Journal of Physics: Conference Series **527** (2014) 012015 doi:10.1088/1742-6596/527/1/012015

Relativistic distribution function for particles with spin at local thermodynamical equilibrium

E Grossi[1]

[1] Università di Firenze and INFN Sezione di Firenze, Florence Italy

E-mail: grossi@fi.infn.it

Abstract. We present an extension of relativistic single-particle distribution function for weakly interacting particles at local thermodynamical equilibrium including spin degrees of freedom, for massive spin 1/2 particles. We infer, on the basis of the global equilibrium case, that at local thermodynamical equilibrium particles acquire a net polarization proportional to the vorticity of the inverse temperature four-vector field. The obtained formula for polarization also implies that a steady gradient of temperature entails a polarization orthogonal to particle momentum. The single-particle distribution function in momentum space extends the so-called Cooper-Frye formula to particles with spin 1/2 and allows to predict their polarization, and particularly of Lambda hyperons, in relativistic heavy ion collisions at the freeze-out.

1. Introduction

The single-particle distribution function is the main quantity in kinetic theory and its form at local thermodynamical equilibrium for relativistic, weakly interacting, gases is well known. For spinless particles, it is simply the Bose-Einstein distribution with x-dependent values of temperature and chemical potential (which can be defined as Bose-Jüttner distribution):

$$f(x,p) = \frac{1}{\mathrm{e}^{\beta(x)\cdot p-\xi(x)} - 1} \tag{1}$$

where $\beta = \frac{1}{T_0}u$ is the inverse temperature four-vector,T_0 being the proper temperature measured by a comoving thermometer with the four-velocity u and $\xi = \mu_0/T_0$ is the ratio between the proper chemical potential μ_0 and T_0. The above formula has, as straightforward consequence, the invariant momentum spectrum at local thermodynamical equilibrium, the so-called Cooper-Frye formula [1]:

$$\varepsilon\frac{\mathrm{d}N}{\mathrm{d}^3\mathrm{p}} = \int_\Sigma \mathrm{d}\Sigma_\mu p^\mu f(x,p) \tag{2}$$

where Σ is a space-like 3-dimensional hypersurface. This formula is widespreadly used in e.g. relativistic heavy ion collisions to calculate hadronic spectra at the end of the hydrodynamical stage.

The distribution (1), multiplied by a degeneracy factor $(2S+1)$, is also used for particles with spin (for fermions replacing the -1 with $+1$ in the denominator) being understood that f means the *total* particle density in phase space, i.e. summed over polarization states. However, in general, particles may not evenly populate the various polarization states and one may then

Journal of Physics: Conference Series **527** (2014) 012015 doi:10.1088/1742-6596/527/1/012015

wonder what is the appropriate extension of (1) in this case. Indeed, in this work, we will answer this question and provide a generalization of (1) and (2) including the spin degrees of freedom. We will argue that a non-even population of the polarization states arises when the inverse temperature four-vector field has a non-vanishing antisymmetric part of its gradient and calculate the polarization vector for massive spin 1/2 particles. Phenomenologically, this extension may have several interesting applications. For instance, it would make possible to predict the value of particle polarization in relativistic heavy ion collisions [2, 3, 4, 5, 6, 7] at the hydrodynamical decoupling, provided that local thermodynamical equilibrium applies to spin degrees of freedom as well. The detailed analysis and calculation can be found in [8].

2. Single particle distribution function with spin

At global thermodynamical equilibrium with finite angular momentum density the density operator is well known [9, 10] and reads:

$$\widehat{\rho} = \frac{1}{Z}\exp[-\widehat{H}/T + \mu\widehat{Q}/T + \boldsymbol{\omega}\cdot\widehat{\mathbf{J}}/T]\mathsf{P}_V \tag{3}$$

where $\boldsymbol{\omega}$ is a constant fixed vector whose physical meaning is that of an angular velocity and T is the global temperature, that is the temperature of a thermostat in contact with the system or that measured by a thermometer at rest with respect to the external inertial observer. The P_V operator is the projector operator onto localized states [11] which is needed in order to avoid the relativistic singularity at $r = c/\omega$. For this distribution, it has been shown the single-particle distribution function of an ideal relativistic Boltzmann gas of particles with spin S can be obtained from just statistical mechanics arguments [11]:

$$f(x,p)_{rs} = \mathrm{e}^{\xi}\,\mathrm{e}^{-\beta\cdot p}\;\frac{1}{2}\left(D^S([p]^{-1}\mathsf{R}_{\hat{\boldsymbol{\omega}}}(i\omega/T)[p]) + D^S([p]^{\dagger}\mathsf{R}_{\hat{\boldsymbol{\omega}}}(i\omega/T)[p]^{\dagger-1})\right)_{rs} \tag{4}$$

where $\beta = \frac{1}{T}(1, \boldsymbol{\omega}\times\mathbf{x})$ is the inverse temperature four-vector; λ is the fugacity; $[p]$ is the SL(2,C) matrix corresponding to the Lorentz transformation taking the time unit vector $\hat{t}$ into $\hat{p}$ (so-called standard transformation); D^S stands for the $(S,0)$ irreducible representation of SL(2,C); R is the SL(2,C) corresponding of a rotation, which is calculated for an imaginary angle $i\omega/T$.

For $S = 1/2$ case this formula can be express using the Dirac spinors $u(p)$ and $v(p)$, thanks to the Weyl's representation, with the normalization $\bar{u}(p)_r u(p)_s = 2m\delta_{rs}$ and with $C = i\sigma_2$ ($\boldsymbol{\sigma}$ being Pauli matrices) [12, 13] read:

$$\begin{pmatrix} u_+(p) \\ u_-(p) \end{pmatrix} = \sqrt{m}\begin{pmatrix} D^S([p]) \\ D^S([p]^{\dagger-1}) \end{pmatrix} \qquad \begin{pmatrix} v_+(p) \\ v_-(p) \end{pmatrix} = \sqrt{m}\begin{pmatrix} D^S([p]C^{-1}) \\ D^S([p]^{\dagger-1}C) \end{pmatrix} \tag{5}$$

and $D^{1/2}(\mathsf{R}_{\hat{\boldsymbol{\omega}}}(i\omega/T)) = \exp[(-\omega/T)\sigma_3/2]$ one can rewrite the (4), for spin 1/2 particles in the Boltzmann limit as:

$$f_{rs}(x,p) = \mathrm{e}^{\xi}\,\mathrm{e}^{-\beta\cdot p}\;\frac{1}{2m}\bar{u}_r(p)\exp[(\omega/T)\,\Sigma_z]u_s(p)\ , \tag{6}$$

being

$$\Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}\ , \tag{7}$$

and similarly for antiparticles. The extension to Fermi-Dirac statistics of this formula is not a straightforward calculation like in the non-rotating case, hence we make an *ansatz* about this

Journal of Physics: Conference Series **527** (2014) 012015 doi:10.1088/1742-6596/527/1/012015

extension which reproduces the usual Fermi-Dirac distribution in the non-rotating case and, at the same time, has the (4) for $S = 1/2$ case as its Boltzmann limit.

$$
\begin{aligned}
f(x,p)_{rs} &= \frac{1}{2m}\bar{u}_r(p)\left(\exp[\beta\cdot p-\xi]\exp\left[-\frac{1}{2}\varpi_{\mu\nu}\Sigma^{\mu\nu}\right]+I\right)^{-1}u_s(p) \\
\bar{f}(x,p)_{rs} &= -\frac{1}{2m}\bar{v}_s(p)\left(\exp[\beta\cdot p+\xi]\exp\left[\frac{1}{2}\varpi_{\mu\nu}\Sigma^{\mu\nu}\right]+I\right)^{-1}v_r(p)
\end{aligned}
\tag{8}
$$

whereas ϖ is the covariant form for the angular velocity:

$$
\varpi_{\mu\nu} = (\omega/T)(\delta^1_\mu\delta^2_\nu - \delta^1_\nu\delta^2_\mu) = \sqrt{\beta^2}\Omega_{\mu\nu} \tag{9}
$$

and $\Omega_{\mu\nu}$ turns out to be the acceleration tensor of the Frenet-Serret tetrad of the β field lines [11]. The above second equality holds for a rigid velocity field only [11, 14].

3. Single particle distribution function at local thermodinamical equilibrium

In quantum relativistic statistical mechanics, local thermodinamical equilibrium density reads:

$$
\widehat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}}\exp\left[-\int {\rm d^3x}\;\beta_\nu(x)\widehat{T}^{0\nu}(x) - \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{\mathcal{S}}^{0,\mu\nu}(x) - \xi(x)\widehat{j}^{0}(x)\right] \tag{10}
$$

and it is obtained by maximizing entropy $S = -\,{\rm tr}[\widehat{\rho}\log\widehat{\rho}]$ with the constraints of given local values of mean energy-momentum, angular momentum and charge density [15, 16]. As entropy is not conserved in nonequilibrium situation, the above operator breaks covariance (there cannot be invariant spatial integrals of non-conserved currents) and it is time dependent. This operator is used in derivations of the relativistic Kubo formulae of transport coefficients [17] and, in comparison with usual formulations, it has an additional term involving the spin tensor, obtained in Ref. [18]. In principle, all quantities at local thermodynamical equilibrium in quantum relativistic statistical mechanics should be calculated using (10) as density operator, including the covariant Wigner function of the Dirac field. However, the full calculation is quite complicated and goes beyond the scope of this work. At the lowest order of approximation, however, we know that the single particle distribution function must yield the same formal expression at global thermodynamical equilibrium with space-time dependent intensive thermodynamics functions, that is space-time dependent β, ϖ and ξ. Hence, the single-particle distribution functions (4) for $S = 1/2$ case become:

$$
\begin{aligned}
f(x,p)_{rs} &= \frac{1}{2m}\bar{u}_r(p)\left(\exp[\beta(x)\cdot p-\xi(x)]\exp\left[-\frac{1}{2}\varpi(x)_{\mu\nu}\Sigma^{\mu\nu}\right]+I\right)^{-1}u_s(p) \\
\bar{f}(x,p)_{rs} &= -\frac{1}{2m}\bar{v}_s(p)\left(\exp[\beta(x)\cdot p+\xi(x)]\exp\left[\frac{1}{2}\varpi(x)_{\mu\nu}\Sigma^{\mu\nu}\right]+I\right)^{-1}v_r(p)
\end{aligned}
\tag{11}
$$

where β is the inverse temperature four-vector, ξ is the ratio between comoving chemical potential and temperature, $\Sigma^{\mu\nu} = (i/4)[\gamma^\mu,\gamma^\nu]$ are the generators of Lorentz transformations of 4-components spinors, $u(p)$ and $v(p)$ are the spinors solutions of the free Dirac equation.

Using this distribution functions we obtain the free Dirac part of the canonical stress-energy tensor as:

$$
T^{\mu\nu}(x) = \int \frac{{\rm d^3p}}{\varepsilon}\; p^\mu p^\nu \sum_r (f_{rr} + \bar{f}_{rr}) \tag{12}
$$

and the current:

$$
j^\mu(x) = \frac{1}{2}\int \frac{{\rm d^3p}}{\varepsilon}\; p^\mu \sum_r \left[{\rm tr}\, f_{rr} - {\rm tr}\,\bar{f}_{rr}\right] \tag{13}
$$

both involving the traces of f_{rs}, $\bar{f}_{rs}$. If ϖ, which is adimensional in natural units, is small enough such that one can write the expansion:

$$\left(\mathrm{e}^{\beta(x)\cdot p\mp\xi(x)}\exp\left[\mp\frac{1}{2}\varpi(x):\Sigma\right]+I\right)^{-1}=\sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\beta(x)\cdot p\pm n\xi(x)}\exp\left[\pm\frac{n}{2}\varpi(x):\Sigma\right] \quad (14)$$

where : is a shorthand for the 2 tensor contraction, i.e. $\varpi(x):\Sigma=\varpi^{\mu\nu}(x)\Sigma_{\mu\nu}$, then it is possible to obtain an approximate expression of those traces for $\varpi_{\mu\nu}\ll 1$. Note that at full rotational equilibrium this condition amounts to require $\hbar\omega/KT\ll 1$ (natural units purposely restored) which is a normally fulfilled condition. Hence:

$$\begin{aligned}\mathrm{tr}\left(\exp\left[\pm\frac{n}{2}\varpi(x):\Sigma\right]\right) &\simeq \mathrm{tr}\left(I\pm\frac{n}{2}\varpi(x):\Sigma+\frac{n^2}{4}\varpi(x):\Sigma\,\varpi(x):\Sigma\right)\\ &= 4+\frac{n^2}{4}\varpi(x)^{\lambda\rho}\varpi(x)^{\sigma\tau}\,\mathrm{tr}\left(\Sigma_{\lambda\rho}\Sigma_{\sigma\tau}\right) \quad (15)\end{aligned}$$

where the tracelessness of Σ matrices has been used. By using known formulae for the traces of γ matrices it can be shown that:

$$\mathrm{tr}\left(\Sigma_{\lambda\rho}\Sigma_{\sigma\tau}\right)=g_{\lambda\sigma}g_{\rho\tau}-g_{\lambda\tau}g_{\rho\sigma}$$

whence the equation (15) becomes:

$$\mathrm{tr}\left(\exp\left[\pm\frac{n}{2}\varpi(x):\Sigma\right]\right)\simeq 4+\frac{n^2}{2}\varpi(x):\varpi(x)\;. \quad (16)$$

Therefore, we have:

$$\begin{aligned}\sum_r f_{rr} &\simeq \sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\beta(x)\cdot p+n\xi(x)}\left(4+\frac{n^2}{2}\varpi(x):\varpi(x)\right)\\ &= 4n_F+\frac{1}{2}n_F(1-n_F)(1-2n_F)\varpi(x):\varpi(x) \quad (17)\end{aligned}$$

where:

$$n_F=\frac{1}{\mathrm{e}^{\beta(x)\cdot p-\xi(x)}+1}$$

and similarly for $\sum_r \bar{f}_{rr}$ with the replacement $\xi\to-\xi$. One can then plug the equation (17) and its corresponding for $\sum_r \bar{f}_{rr}$ into equations (12) and (13) to obtain, e.g., for the charge density:

$$j^0(x)=2\int \mathrm{d}^3\mathrm{p}(n_F-\bar{n}_F)+\varpi(x):\varpi(x)\frac{1}{4}\int \mathrm{d}^3\mathrm{p}[n_F(1-n_F)(1-2n_F)-\bar{n}_F(1-\bar{n}_F)(1-2\bar{n}_F)]\;.$$

For $\varpi=0$ one recovers the usual expression; it is worth noting that the lowest order correction to charge density and stress-energy tensor is quadratic in ϖ, i.e. in $\hbar\omega/KT$ at equilibrium, then it's a small contribution.

While the general physical meaning of the fields β and ξ can be easily inferred from the equilibrium limit (β is the local inverse temperature four-vector field and ξ the ratio between the comoving chemical potential $\mu_0(x)$ and the local comoving temperature $T_0(x)=1/\sqrt{\beta^2}$), $\varpi_{\mu\nu}(x)$'s expression cannot be uniquely obtained from the equilibrium distribution. The reason of this ambiguity is that at rotational equilibrium, the tensor ϖ is:

$$\varpi_{\mu\nu}=\sqrt{\beta^2}\Omega_{\mu\nu} \quad (18)$$

where $\Omega_{\mu\nu}$ is the acceleration tensor of the Frenet-Serret tetrad of the β field lines [11] and [14]:

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \tag{19}$$

that is the relativistic generalization of angular velocity of fluid. In a nonequilibrium situation, the right hand sides of equations (19) and (18) differ and it is not obvious which one applies, perhaps neither. However, if the system is not too far from equilibrium, ϖ cannot be too distant from the right hand side of (19). Particularly, the difference must be of the 2nd order in the gradients of the β field (for instance: $(\partial \cdot \beta)(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$ or second order gradients like $\partial_\mu \partial_\nu \beta^\lambda$) which vanish at equilibrium [14]. For the lowest-order formulation of relativistic hydrodynamics, the expression (19) is sufficient to determine the expression of spin-related quantities.

4. Polarization

The main consequence of the distribution functions (11) is that spin 1/2 particles get polarized at local thermodynamical equilibrium. The polarization four-vector, for a particle with mass m and four-momentum p is defined as:

$$\Pi_\mu = -\frac{1}{2}\epsilon_{\mu\rho\sigma\tau} S^{\rho\sigma} \frac{p^\tau}{m} \tag{20}$$

where $S^{\rho\sigma}$ is the mean value of the *total* angular momentum operator of the single particle. However the Levi-Civita tensor makes the orbital part of the total angular momentum irrelevant, so we are left with the spin-tensor density in the phase space $\mathcal{S}^{0,\rho\tau}(x,p)$ contribution only:

$$\langle \Pi_\mu(x,p) \rangle = -\frac{1}{2}\frac{1}{\sum_r f_{rr}} \epsilon_{\mu\rho\sigma\tau} \frac{\mathrm{d}\mathcal{S}^{0,\rho\sigma}(x,p)}{\mathrm{d}^3 p} \frac{p^\tau}{m} \ . \tag{21}$$

The *canonical* spin tensor can be computed using (11):

$$\mathcal{S}^{\lambda,\mu\nu}(x) = \frac{1}{2}\int \frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon} \sum_{rs} \Big(f_{rs}(x,p)\bar{u}_s(p)\{\gamma^\lambda, \Sigma^{\mu\nu}\} u_r(p)\Big) - \Big(\bar{f}_{rs}(x,p)\bar{v}_r(p)\{\gamma^\lambda, \Sigma^{\mu\nu}\} v_s(p)\Big) \ . \tag{22}$$

We can write the spin tensor as:

$$\mathcal{S}^{\lambda,\mu\nu} = \frac{1}{2}\int \frac{\mathrm{d}^3\mathrm{p}}{\varepsilon} \ \Big(p^\lambda \Theta^{\mu\nu} + p^\nu \Theta^{\lambda\mu} + p^\mu \Theta^{\nu\lambda} + p^\lambda \bar{\Theta}^{\mu\nu} + p^\nu \bar{\Theta}^{\lambda\mu} + p^\mu \bar{\Theta}^{\nu\lambda} \Big) \tag{23}$$

where

$$\Theta^{\mu\nu} \equiv \sum_r \left(f_{rs}\bar{u}_s(p)\Sigma^{\mu\nu} u_r(p) \right) \qquad \bar{\Theta}^{\mu\nu} \equiv -\,\mathrm{tr}\left(\bar{f}_{rs}\bar{v}_r(p)\Sigma^{\mu\nu} u_s(p) \right) \ .$$

Altogether, the form (23) of the canonical spin tensor only depends on the fact that the distribution function $f_{rs}\bar{f}_{rs}$ are a superposition of an even number of γ matrices. The full antisymmetry of the indices is now apparent, although it was already contained in the operator definition ensuing from the properties of γ matrices. Thanks to trace cyclicity, Θ can be written as a derivative with respect to the ϖ tensor:

$$\begin{aligned}
\Theta^{\mu\nu} &= \sum_{n=1}^{\infty} (-1)^{n+1} \mathrm{e}^{-n\beta(x)\cdot p + n\xi(x)} \,\mathrm{tr}\left(\exp\left[\frac{n}{2}\varpi(x) : \Sigma\right] \Sigma^{\mu\nu} \right) \\
&= \sum_{n=1}^{\infty} (-1)^{n+1} \mathrm{e}^{-n\beta(x)\cdot p + n\xi(x)} \frac{1}{n}\frac{\partial}{\partial \varpi_{\mu\nu}} \mathrm{tr}\left(\exp\left[\frac{n}{2}\varpi(x) : \Sigma\right] \right) \\
&= \frac{\partial}{\partial \varpi_{\mu\nu}} \mathrm{tr}\left(\log\left\{ I + \mathrm{e}^{-\beta(x)\cdot p + \xi(x)} \exp\left[\frac{1}{2}\varpi(x) : \Sigma\right] \right\} \right)
\end{aligned} \tag{24}$$

Journal of Physics: Conference Series **527** (2014) 012015 doi:10.1088/1742-6596/527/1/012015

where the (14) has been used. Then, using the approximation (16), the equation (24) becomes:

$$\begin{aligned}\Theta^{\mu\nu} &\simeq \sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\beta(x)\cdot p+n\xi(x)}\frac{1}{n}\frac{\partial}{\partial\varpi_{\mu\nu}}\left(4+\frac{n^2}{2}\varpi(x):\varpi(x)\right)\\ &= \sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\beta(x)\cdot p+n\xi(x)}n\varpi(x)^{\mu\nu} = n_F(1-n_F)\varpi(x)^{\mu\nu}\ . \end{aligned} \tag{25}$$

Likewise, it can be shown that:

$$\bar{\Theta}^{\mu\nu} \simeq \bar{n}_F(1-\bar{n}_F)\varpi(x)^{\mu\nu}\ . \tag{26}$$

Not far from equilibrium, for small value of $\varpi^{\mu\nu}$, the polarization vector becomes:

$$\langle\Pi_\mu(x,p)\rangle = -\frac{1}{4\sum_r f_{rr}}\epsilon_{\mu\rho\sigma\tau}\frac{1}{\varepsilon}\left(p^0\Theta^{\rho\sigma}+p^\sigma\Theta^{0\rho}+p^\rho\Theta^{\sigma 0}\right)\frac{p^\tau}{m} = -\frac{1}{2\sum_r f_{rr}}\epsilon_{\mu\rho\sigma\tau}\ \Theta^{\rho\sigma}\frac{p^\tau}{m} \tag{27}$$

and at the lowest orther in ϖ becomes:

$$\langle\Pi_\mu(x,p)\rangle \simeq \frac{1}{16}\epsilon_{\mu\rho\sigma\tau}\ (1-n_F)\left(\partial^\rho\beta^\sigma-\partial^\sigma\beta^\rho\right)\frac{p^\tau}{m} = \frac{1}{8}\epsilon_{\mu\rho\sigma\tau}\ (1-n_F)\partial^\rho\beta^\sigma\frac{p^\tau}{m} \tag{28}$$

where $n_F = 1/(\exp(\beta(x)\cdot p-\xi(x))+1)$ is the Fermi-Dirac distribution function.

The above formula has the remarkable consequence that quasi-free particles get polarized not only in a vorticous flow (what was pointed out in previous works [19]), but also in a steady temperature gradient without velocity flow, i.e. when $\nabla\beta^0 \neq 0$.In the non-relativistic the predicted polarization reads:

$$\Pi = (\Pi^0,\mathbf{\Pi}) = (1-n_F)\frac{\hbar p}{8mKT^2}(0,\nabla T\times\hat{\mathbf{p}})$$

which is usually tiny but could be relevant in some extreme situations.

It may be of interest, e.g. for relativistic heavy ion collisions, to calculate the space-integrated mean polarization vector. For a three-dimensional spacelike hypersurface Σ, one has:

$$\langle\Pi_\mu(p)\rangle \equiv \frac{\displaystyle\int \mathrm{d}\Sigma_\lambda\ \frac{p^\lambda}{\varepsilon}(-1/2)\epsilon_{\mu\rho\sigma\tau}\frac{\mathrm{d}\mathcal{S}^{0,\rho\sigma}}{\mathrm{d}^3\mathrm{p}}\frac{p^\tau}{m}}{\displaystyle\int \mathrm{d}\Sigma_\lambda\ \frac{p^\lambda}{\varepsilon}\sum_r f_{rr}(x,p)} \simeq \frac{1}{8}\epsilon_{\mu\rho\sigma\tau}\frac{p^\tau}{m}\frac{\displaystyle\int \mathrm{d}\Sigma_\lambda\ p^\lambda\ n_F(1-n_F)\partial^\rho\beta^\sigma}{\displaystyle\int \mathrm{d}\Sigma_\lambda\ p^\lambda n_F}\ . \tag{29}$$

For antiparticles, one gets the same formula, with $\bar{n}_F = 1/(\exp(\beta(x)\cdot p+\xi(x))+1)$ replacing n_F. Here an important comment is in order: the fact that local thermodynamical equilibrium implies the same orientation for the polarization vector of particles and antiparticles (unlike e.g. in the electromagnetic field) is a general outcome and does not depend on the introduced approximations. It stems from the fact that the spin tensor, as well as the angular momentum, is a charge-conjugation even operator, or, more simply stated, that thermal and mechanical effects do not "see" the internal charge of the particles.

This formula (29) may be used to predict the polarization of particles, in particular Λ hyperons, produced at the freeze-out (primary particles) in a relativistic heavy-ion collision, after the hydrodynamical evolution.

In a heavy ion collision the polarization could exist also in plasma phase if it were described as weakly interacting particles, since is an effect of the local equilibrium. Nevertheless it's not

Journal of Physics: Conference Series **527** (2014) 012015 doi:10.1088/1742-6596/527/1/012015

the case for QGP and it's unknown the effect of strong coupling on the polarization. However at the freeze-out it's assumed, to compute the spectra that the fluid instantaneously produce free streaming hadrons distributed with a free particle-distribution function, then we can think that also happened with the spin's degrees of freedom. In a more realistic model of the particle production the interaction between hadrons turn of slowly and this polarization effect could be modified.

The polarization of the primary particles emitted is proportional to the thermal vorticity (19) at freeze-out hypersurface and could give an observable contribution [20].

The observation of this effect would be a confirmation of the achievement of the local thermodynamical equilibrium also to spin degrees of freedom and would also indicate that vorticity and circulation may persist in the hydrodinamical evolution up to freeze-out.

References

[1] Cooper F and Frye G 1974 *Phys. Rev. D* **10** 186
[2] Baznat M, Gudima K, Sorin A, and Teryaev O 2013 *Phys. Rev. C* **88** 061901
[3] Huang X G, Huovinen P, and Wang X N 2011 *Phys. Rev. C* **84** 054910
[4] Gao J H, Chen S W, Deng W T, Liang Z T, Wang Q, and Wang X N 2008 *Phys. Rev. C* **77** 044902
[5] Gao J H, Liang Z T, Pu S, Wang Q, and Wang X N 2012 *Phys. Rev. Lett.* **109** 232301
[6] Betz B, Gyulassy M, and Torrieri G 2007 *Phys. Rev. C* **76** 044901
[7] Csernai L, Magas V K, and Wang D J 2013 *Phys. Rev. C* **87** 034906
[8] Becattini F, Chandra V, Del Zanna L, and Grossi E 2013 *Annals Phys.* **338** 32
[9] Landau L and Lifshitz L 1980 *Statistical Physics, Part 1* (Oxford, England: Pergamon Press)
[10] Vilenkin A 1980 *Phys. Rev. D* **21** 2260
[11] Becattini F and Tinti L 2010 *Annals Phys.* **325** 1566
[12] Moussa P and Stora R 1966 *Angular Analysis of Elementary Particle Reactions*, in Proceedings of the 1966 International School on Elementary Particles, Hercegnovi (Gordon and Breach, New York, London)
[13] Becattini F and Ferroni L 2007 *Eur. Phys. J. C* **52** 597
[14] Becattini F 2012 *Phys. Rev. Lett.* **108** 244502
[15] Zubarev D N 1971 *Non Equilibrium Statistical Thermodynamics* (Nauk, Moscow english translation: New York, Consultant Bureau, 1974)
[16] Zubarev D N , Morozov V G, and Röpke G 1997 *Statistical Mechanics of Nonequilibrium Processes. Volume 2: Relaxation and Hydrodynamic Processes* (Akademie Verlag, Berlin)
[17] Hosoya A, Sakagami M, and Takao M 1984 *Annals Phys.* **154**, 229
[18] Becattini F and Tinti L 2011 *Phys. Rev. D* **84** 025013
[19] Becattini F and Piccinini F 2008 *Annals Phys.* **323** 2452
[20] Becattini F, Csernai L, and Wang D J 2013 *Phys. Rev. C* **88** 034905