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# POD Analysis of a Wind Turbine Wake in a Turbulent Atmospheric Boundary Layer

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**Abstract.** The wake of a single wind turbine is modeled using an actuator disk model and large eddy simulations. As inflow condition a numerically generated turbulent atmospheric boundary layer is used.
The proper orthogonal decomposition (POD) is applied to a plane perpendicular to the main flow in the far wake of the turbine. Reconstructions of the field are investigated depending on the numbers of POD modes used. Even though a great number of modes is needed to recover a great part of the turbulent kinetic energy, our results indicate that relevant aspects of a wake flow can be recovered using only a few modes. Particularly, the dynamics of the average velocity over a potential disk in the wake can partially be captured using only three modes.

## 1. Introduction

It is a well known fact, that the wakes of wind turbines strongly impact the power output in wind farms and the loads acting on the turbines. Advanced understanding and modeling of wakes is therefore a crucial step for the understanding of wind farm dynamics and layout optimization.

To describe the mean field of the wake, different analytical mean flow models have been developed using different underlying approximations [1, 2, 3]. These models however, exclude the dynamical aspects of the wake which are commonly expected to be relevant for the loads and power production of a turbine in a wake flow.
A first step to include dynamical behavior is to take the so called meandering of the velocity deficit into account. This is commonly done through assuming that the meandering is mainly caused by the large scale dynamics of the atmospheric boundary layer (ABL) [4, 5, 6]. These approaches however, do not take into account the temporal dynamics of the deficit structure which can e.g. lead to very high loads. Furthermore, it is not fully clear whether the meandering can always be treated as a passive tracer driven by the large scale dynamics.

A more detailed approach can be done using large eddy simulations (LES) with actuator disk or actuator line models to represent the turbines [7, 8, 9]. Despite their simplifications, these simulations still suffer from rather long computational times which strongly limits their efficiency for practical purposes such as wind farm optimization.
It is therefore essential, to develop computationally efficient wake models including the dynamical behavior.

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One approach to simplification, sometimes used in fluid dynamics, is to use the proper orthogonal decomposition to develop reduced order models for specific flow problems [10, 11, 12]. Andersen et al. [13] applied the POD to the flow in an infinitely long row of turbines using data from LES simulations with actuator line models. They could extract well-defined partially symmetric POD modes as a first step of building a reduced order model of wakes in a long row of turbines. Furthermore, their results indicate that the low frequency dynamics are not only related to the large scale movement of the ABL, but are an inseparable part of the wake dynamics.

In this work, we apply the POD to LES data of a single wake of a wind turbine modeled by an actuator disk. Unlike Andersen et al. [13], we analyze a single wake situation and include a more realistic inflow via modeling a turbulent ABL in the case of a neutrally stable atmosphere. This raises our first question, whether meaningful POD modes can be obtained in this specific setting. Furthermore, we investigate whether only a few modes can already capture relevant aspects of the wake dynamics. For this purpose, we examine the average velocity over a disk in the wake flow as a quality measure for our low order descriptions of the wake. This effective velocity can be used as an input to estimating the power output of a sequential turbine in the wake and also give first indications on loads such as the thrust.

## 2. LES DATA

As described in the introduction, we investigate a wake of a single turbine in a turbulent ABL (Fig. 1). Large eddy simulations have been performed using the parallelized LES model PALM [14]. The generated data has a spatial resolution of 4 m and is sampled (written out) at a rate of 2 Hz with a time series length of 11750 s excluding the first 500 s of the simulation. The wind turbine is modeled by an actuator disk approach following [7] with a rotor diameter of $D = 104$ m, a hub height of 160 m and a thrust coefficient of $C_T = 0.75$.
The full domain used for the simulations is 8188 m x 2044 m x 512 m, so that the wake is almost by the boundary conditions at the top and in $y-$ direction.

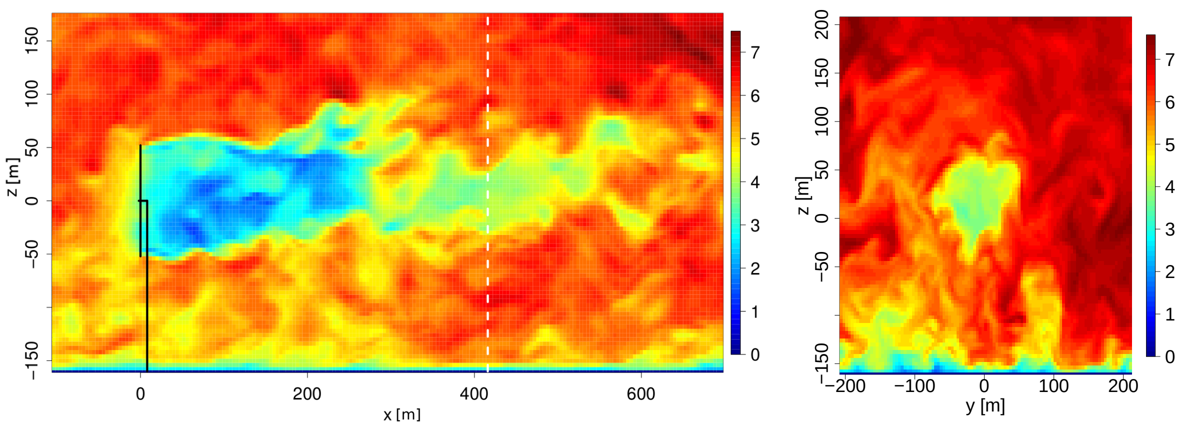


**Figure 1.** Snapshots of the analyzed LES data. left: x-z plane ($y = 0$ m), right: y-z plane ($x = 4$D). The color denotes the velocity component $u$ in the main wind direction in $\frac{m}{s}$.

A stationary and fully turbulent ABL is generated in a pre-run with smaller domain and a simulation time of 12 h. The main run with the full domain ( is initialized with the final results of the pre-run. Furthermore, a turbulence recycling is applied [15, 16].

For our analysis, we focus on data in a plane perpendicular to the main flow direction (white dashed line in Fig. 1) 4 D downstream of the turbine. In this distance we expect only minor differences between the actuator disk model used here and more sophisticated actuator line models [16]. Furthermore, we confine our analysis on the main velocity component yielding a the spatio-temporal data field u(y,z,t). A snapshot for $t = 50$s can be seen in Fig. 1 (left).

## 3. POD Theory

The main idea of the POD is to use spatial correlations in the flow to obtain a low-dimensional description of the data. The flow field is decomposed into a superposition of different spatial modes which have been obtained through a diagonalization procedure of the covariance operator of the field. In the following, we briefly summarize the mathematical definition and important properties of the POD. Details can be found e.g. in [10].

For a scalar (mean free) field $u(r,t)$ (in our case $r$ is a 2d vector $(y,z)$) the POD is given by:

$$u(r,t) = \sum_j a_j(t)\phi_j(r) \text{ with } a_j(t) = (\phi_j|u) := \int dr \phi_j^*(r) u(r,t) \ , \tag{1}$$

where $\phi_j$ are the orthogonal eigenfunctions (POD modes) of the covariance operator :

$$\int dr' \langle u(r,t)u^*(r',t)\rangle_t \phi_j(r') = \lambda_j \phi_j(r) \text{ with } \lambda_1 > \lambda_2 > ... \tag{2}$$

Note that in practice Eq.(2) is usually approximated by the eigenvalue problem of the discretized covariance matrix $C_{ij} = \langle u_i(t) u_j^*(t)\rangle_t$, where $u_k$ is the value of $u$ at the kth grid point.

We can now approximate the original field $u(r,t)$ by using only the first N Modes:

$$\hat{u}^{(N)}(r,t) := \sum_{j=1}^{N} a_j(t)\phi_j(r) \ , a_j(t) = (\phi_j|u) \tag{3}$$

Due to Parseval's relation

$$\langle \|u(r,t)\|_2^2\rangle_t := \langle \int dr |u(r,t)|^2\rangle_t = \sum_{j=1}^{\infty} \langle |a_j|^2\rangle_t \text{ and } \langle |a_j(t)|^2\rangle_t = \lambda_j \ , \tag{4}$$

the j$^{\text{th}}$ eigenvalue can be interpreted as a measure for the kinetic energy contained in the j$^{\text{th}}$ mode yielding the reconstruction error:

$$\langle \|u - \hat{u}^{(N)}\|_2^2\rangle = \sum_{j=N+1}^{\infty} \lambda_j \ . \tag{5}$$

Furthermore,the ratio of kinetic energy recovered by the reconstruction is:

$$R_{kin} := \frac{\sum_{j=1}^{N} \lambda_j}{\sum_{j=1}^{\infty} \lambda_j} \ . \tag{6}$$

Another important property of the POD is that it is the optimal orthogonal decomposition with respect to the kinetic energy. This means for $N$ functions of another orthogonal basis the reconstruction error in Eq.(5) is always greater than for the first $N$ POD modes.

In the following $u(r,t)$ has a nonzero mean field which is removed before the POD and added after a possible reconstruction. We also write $u^{(0)} := \langle u(r,t)\rangle_t$ for steady mean field of $u(r,t)$.

For many applications the so called method of snapshots (see e.g. [10] ) is used instead of directly solving Eq. (2). Here, we solve the direct problem, since the number of time steps used for the analysis is of the same order as the number of grid points. Therefore, the method of snapshots is not more efficient.

## 4. POD Analysis

### *4.1. Preprocessing*

Since we aim to describe the wake dynamics in an ABL separately from the ABL itself, we apply two preprocessing steps which are described in the following.

As a first step, we subtract the mean field of the flow without turbine (Fig. 2a) from $u(y,z,t)$. This lead to a slightly better separation from the ABL structures (Fig. 2c). Additionally, we change the sign of $u$ which is of minor importance here.
Second, we extract the deficit simply by using a (temporally local) relative threshold. This means that we set all values weaker than 40% of the current deficit maximum to zero (Fig. 2d). The results presented here are relatively robust against the choice of the threshold with similar results choosing $10\% - 50\%$. Also it is important to point out that even though we choose a relatively strong threshold, we are able to reconstruct aspects of the full field and not only the preprocessed one (see Sec. 4.3). However, the extracted POD modes do change with a threshold chosen, but a deep investigation and explanation of this effect is beyond the scope of this paper.

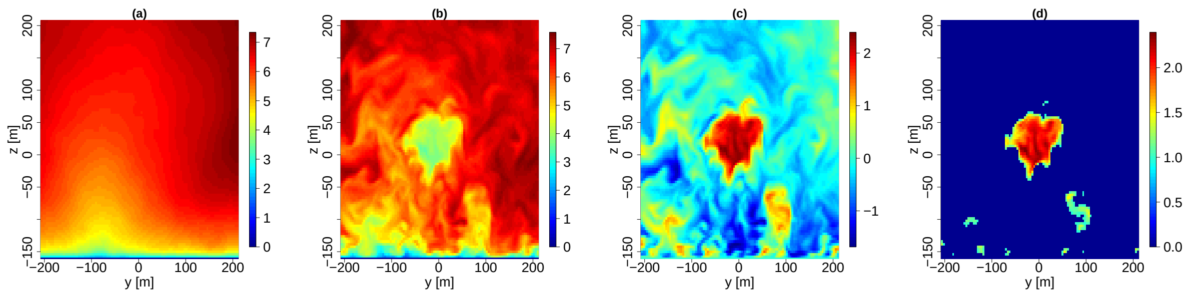


**Figure 2.** Preprocessing: Mean field without turbine (a), Wake $u(y,z)$ at distance $x = 4D$ and $t = 50\ s$ (b), After subtracting mean field without turbine (c), After applying threshold (d)

### *4.2. POD Modes*

Next, we apply the POD to the preprocessed field and describe the obtained POD modes and corresponding eigenvalues.

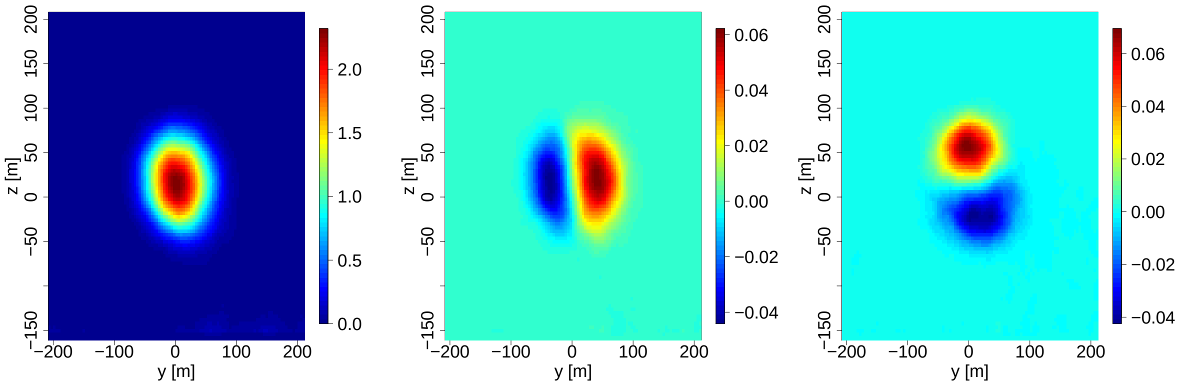


**Figure 3.** Left: mean, Middle: POD mode 1, Right: POD mode 2

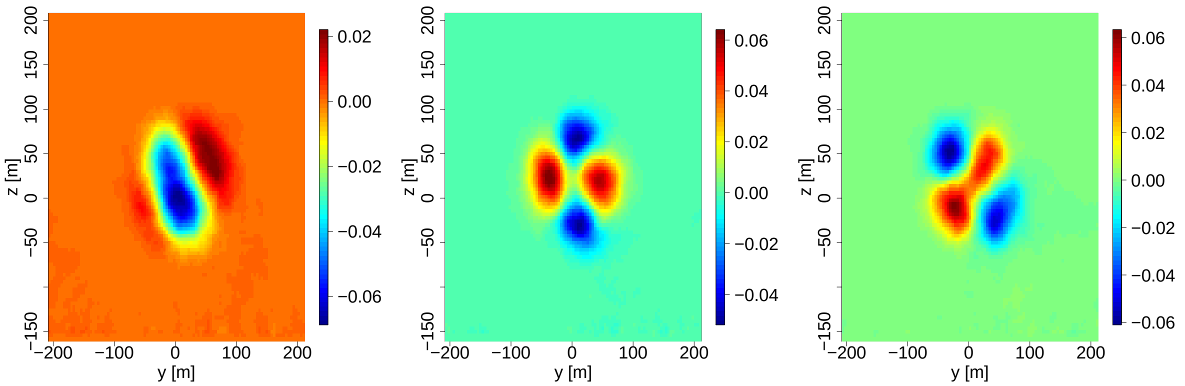


**Figure 4.** POD modes 3-5

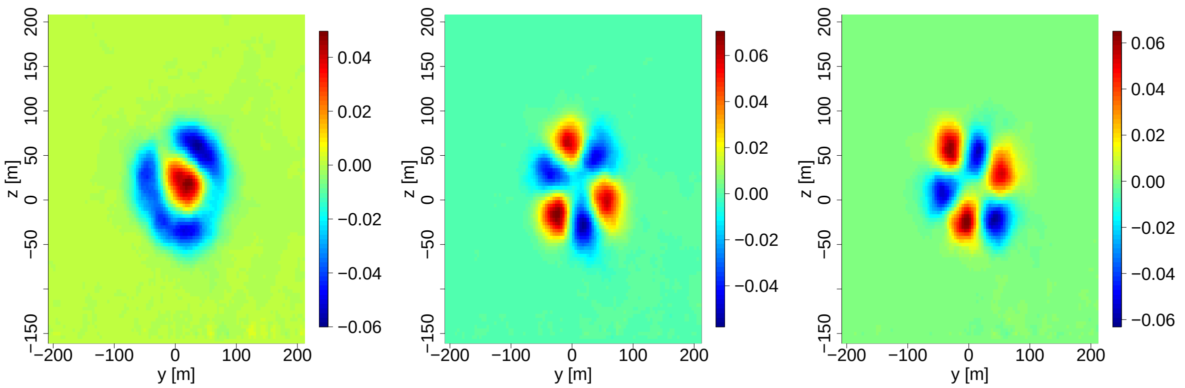


**Figure 5.** POD modes 6-8

We see that well-defined modes are found (Fig. 3 - 5) which are remarkably similar to the ones obtained by Andersen et al. [13]. The dipole-like (1,2) and quadrupole-like (4,5) mode pairs occurring in both works indicate a statistical invariance of the field under rotations [10]. Thus,

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even though the ABL breaks this rotational symmetry the wake dynamics seem to partially retain some of its symmetric behavior.
There is an overall trend from larger to smaller structures with the mode number with some exceptions in the first 10 Modes. Thus, as typical in turbulent flows we have a slow decay of energy with decreasing scale (Fig. 6).

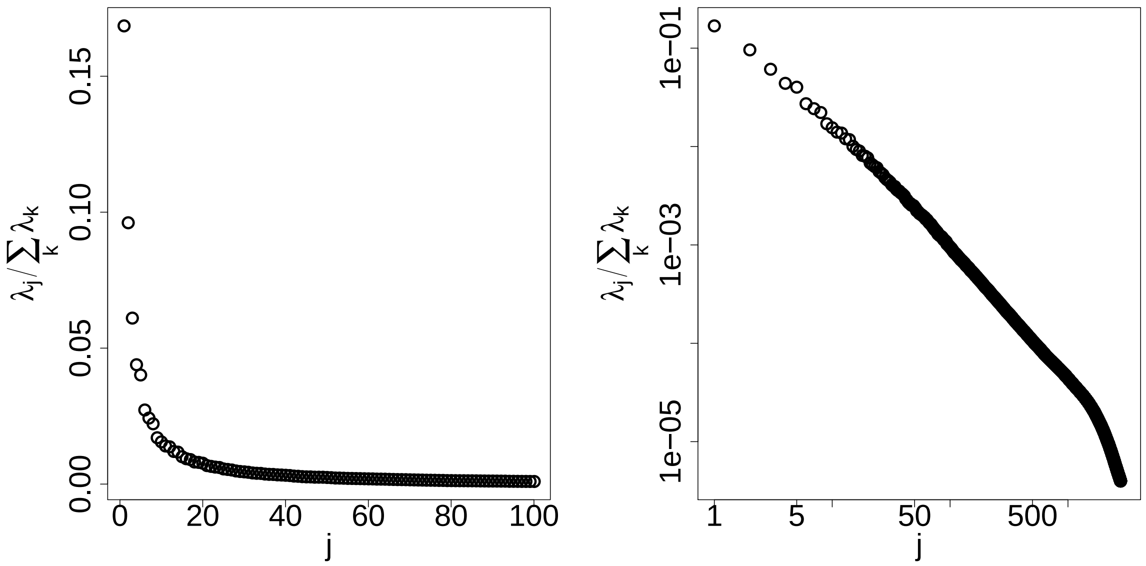


**Figure 6.** Normalized POD eigenvalues: linear axes (left), logarithmic axes (right)

### *4.3. POD Reconstruction*

We now use the obtained modes to reconstruct the LES data using different numbers of modes. Even though the modes have been extracted for the field after applying the threshold procedure (Sec. 4.1), we can in principle reconstruct either the original field or the field after threshold application. This can be done by inserting the chosen field into Eq.(3) (as $u$). Here, we focus on the results when reconstructing the original field. This way, we can show that the modes cannot only describe a preprocessed field which is partially determined by its threshold parameter but also aspects of the full field.

Examining the reconstructed snapshots (Fig. 7, 8), we see that more modes obviously lead to a more accurate description of the wake structure. To recover the spatial small scale structures of the wake very many POD modes are necessary. However, pure visualization does not offer many clues on the number of modes necessary to obtain a useful low order description. Therefore, we need to define quantitative measures to draw conclusions on the quality of reconstruction.

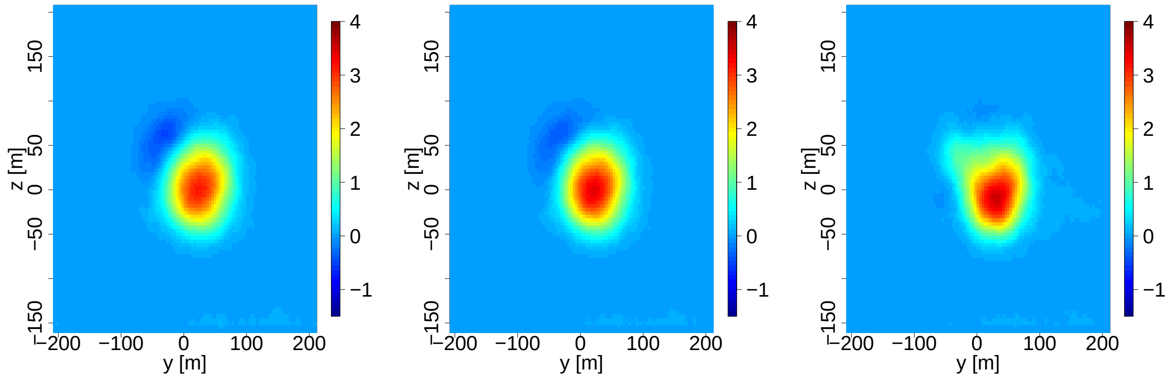


**Figure 7.** POD reconstruction ($t$ = 500 s): Mean + 2 modes (left), Mean + 3 modes (middle), Mean + 6 modes (right)

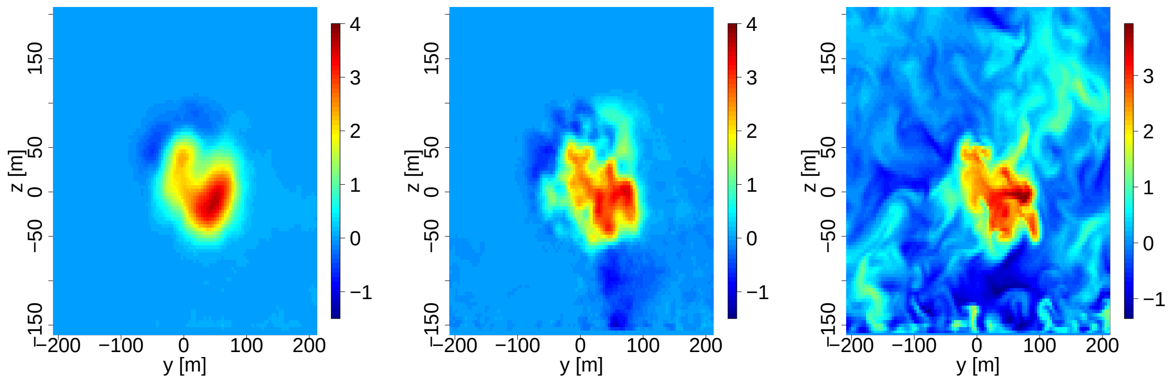


**Figure 8.** POD reconstruction ($t$ = 500 s): Mean + 10 modes (left), Mean + 100 modes (middle), Original (right)

A first and obvious measure is the percentage of (the fluctuating part of the) kinetic energy captured by the reconstruction (see Eq.(6). The slow decay of the POD spectrum obviously implies, that we need a lot of modes to recover a great part of the kinetic energy. E.g. we need around 100 Modes to recover 80% of the kinetic energy (Fig. 9). This big number is obviously inconvenient for a low order description of the wake. Consequently, this raises the question, whether only a few modes can still recover other important aspects of the wake flow. To answer this question other quality measures have to be used.

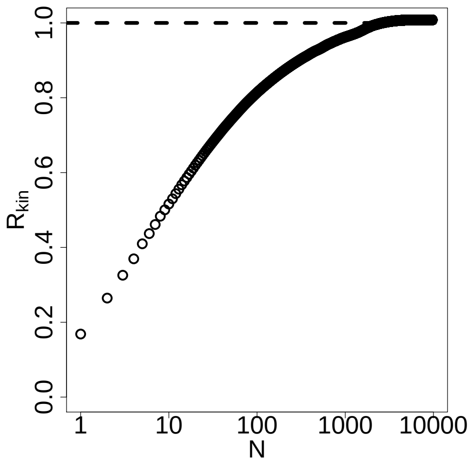


**Figure 9.** Percentage of kinetic energy recovered by reconstruction vs number of used modes N

### *4.4. Recover the Effective Velocity for a Potential Turbine in the Wake*

Instead of using the kinetic energy of the flow, we introduce another measure for the reconstruction quality which we call the effective velocity. By effective velocity we mean the average velocity over a virtual turbine (disk) in the flow (Fig. 10). This average can be used for a very rough approximation of the power output of a virtual turbine [17, 18] e.g. by inserting $u_{\text{eff}}$ into a specific power curve .

We can now compare the effective velocity of the full field with the effective velocity of different reconstructions. While the first two modes completely fail to capture the dynamics of $u_{\text{eff}}$, including the third mode already yields an impressively good description (Fig. 10). This sudden change can also be seen by looking at

$$R_{\text{eff}}(N) := 1 - \frac{\|u_{\text{eff}} - u_{\text{eff}}^{(N)}\|_2^2}{\|u_{\text{eff}} - u_{\text{eff}}^{(0)}\|_2^2} \,, \tag{7}$$

as a measure for the quality of a reconstruction (Fig. 11. Obviously, $R_{\text{eff}}$ is 1 for perfect reconstruction and 0 for a trivial reconstruction with the steady mean field $u^{(0)}$. After the sudden change from 2 to 3 modes further noticeable improvement is possible by including modes 4-6 reaching an almost perfect reconstruction with 7 Modes.

This fast convergence shows that physically relevant quantities such as $u_{\text{eff}}$ can possibly be recovered even though a the kinetic energy is only weakly reproduced (compare Fig. 11 and Fig. 9). For 7 modes we recover below 50% of the kinetic energy but reach above 98% for $R_{\text{eff}}$.

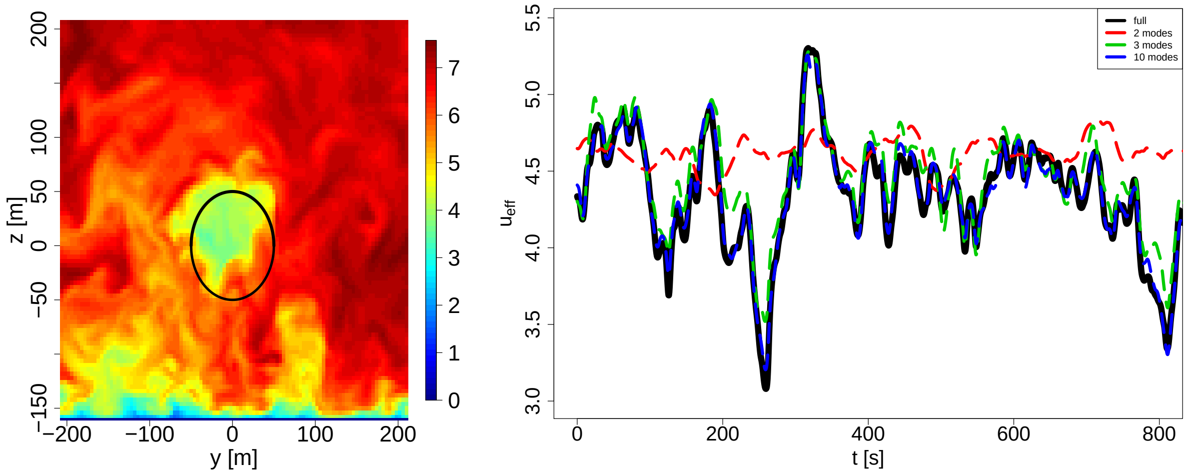


**Figure 10.** Left: Snapshot of the full field at $t = 50$ s. $u(y, z)$ is averaged of the area in the black circle to obtain the effective velocity $u_{\mathrm{eff}}$.
Right: $u_{\mathrm{eff}}$ for different numbers of POD modes used for reconstruction.

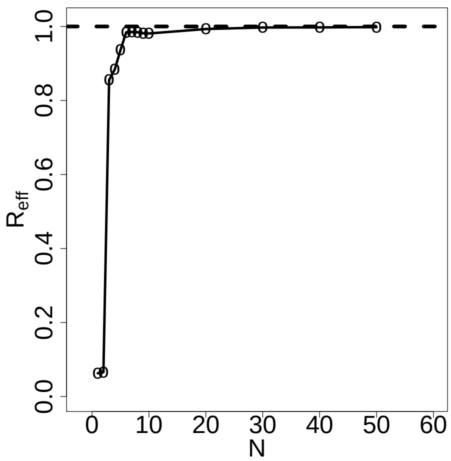


**Figure 11.** Quality of reconstruction with respect to the effective velocity: $R_{\mathrm{eff}}$ (Eq. (7)) versus the number of modes used for reconstruction.

## 5. Conclusions

We have applied the POD to data generated by LES simulations of an actuator disk in an atmospheric boundary layer. With an appropriate preprocessing to separate the wake from the ABL structures it was possible to extract well-defined pod modes. These modes turned out to be similar to the modes obtained by Andersen et al. [13] for the case of an infinitely long row of turbines without ABL indicating that wake dynamics can generally be described by similar kinds of modes.

Even though we needed a lot of modes to recover a great amount of the kinetic energy of the wake it seems to be still possible to capture important aspects of the wake by only a few modes. Particularly, the dynamics of the effective velocity, experienced by a sequential turbine in the wake, could be approximated relatively well by only three modes. This would mean a reduction from a full spatio-temporal field to only three time-series which represent the weights of the corresponding pod modes. Therefore, these results strongly support the idea of low order modeling of wakes through modal decompositions.

Further research is obviously necessary, particularly by applying these ideas to more realistic LES simulations, lab experiments (e.g. using high speed PIV) and field measurements (e.g. using lidar data). Furthermore, our investigation has to be extended to different quality measures and for varying distances to the turbine.
Our results immediately raise the question of a physical meaning of the POD modes to explain e.g. the suddenly good description of the effective velocity with only three modes. Preliminary results indicate that the modes can actually be interpreted by investigating their connection to physically meaningful quantities such as the position of the wake (wake meandering) and the average spatial amplitude.
Another open question is whether spatial modes can be chosen in a different manner and not by POD. The POD is optimized with respect to kinetic energy (Sec. 3) and not to other measures such as the effective velocity. Choosing the POD modes just in a different order or even extract different modes could possibly improve our results.
To obtain complete model from a low order description, we actually need to model the temporal evolution of the expansion coefficients of modes. Projecting the Navier-Stokes-Equations on the extracted modes, as done in [11, 12] is difficult, since the extracted modes are two-dimensional but are embedded in fully three-dimensional dynamics. In contrast to Andersen et al. [13], we also did not observe any pronounced low frequencies which could maybe used for a spectral model of the expansion coefficients. Our ongoing research is therefore concerned with modeling these coefficients of the modes as stochastic processes. These stochastic models combined with the POD modes can in principle be used to build a stochastic wake model.

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