

Overview of hydrodynamic modeling of heavy ion collision with ECHO-QGP

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Abstract. ECHO-QGP is a numerical code, aimed to reproduce the dynamics of matter produced in heavy ions collisions space following its evolution through the phase transition from QGP to HG, using viscous hydrodynamic principles. Even though ECHO-QGP is being used for nuclear physics applications, its beginnings were designed for astrophysics purposes, treating general relativistic magneto-hydrodynamics. For this reason it praises second-order treatment of causal relativistic viscosity effects in every general relativity metrics and will be extended to include evolution of the electromagnetic fields coupled to the plasma.

1. Introduction

The idea of applying ideal hydrodynamics and its to strongly interacting matter goes back to 1953, when Landau for the first time presented it in [1]. Only few years ago though, with the first RHIC data [2], this approach has been proved to predict quantitatively experimental observables [3]. The validity of hydrodynamics has been later confirmed with the LHC data [4,5], showing evidences of QGP formation, early thermalisation and presence of viscosity but with very small ratio η/s . In the low p_T region, hydrodynamics well reproduces transverse momentum spectra of the emitted hadrons in central and semicentral collisions, and the anisotropy in momentum space generated at very early stages of the collision, detectable through the elliptic flow v_2 . Nevertheless it still fails in describing other features such as two-particle correlation.

In this context, we present ECHO-QGP which is a numerical code for the description of heavy ion collisions in a (3+1)-D space and, as an outcome of our collaboration, foreseen to reproduce the above described features.

In the first section of the present work we will explain the formalism and the basics hydrodynamic equations used in ECHO-QGP and in the following we will describe the accessory theories used as setup in ECHO-QGP. In the third we explain the last stage of the simulation: the conversion of hydrodynamic fields in particles. In the last section we draw our conclusions and propose future outlooks. The present work does not include all the formal derivation of equations, the algorithmic details, and the description of the many tests ECHO-QGP has undergone, which can be found in [6–8].

2. Relativistic hydrodynamics

In order to describe the space time evolution of a fluid, we need a set of equations composed by the conservation law of the momentum-energy tensor $T^{\mu\nu}$, together with a continuity equation



for each conserved current N^μ :

$$d_\mu N^\mu = 0, \quad (1)$$

$$d_\mu T^{\mu\nu} = 0. \quad (2)$$

For a perfect fluid this set of (five) equations contains just six unknowns: charge density n , energy density e , pressure P , and the fluid four velocity u^μ . The last equation needed to close up this set is a complete equation of state (EOS).

For the case of viscous hydrodynamics, not only the energy-momentum tensor becomes dependent on the viscous fields, but we must take into account the evolution of the shear and bulk part of the viscous-stress tensor (respectively $\pi^{\mu\nu}$ and Π). The shear viscous tensor is defined as $\pi^{\mu\nu} = [\frac{1}{2}(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha) - \frac{1}{3}\Delta^{\mu\nu} \Delta_{\alpha\beta}]T^{\alpha\beta}$, and it satisfies the orthogonality ($\pi^{\mu\nu}u_\nu = 0$) and traceless ($\pi^\mu_\mu = 0$) conditions. In the most general case the expressions for the energy momentum tensor and conserved current are:

$$N^\mu = nu^\mu + V^\mu, \quad (3)$$

$$T^{\mu\nu} = eu^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + w^\mu u^\nu + w^\nu u^\mu, \quad (4)$$

where we included the isotropic pressure ($P + \Pi = \frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu}$); the energy-momentum flow orthogonal to u^μ ($w^\mu = -\Delta^\mu_\alpha T^{\alpha\beta}u_\beta$) and the particles diffusion flux ($V^\mu = \Delta^\mu_\alpha N^\alpha$).

Since for this work we left out the case of finite baryon density, it is very convenient (and commonly adopted) the Landau frame, which allows to neglect both the continuity equation and set $w^\mu = 0$.

All the conservation laws are split in the direction parallel and orthogonal to u^μ , so we can decompose the covariant derivative of the fluid velocity *irreducible tensorial parts* as

$$d_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu} - u_\mu Du_\nu + \frac{1}{3}\Delta_{\mu\nu}\theta, \quad (5)$$

where we define the (transverse, traceless, and symmetric) *shear tensor* $\sigma_{\mu\nu}$, the (transverse, traceless, and antisymmetric) *vorticity tensor* $\omega_{\mu\nu}$, and the *expansion scalar* θ . Using the definition (5) and after a bit of manipulation, the conservation laws respectively for energy and momentum, bulk and shear viscous parts of stress tensor (including terms up to second-order in the velocity gradients), can be expressed as

$$De = -(e + P + \Pi)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} \quad (6) \quad (e + P + \Pi)Du_\nu + \nabla_\nu(P + \Pi) + \Delta^\beta_\nu \nabla_\alpha \pi^\alpha_\beta + Du^\mu \pi_{\mu\nu} = 0, \quad (8)$$

$$D\Pi = -\frac{1}{\tau_\Pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta, \quad (7) \quad D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta + \mathcal{I}_1^{\mu\nu} + \mathcal{I}_2^{\mu\nu}, \quad (9)$$

where the source term $\mathcal{I}_1^{\mu\nu} = (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)Du_\lambda$ comes from the orthogonal projection, while $\mathcal{I}_2^{\mu\nu} = -\lambda(\pi^{\mu\lambda}\omega^\nu_\lambda + \pi^{\nu\lambda}\omega^\mu_\lambda)$ is the vorticity contribution term.

Once the evolution equations are explicitly displayed, we need of complementary theories to fix the values of the viscosity coefficients (η , ζ), the shear and bulk relaxation times (τ_π and τ_Π) and the other second-order transport parameter ($\lambda \equiv \lambda_2/\eta$). In our analysis we exploited the $\mathcal{N} = 4$ Super-symmetric Yang Mills theory [10] for the the parameter λ_2 which is still unknown in a non-perturbative domain in QCD. Given RHIC results [11] on elliptic flow measurements fitted with Glauber type initial conditions [12], we use a value for η/s variable within the range $0.08 \leq \eta/s \leq 0.16$, while concerning the bulk viscosity, we used the relation $\zeta = 2\eta(\frac{1}{3} - c_s^2)$, referring to the study of strongly coupled gauge theories [13]. For the shear and bulk relaxation times values τ_π and τ_Π , we follow the same choice made by other groups [14, 15]: $\tau_\pi = \tau_\Pi = \frac{3\eta}{sT}$ which are extracted from kinetic theory [16, 17] and from AdS-CFT [10, 18]. The vorticity contribution $\omega_{\mu\nu}$ in the present analysis will be mostly ignored by fixing $\lambda = 0$.

2.1. Implementation in ECHO-QGP

Our aim in this section is to write all the conservation laws in a conservative form suitable for a numerical code, and in particular for ECHO-QGP. In order to do so, we must isolate the time dependency, and reduce our set of equations in matricial form:

$$\partial_0 \mathcal{U} + \partial_k \mathcal{F}^k = \mathcal{S} \quad (10)$$

This expression must be suitable for every general relativity diagonal metrics. We can eventually write our set of 12 equations in the matricial form (of equation 10) by manipulating equations (3)-(9) and obtaining:

$$\text{conservative variables } \mathcal{U} = |g|^{\frac{1}{2}} \begin{pmatrix} N \equiv N^0 \\ S_i \equiv T_i^0 \\ E \equiv -T_0^0 \\ N\Pi \\ N\pi^{ij} \end{pmatrix}; \quad \text{fluxes } \mathcal{F}^k = |g|^{\frac{1}{2}} \begin{pmatrix} N^k \\ T_i^k \\ -T_0^k \\ N^k\Pi \\ N^k\pi^{ij} \end{pmatrix}; \quad (11)$$

$$\text{source terms } \mathcal{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2}T^{\mu\nu}\partial_i g_{\mu\nu} \\ -\frac{1}{2}T^{\mu\nu}\partial_0 g_{\mu\nu} \\ n[-\frac{1}{\tau_\pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta] \\ n[-\frac{1}{\tau_\pi}(\pi^{ij} + 2\eta\sigma^{ij}) - \frac{4}{3}\pi^{ij}\theta + \mathcal{I}_0^{ij} + \mathcal{I}_1^{ij} + \mathcal{I}_2^{ij}] \end{pmatrix} \quad (12)$$

The algorithm in ECHO-QGP, which takes care of solving this set of equations can be summarized in three stages: to retrieve primitive variables; to compute fluxes; to evolve the time step. The retrieving of the primitive variables is performed applying the orthogonality conditions for the shear viscous stress tensor, and subsequently inverting the equations (10)-(12).

The fluxes are computed by recovering the primitive variables at first, obtaining left and right states at cell interfaces, and then for each component and at each intercell, upwind fluxes are worked out (HLL two-state formula). Since the velocities are both primitive variables and necessary to extrapolate the fluxes, these two tasks are concatenated one into another, to obtain the final set of primitive variables at the current time step. The evolution equations are updated in time via a second or third order Runge-Kutta time-stepping routine.

ECHO-QGP has been widely tested, and proved agreement to analytic solutions, semi-analytic solutions and other publicly available numerical codes (See for further details the agreement between ECHO-QGP [6] and the tests in [19–26]).

2.2. Initial conditions

The parameters listed in this section, together with the shape of the initial distribution for energy density or entropy density, fluid velocities and viscous-stress tensor, are the choices we made to perform the current work, but we would like to underline that any set of parameters or functional form can easily be implemented by the user in ECHO-QGP.

The current version of the ECHO-QGP, uses both optical or Monte Carlo Glauber initial conditions.

For the first choice, ECHO-QGP provides an initial profile for the energy density distribution (or alternatively for the entropy density), which in the transverse plane can be written as

$$e(\tau_0, \mathbf{x}; b) = e_0 \left[(1 - \alpha) \frac{n_{\text{part}}(\mathbf{x}; b)}{n_{\text{part}}(\mathbf{0}; 0)} + \alpha \frac{n_{\text{coll}}(\mathbf{x}; b)}{n_{\text{coll}}(\mathbf{0}; 0)} \right], \quad (13)$$

where the soft n_{part} and the hard n_{coll} contributions are parametrized by the parameter α running from 0 to 1. In this scheme, e_0 is $e_0 = e(\tau_0, \mathbf{x} = 0; b = 0)$; and the density of participants and of binary collisions of the colliding nuclei A and B , can be written in the transverse plane as

$$n_{\text{coll}}(\mathbf{x}; b) = AB \sigma_{\text{in}}^{NN} \hat{T}_A(\mathbf{x} + \mathbf{b}/2) \hat{T}_B(\mathbf{x} - \mathbf{b}/2) \quad n_{\text{part}}(\mathbf{x}; b) \equiv n_{\text{part}}^A(\mathbf{x}; b) + n_{\text{part}}^B(\mathbf{x}; b)$$

$$n_{\text{part}}^{A(B)}(\mathbf{x}; b) = A(B) \hat{T}_{A(B)}(\mathbf{x}_{(-)}^+ \mathbf{b}/2) \left\{ 1 - [1 - \hat{T}_{B(A)}(\mathbf{x}_{(+)}^- \mathbf{b}/2) \sigma_{\text{in}}^{NN}]^{B(A)} \right\}$$

We denoted the inelastic nucleon-nucleon cross-section as σ_{in}^{NN} , and $\hat{T}_{A/B}(\mathbf{x})$ is the usual nuclear thickness function [27]. To extend the initialization to the (3+1)-D space we used a profile given by a function that factorizes the dependency along the longitudinal and transverse plane, following the indications in [28, 29]. Our choice for the energy density profile is a gaussian curve, with σ_η width, flattened in the central region $|\eta_s| \leq \Delta_\eta$. Given a beam rapidity Y_b , the factorization of the initial energy density profile reads:

$$e(\tau_0, \mathbf{x}, \eta_s; b) = \tilde{e}_0 \theta(Y_b - |\eta_s|) f^{\text{PP}}(\eta_s) F(\mathbf{x}; b) \quad (14)$$

$$F(\mathbf{x}; b) = \left[\alpha n_{\text{coll}}(\mathbf{x}; b) + (1 - \alpha) \left(\frac{Y_b - \eta_s}{Y_b} n_{\text{part}}^A(\mathbf{x}; b) + \frac{Y_b + \eta_s}{Y_b} n_{\text{part}}^B(\mathbf{x}; b) \right) \right] \quad (15)$$

$$f^{\text{PP}}(\eta_s) = \exp \left[-\theta(|\eta_s| - \Delta_\eta) \frac{(|\eta_s| - \Delta_\eta)^2}{\sigma_\eta^2} \right]. \quad (16)$$

The Glauber Monte Carlo routine instead, allows to perform event-by-event simulations. It shapes the energy density profile sampling the positions of nucleons of each nucleus from a Woods-Saxon distribution and then it reallocates them into the respective center-of-mass frame. The impact parameter is sampled in the range $b \in [0, b_{\text{max}}]$, with respect to the distribution $dP = 2\pi b db$. The criterion of collision between two nucleons, i (from nucleus A) and j (from nucleus B), is $(x_i - x_j)^2 + (y_i - y_j)^2 < \sigma_{\text{NN}}/\pi$, the event is accepted if at least one binary nucleon-nucleon collision happened. As in the optical Glauber model, we combine the soft and the hard components to the collision, but this time every participant nucleon or collision is a source of energy density:

$$e(\tau_0, \mathbf{x}) = \frac{K}{2\pi\sigma} \left\{ (1 - \alpha) \sum_{i=1}^{N_{\text{part}}} \exp \left[-\frac{(\mathbf{x} - \mathbf{x}_i^{\text{part}})^2}{2\sigma^2} \right] + \alpha \sum_{i=1}^{N_{\text{coll}}} \exp \left[-\frac{(\mathbf{x} - \mathbf{x}_i^{\text{coll}})^2}{2\sigma^2} \right] \right\}. \quad (17)$$

where the parameter σ plays the role of the variance of gaussian stretching of the source.

The dependence along the longitudinal direction can be inserted *a posteriori* as in the optical-Glauber initialization of equation (16).

At $\tau = \tau_0$ we set, as initial condition, zero transverse flow velocities and a longitudinal flow given by the Bjorken's solution ($Y = \eta_s$, Y being the fluid rapidity). Finally, note that even if $u^\mu \equiv (1, 0, 0, 0)$ in the LRF at τ_0 , in Bjorken coordinates the derivatives appearing in θ and $\sigma^{\mu\nu}$ in Ref. [6] do not all vanish, thus Π and the components of $\pi^{\mu\nu}$ must be initialized somehow. We use the first-order expressions $\Pi = -\zeta\theta$ and $\pi^{\mu\nu} = -2\sigma^{\mu\nu}$, thus $\Pi = -\zeta/\tau$, and $2\pi^{xx} = 2\pi^{yy} = -\tau^2 \pi^{\eta\eta} = \frac{4}{3}\eta/\tau$, at $\tau = \tau_0$, with all other components set to zero.

2.3. Equation of state

The code is already designed to handle any form for $P = \mathcal{P}(e)$. Even if it is not a complete EOS, it is the most convenient choice of parametrization of the EOS in case of vanishing baryon density. We refer as EOS-I to the ideal, ultrarelativistic, non-interacting QGP with 3 light flavors ($g = 37$) which has been included in ECHO-QGP in its analytical form:

$$P = e/3 = \frac{g\pi^2}{90} T^4, \quad c_s^2 = \frac{1}{3}$$

ECHO-QGP contains other, more realistic, QCD EOS's too; in particular a EOS by Laine and Schröder [30], arising from a weak-coupling QCD calculation with realistic quark masses and already used in UVH2+1 by Luzum and Romatschke [31, 32], and a second one obtained by matching a Hadron-Resonance-Gas EOS at low temperature with the continuum-extrapolated lattice-QCD results by the Budapest-Wuppertal collaboration [33, 34].

3. Decoupling

In order to compare to actual data we need to produce quantities related to the experiments, which are not naturally arising from the hydrodynamics evolution itself. The variety of techniques to generate particles from the hydro stage is wide, and has a long history. One of the currently most used technique of decoupling comes from Ref. [35, 36], which basically exploit the fact that, since the particle mean free paths strongly depend on the temperature of the medium, it can be assumed that below a certain temperature T_{freeze} (or equivalently “outside” a constant-temperature hypersurface) particles stop interacting and propagate as free streaming particles. Once determined the oriented hypersurface of constant temperature Σ , the total emission of primary particles flowing through it becomes the sum of all its points, that behave like sources. Referring to the particle species as i , the Cooper-Frye formula reads

$$E \frac{d^3 N_i}{dp^3} = \frac{d^3 N_i}{dy p_T dp_T d\phi} = \frac{g_i}{(2\pi)^3} \int_{\Sigma} \frac{-p^\mu d^3 \Sigma_\mu}{\exp\left(-\frac{u^\mu p_\mu + \mu_i}{T_{freeze}}\right) \pm 1} \quad (18)$$

From the numerical point of view, the tough task for this recipe is to correctly find the hypersurface. There are refined techniques to do so, like CORNELIUS [37] or MUSIC's [38], but as a starting point, we choose to perform the simplest way of reconstructing it. This choice translates in a “pixelized” hypersurface, meaning that it can be orthogonal to only one direction at time:

$$d^3 \Sigma_\mu = \begin{pmatrix} dV_{\perp\tau} \\ dV_{\perp x} \\ dV_{\perp y} \\ dV_{\perp\eta} \end{pmatrix} = \begin{pmatrix} \tau \Delta x \Delta y \Delta \eta_s s^\tau \\ \tau \Delta y \Delta \eta_s \Delta \tau s^x \\ \tau \Delta \eta_s \Delta \tau \Delta x s^y \\ \frac{1}{\tau} \Delta \tau \Delta x \Delta y s^\eta \end{pmatrix} \quad s^\mu \equiv -\text{sign}\left(\frac{\partial T}{\partial x^\mu}\right) \quad (19)$$

where the aim of the vector s^μ is to ensure that the hypersurface is oriented towards the colder region. We scan the whole space-time grid of the hydrodynamics evolution and select those adjacent cells (say A and B) where the temperature changes in such a way that: $(T_A - T_{freeze})(T_{freeze} - T_B) > 0$. An example of how the hypersurface comes across is displayed in figure 1. Here, even using a modestly refined technique, the hypersurface is very smooth in the case of optical Glauber initial conditions. Using this discretization, the Cooper-Frye formula (18), reduces to:

$$\frac{d^3 N_i}{p_T dp_T dy d\phi} = \frac{g_i}{(2\pi)^3} \sum_{T_{freeze}} \frac{m_T \cosh(\eta_s - y) dV^{\perp\tau} + p_T (\cos \phi dV^{\perp x} + \sin \phi dV^{\perp y}) - m_T \sinh(\eta_s - y) \tau dV^{\perp\eta_s}}{\exp[(u^\tau m_T \cosh(\eta_s - y) - p_T (u^x \cos \phi - u^y \sin \phi) + \tau u^\eta m_T \sinh(\eta_s - y) - \mu_i)/T] \pm 1} \quad (20)$$

where we explicitly wrote, in Bjorken coordinates the particle momentum and transverse mass:

$$p^\mu = (m_T \cosh(y - \eta); p_T \cos \phi, p_T \sin \phi, m_T \sinh(y - \eta)), \quad m_T = (p_T^2 + m^2)^{1/2} \quad (21)$$

Despite this algorithm is still at a much less refined stage, producing only primary spectra and using a rough reconstruction of the hypersurface, it has been tested against a well-known code such as AZHYDRO [39, 40], and proved a good agreement.

Anyway, the corrections to the primary particle spectra related to the decay of unstable particles have been shown to be significant and they must be included to reproduce the experimental data [39, 41]. Moreover, recent studies showed how the best way to reproduce data is to combine hydrodynamics and transport model [28, 37, 42–49]: in these works the Cooper-Frye prescription is not used to perform the freeze-out directly, but it is just exploited as a way of changing the physical description, leaving the hadronization task to the transport models.

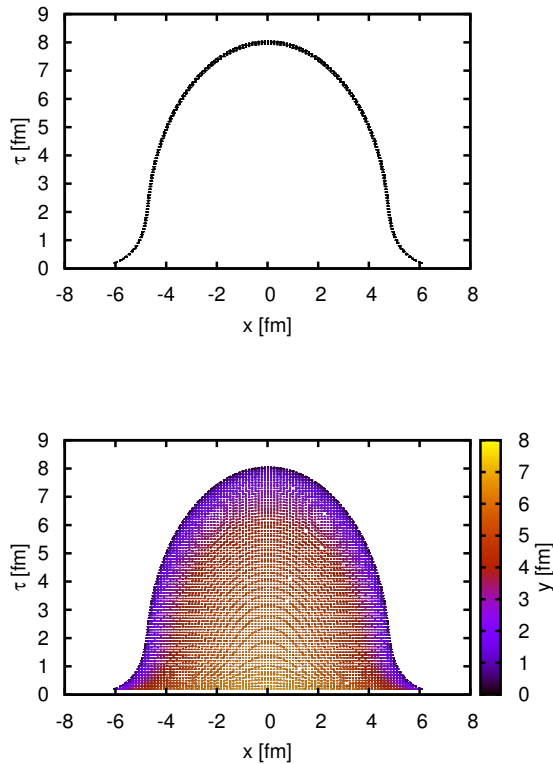


Figure 1. (Color online) Hypersurface shape in Bjorken coordinates, in the upper panel for zero value of the y coordinate, in the lower one with a map coloring for positive y coordinates.

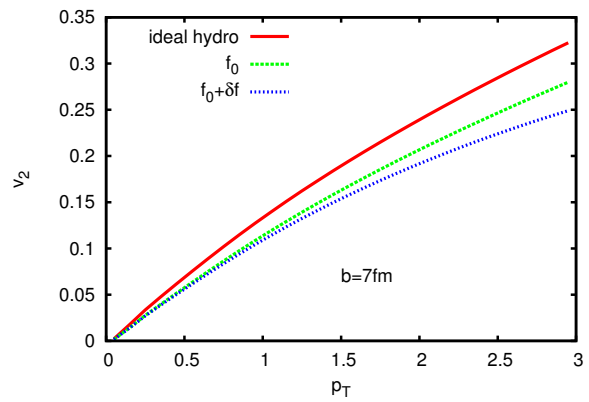


Figure 2. (Color online) Elliptic flow suppression due to the inclusion of viscosity only in the hydro stage (green line, f_0) and in the decoupling $f_0 + \delta f$). We quantitatively agree with other groups on the overall contribution of the inclusion of viscosity (about 20% at $p_T \simeq 1.5$ GeV).

In order to extend the Cooper-Frye prescription to the presence of viscosity, the particle distribution must be corrected by a small deviation $f(x, p) = f_0(x, p) + \delta f(x, p)$. For the present work we neglect in δf the contribution of the bulk viscosity, whose formalization is still controversial, but we include the shear viscosity contribution as in [50, 51], letting it be:

$$\delta f(x, p) = f_0(1 \pm f_0) \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2T^2(e + p)} \quad (22)$$

As shown in fig. (2), the inclusion of the viscous contribution nicely reproduces the elliptic flow suppression of about 20% at 1.5 GeV transverse momentum in agreement with the results in [52–56] but the issue on why some groups find the most part of the suppression due to the hydro evolution and others to the decoupling process is still to be understood.

4. Summary and outlook

Although its late birth, ECHO-QGP is answering to all the diagnostics at best, and in the next future our collaboration aim to achieve and overdue the state of the art in all its features. In the perspective of distributing the code, we must point out that at the current stage it allows many customization, including different metrics, the possibility of completely switching off the viscosity (not only set very small parameters), a quite wide range of initial condition possibilities and the inclusion of user-defined equation of state. We are currently working on the implementation of the event-by-event decoupling, including a more refined technique of detecting the hyper-surface, and the inclusion of non vanishing initial fluid four velocity profile. As a further item to address, we would like to include in ECHO-QGP the possibility of dealing with a finite-density EOS and finally, to recover the possibility of evolving also electromagnetic fields, which would characterize uniquely ECHO-QGP among over hydrodynamical codes.

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