon density for different values of the coupling $x_{\sigma\Delta}$ (and $x_{\omega\Delta} = 1$). We observe that the system does not satisfied the mechanical stability condition (12) for values of $x_{\sigma\Delta} \geq 1.25$.

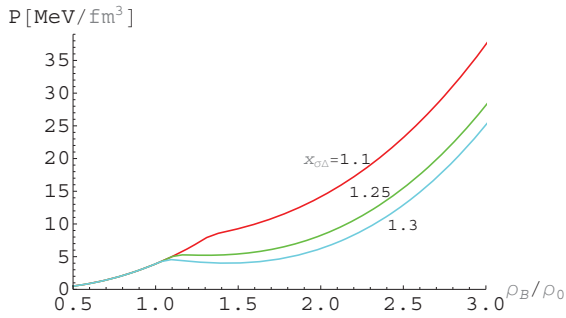


Figure 4. (Color online) Pressure as a function of baryon density for β -stable matter at zero temperature for various values of the coupling $x_{\sigma\Delta}$.

In figure 5, the electric charge chemical potential isobars μ_C are reported as a function of the net electric charge density for $x_{\sigma\Delta} = 1.25$ and at different values of pressure (the curves labeled *a* through *e* have pressure $P=3, 4, 5, 8, 10$ MeV/fm³, respectively). It is possible to see that the chemical stability condition (15) is not always satisfied for fluctuations of the net ρ_C . In this case the β -stable hadronic EOS is unstable in a range of baryon density between $0.9 \div 2.5 \rho_0$ and a mixed phase with two hadronic phases take place. It is important to outline that in this case the chemical instability condition is realized at lower pressure (and lower baryon density) with respect to the region in which the mechanical instability condition is realized. This matter of fact stresses the importance of considering both stability conditions for a binary system with two conserved charge.

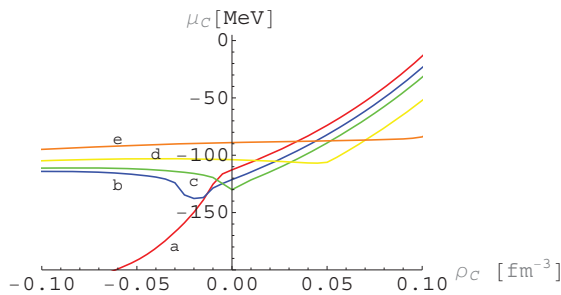


Figure 5. (Color online) Electric charge chemical potential isobars as a function of the charge density ρ_C for $x_{\sigma\Delta} = 1.25$. The curves labeled *a* through *e* have pressure $P=3, 4, 5, 8, 10$ MeV/fm³, respectively.

As in the previous section, from the analysis of the two chemical potential isobars, it is possible to construct the binodal surface relative to the hadronic mixed phase at different baryon and electric charge concentrations.

The binodal section for $x_{\sigma\Delta} = 1.25$ is reported in figure 6. In this case the system has a fixed (global) net $\rho_C = 0$. Therefore, during the isothermal compression, the system in the point *A* (at about $\rho_B \approx 0.9 \rho_0$) becomes unstable and starts to be energetically favorable the separation into two phases. An infinitesimal Δ -dominant phase at greater baryon density appears in *B*,

at the same pressure. During the phase transition, each phase evolves towards a configuration with different $y = \rho_C/\rho_B$ because, although the electric charge asymmetry is globally conserved, this is not true for the single phase and it is energetically favorable to separate it into a phase at greater density with negative ρ_C , due to the presence of Δ^- particles (left branch) and a phase at lower density with $\rho_C > 0$ (right branch) rather than into two phases with equal charge fraction. Finally, the system ends the phase transition in the point C (at about $\rho_B \approx 2.5 \rho_0$), leaving the phase with $\rho_C > 0$ in the point D .

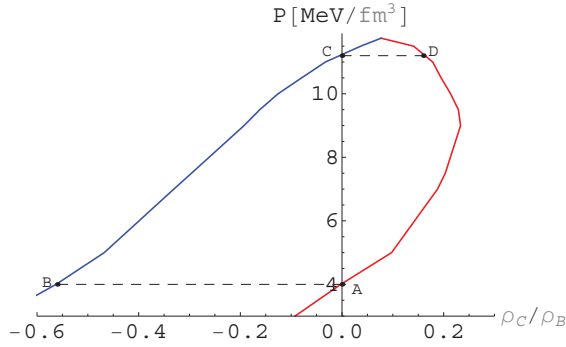


Figure 6. (Color online) Binodal section at zero temperature and $x_{\sigma\Delta} = 1.25$.

The above phase transition, strongly favors the formation of Δ^- particles at lower baryon density and implies a sensible variation in the structure of the CS. This matter of fact can be observed in figure 7, where we display the baryon density dependence of the particle's fractions. It is remarkable that the early appearance of Δ resonances, the first one being the Δ^- , considerably shifts the onset of hyperons which start to form at densities of $\sim 5 \rho_0$.

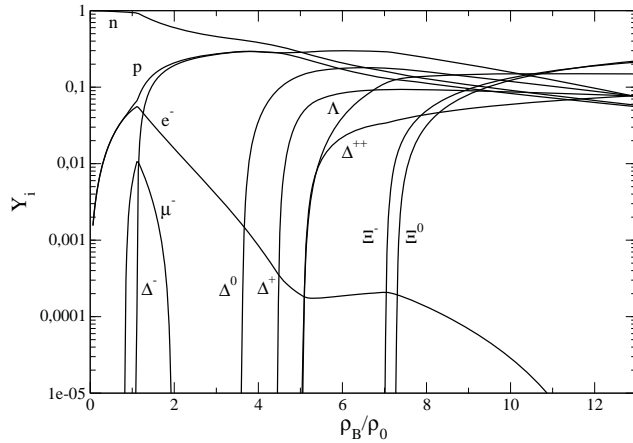


Figure 7. Particles fractions as functions of baryon density, for $x_{\sigma\Delta} = 1.25$, $x_{\omega\Delta} = 1$.

5. Can very compact and very massive neutron stars both exist?

In connection with the above results, related to the relevance of Δ -isobars in dense and β -stable nuclear matter, we are going to investigate an interpretation regarding to the possible co-existence of compact and very massive CSs in terms of two separate families of hadronic and quark stars [18].

Unfortunately the precise measurement of the radius of CSs has up to now proved to be extremely difficult, since it is in most cases based on specific assumptions concerning e.g. the atmosphere and the distance of the object under investigation. Different analysis often lead to

opposite conclusions. There have been therefore claims of very small radii, of the order or smaller than about 10 km [32], while other analysis suggest for the same objects significantly larger radii, of the order of 12 km [33]. It is clear that a precise and model independent measurement of the radius of at least a few CSs is crucial to finally provide the necessary information which will allow the extraction of the EOS of stellar matter at large densities.

From the theoretical side the families of nucleonic, hyperonic and hybrid stars, stiff enough to reach $2 M_\odot$, all provide radii which are not too small, typically larger than about 11.5-12 km for the canonical $1.4 M_\odot$ star [3, 4, 34]. This seems to put a theoretical bias against the existence of stars having very small radii. No single EOS exists at the moment which is able to provide at the same time large masses for a few CSs and small radii for others. Since the situation from the observational viewpoint is still rather open, in this contribution we are going to discuss a model which satisfies those two conditions.

It is difficult to have a unique family of CSs allowing both very small radii and very massive configurations because to have small radii the EOS needs to be rather soft. Therefore, large densities are reached in the center of very compact stars, typically of the order of $5 \div 6 \rho_0$ or larger. On the other hand, to have very massive configurations the EOS should be stiff at those same densities. No microscopic mechanism exists to allow a sudden stiffening of the EOS at those large densities. What we discuss is instead a solution based on two families of CSs, one made of hadrons and the other made of deconfined quarks stars (QSs) by assuming that the Bodmer-Witten hypothesis to hold true. While in the literature many papers exist in which two families have been discussed [35, 36], none takes into account the two constraints discussed above.

Concerning the quark matter EOS, we rely to the simple MIT bag model description in which confinement is provided by a bag constant B_{eff} and the perturbative QCD interactions are effectively included in the coefficient a_4 [37]. The total thermodynamical potential reads [5]:

$$\Omega = \sum_{u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\text{eff}}, \quad (16)$$

where μ is the quark chemical potential and Ω_i are the thermodynamical potentials for non-interacting up, down, and strange quarks and electrons. The mass of the strange quark is fixed to 100 MeV while the up and down quarks are considered as massless. As shown in Refs. [5], in this scheme it is possible to obtain stellar configurations up to two solar masses or heavier. Here we will use the following parameters sets: $B_{\text{eff}}^{1/4} = 142$ MeV - $a_4 = 0.9$ (set1), and $B_{\text{eff}}^{1/4} = 127$ MeV - $a_4 = 0.6$ (set2) both taken from [5]. Set1 allows a maximum mass for QSs of $2 M_\odot$, set2 has been implemented to give an example of quark EOS for which the maximum mass reaches $2.4 M_\odot$.

The mass-radius relations for hadronic stars and for QSs are displayed in figure 8. The maximum mass of hadronic stars, containing both Δ resonances and hyperons, is close to $1.5 M_\odot$ for the parameters' sets considered here. When excluding hyperons and Δ -resonances the maximum mass of neutron stars reaches instead a value of $\sim 2 M_\odot$ but with a large radius. The appearance of Δ resonances is crucial to obtain very compact stellar configurations (as also shown in Ref. [15]) with radii down to 8 km: the corresponding mass-radius curves enter the area, framed by the two vertical lines, of very compact objects inferred in Ref. [32]. The appearance of hyperons in the stars provides a further softening of the EOS, reducing the maximum mass of $\sim 0.1 \div 0.2 M_\odot$ (see solid/dashed curves related to hadronic stars). On the other hand, the mass of QSs can reach values compatible with the recent limit of $2 M_\odot$ (solid line) or even higher values (dashed line). Notice that QSs mass-radius relations also enter the area of very compact objects but for masses $\leq 1 M_\odot$: such light stars are difficult to produce in standard supernova simulations and moreover the lightest known neutron star has a mass of $\sim 1.2 M_\odot$.

The interpretation we propose here is that massive stars, $M \geq 1.5 \div 1.6 M_{\odot}$, are QSs with radii $R \geq 11$ km whereas stars with $R \leq 10$ km are composed mainly of nucleons and Δ resonances, with a maximum mass of $\sim 1.5 \div 1.6 M_{\odot}$. The tension between measurements of large masses and small radii could be strengthened if a neutron star more massive than $2 M_{\odot}$ is discovered (see, for example, Ref. [38]), favoring our interpretation of two coexisting families of CSs.

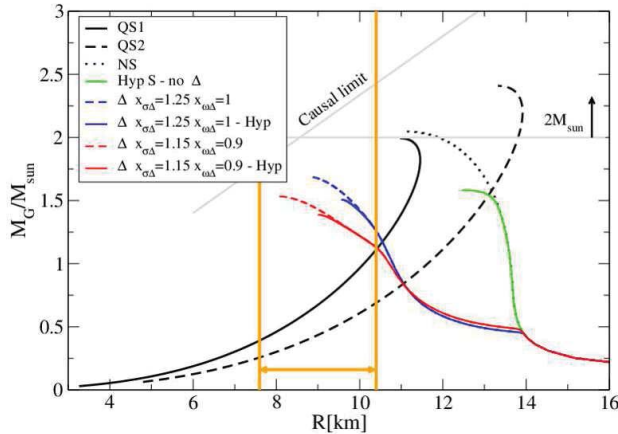


Figure 8. (Color online) Mass-radius relations of QSs (set1 and set2) and of hadronic stars. The two vertical lines correspond to the 1σ limit for the radii extracted from the analysis of quiescent low-mass X-ray binaries [32].

A crucial question concerns the astrophysical scenarios in which hadronic and QSs are formed and how QSs can generate from hadronic stars. In figure 9 we display the gravitational and baryonic masses as functions of the radius for hadronic stars and QSs. On this plot it is possible to construct a path for the formation of QSs from cold hadronic stars accreting matter from a companion. The stellar configuration labeled with B on the solid line represents the hadronic star for which hyperons start to form in the inner core (notice that at the corresponding point on the baryonic mass curves, the branch with hyperons deviates from the branch with only Δ resonances). The larger the mass of the star the larger its hyperon content. Notice that: i) only in the presence of hyperons, which carry strangeness, can droplets of strange quark matter form via nucleation; ii) the star can decay into a QS with the same baryonic mass since this process is energetically favored because the gravitational mass of the configuration D is smaller than the one of B. The energy released in the conversion of a hadronic star into a quark star has been estimated in many papers and can easily reach 10^{53} erg [35, 36, 39, 40].

All the hadronic stellar configurations between B and A can transform into QSs, the probability and velocity of conversion depending on the specific microphysics process of formation of the first droplets of quark matter and on the subsequent expansion of the newly formed phase.

There are many studies in the literature addressing these issues. In the scenario here discussed, conversion of cold hadronic stars, quantum nucleation represents a possible mechanism for the formation of the first quark matter droplet [35, 36, 39, 40]. Once a seed of quark matter is formed, the conversion of the whole hadronic star proceeds very fast, with time scales of the order of ms, due to the development of hydrodynamical instabilities [41, 42]. A detailed study of the conversion process with the new proposed EOSs is mandatory for future works.

Another scenario for the formation of quark stars is related to the supernova explosion of massive progenitors. Large densities can be reached at the moment of the collapse, soon after the bounce, due to the large fallback and hyperons can already appear at this stage, immediately triggering the formation of quark matter. There the energy released in the conversion can help Supernovae to explode [39]. In general the conversion of a hadronic star into a QS will produce

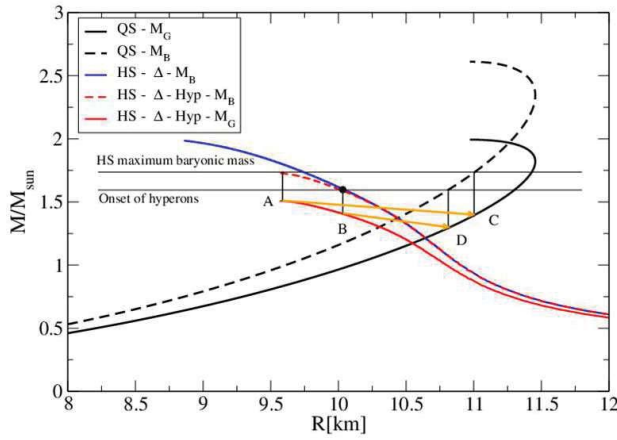


Figure 9. (Color online) Gravitational and baryonic mass-radius relations of QSs (set1) and of hadronic stars (with and without hyperons, for $x_{\sigma\Delta} = 1.25$, $x_{\omega\Delta} = 1$). A is the maximum mass of hadronic stars containing hyperons. B is the gravitational mass for which hyperons start to form. The quark stellar configurations D and C have the same baryonic masses of B and A but smaller gravitational masses.

spectacular transient events such as neutrino and gamma-ray-bursts.

There are many possible observable which could be used to test our proposal in which most of the known neutron stars (with masses close to $\sim 1.4 M_{\odot}$) are hadronic stars while massive stars are more likely QSs (bare or with a crust). We predict that massive CSs also have large radii and, being composed by a different type of matter with respect to the $1.4 M_{\odot}$ stars (in particular regarding strangeness), should show anomalous cooling histories and spinning frequency distributions. Moreover also quasi-periodic oscillations of very massive CSs should differ from the ones of hadronic stars [43].

6. Conclusions

The main goal of this work is to show the possible existence of chemical and mechanical instability for asymmetric and β -stable dense nuclear matter with possible phenomenological consequences in compressed baryonic matter regime, reachable in high energy heavy ion collisions, and in the physics of compact stars. At this scope we have studied the relativistic nuclear EOS with the inclusion of Δ -isobars and by requiring global conservation of baryon and electric charge densities. Similarly to the liquid-gas phase transition in nuclear matter, a nucleon- Δ matter phase transition can also occur at higher densities. We have shown that for asymmetric nuclear matter both mechanical and chemical instabilities take place and, during the phase transition, the two hadronic phases have a different baryon density and electric charge fraction. In the liquid-gas phase transition, the process of producing a larger neutron excess in the gas phase is referred to as isospin fractionation [44]. A similar effects can occur in the nucleon- Δ matter phase transition due essentially to a Δ^- excess in the Δ -matter phase with lower values of y . This feature could be phenomenologically relevant in heavy ion collision experiments and in the bulk properties of protoneutron stars. The presence of the Δ -isobar degrees of freedom plays a crucial role also in β -stable nuclear matter at zero temperature. Also in this case we are in presence of a binary system with two conserved charges (baryon and electric charge numbers) and the thermodynamic instabilities imply a phase transition favoring the formation of Δ^- particles at relative low baryon density ($\rho_B \approx \rho_0$) and shifting the formation of hyperon particles at large baryon densities ($\rho_B \approx 5 \div 6 \rho_0$).

Finally, we have discussed the possible co-existence of very compact and very massive compact stars in terms of two separate families: compact hadronic stars in which an early appearance of Δ resonances is crucial to guarantee the stability of the branch of hadronic stars and very

massive quark stars. In this context, it is proper to remember a well known argument against the co-existence of quark stars and neutron stars, based on the production of strangelets during the merging of two compact stars. If at least one of the two compact stars is a quark star it is possible that strangelets are emitted polluting the whole Galaxy and triggering the conversion of all compact stars into quark stars. However, recent numerical simulation of quark stars' mergers have shown that, in many cases, a prompt collapse to a black hole occurs and no matter is ejected [45]. Another possibility to prevent the strangelets pollution is offered by hydrodynamical simulations [42] in which the burning of a neutron star into a quark stars seems to be uncomplete and a possible thick layer of hadronic matter survives shielding the inner quark matter core.

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