Numerical Simulation of Nonlinear Ultrasonic Waves Due to Bi-material Interface Contact

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Abstract. Boundary integral equations are formulated to investigate nonlinear waves generated by a debonding interface of bi-material subjected to an incident plane wave. For the numerical simulation, the IRK (Implicit Runge-Kutta method) based CQ-BEM (Convolution Quadrature-Boundary Element Method) is developed. The interface conditions for a debonding area, consisting of three phases of separation, stick, and slip, are developed for the simulation of nonlinear ultrasonic waves. Numerical results are obtained and discussed for normal incidence of a plane longitudinal wave onto the nonlinear interface with a static compressive stress.

1. Introduction

Nonlinear ultrasonic nondestructive testing has been developed over the last decade [1], since nonlinear waves are generated by nonlinear elasticity of materials and unbonded interface conditions and are very sensitive to a degradation of material properties at very early stage. However, the mechanism of generation of nonlinear waves has not yet been understood very well from the theoretical and/or numerical point of view.

So far, two dimensional simulations on nonlinear ultrasonic waves have been carried out [2][3]. Also, the axisymmetric problem of a circular crack subjected to normal incidence of a longitudinal wave was solved numerically [4]. However, no full three dimensional analysis has been done. It is, therefore, demanded to conduct numerical simulations with three dimensional realistic models. In this paper, three dimensional boundary integral equations are formulated for a circular interface crack with unbonded conditions in bi-material half spaces. The integral equations are discretized using the IRK based CQ-BEM and numerically solved to investigate the nonlinearity involved in ultrasonic waves scattered by the nonlinear interface crack.

2. Formulation of boundary integral equations

The model of a bi-material, consisting of two semi-infinite domains D and D^{II}, is shown in figure 1. S_h and S_d represent the bonding and debonding areas, respectively, of the bi-material interface. Assuming that an incident plane wave is given in D^I, we may formulate time-domain boundary integral equations, in which reflected and transmitted waves are unknown values. However, the time-domain boundary integral equations for the bi-material have the following disadvantages:

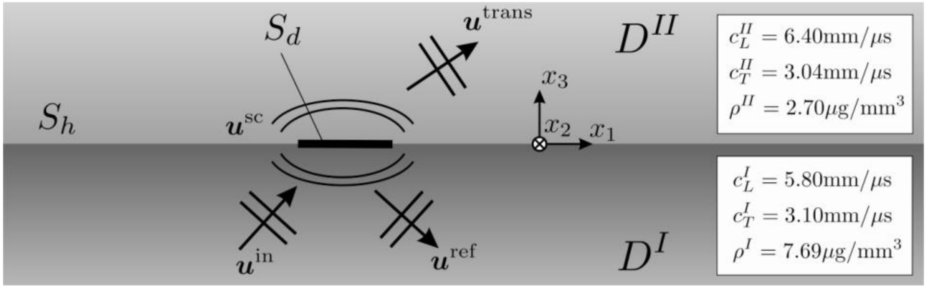

Figure 1. Model for numerical simulation.

- For normal incidence of a plane wave, reflection and transmission occur at all points on the interface from the initial step in time. Due to this fact, truncation errors can be introduced at the edge elements, where the infinite interface is truncated in numerical analysis.
- For oblique incidence, initial conditions that are given by zero displacement and velocity for unknown reflected and transmitted waves at all points in the domains, cannot be satisfied unless an infinite interface is taken into account in the numerical analysis.

Therefore, in this paper, the integral formulation in which unknown variables are only scattered waves from the debonding area, is proposed in order to overcome these difficulties. If the flat interface of infinite extent is perfectly bonded and is subjected to a plane wave incidence, it is easy to calculate analytically the "free field $\boldsymbol{u}^{\mathrm{free}}$", defined as the summation of the incident wave $\boldsymbol{u}^{\mathrm{in};I}$ and the reflected wave $\boldsymbol{u}^{\mathrm{ref};I}$ in D^I, and the transmitted wave field $\boldsymbol{u}^{\mathrm{trans};II}$ in D^{II} as follows:

$$\boldsymbol{u}^{\mathrm{free};I} = \boldsymbol{u}^{\mathrm{in};I} + \boldsymbol{u}^{\mathrm{ref};I} : D^I, \quad \boldsymbol{u}^{\mathrm{free};II} = \boldsymbol{u}^{\mathrm{trans};II} : D^{II} \tag{1}$$

If debonding exists in a local area on the interface, the free field may be disturbed by the wave field $\boldsymbol{u}^{\mathrm{sc}}$ scattered by the debonding area, and the total displacement $\boldsymbol{u}$ can be expressed by $\boldsymbol{u} = \boldsymbol{u}^{\mathrm{sc}} + \boldsymbol{u}^{\mathrm{free}}$. Since the scattered wave $\boldsymbol{u}^{\mathrm{sc}}$ satisfies the initial condition and the radiation condition, the boundary integral equations for $\boldsymbol{u}^{\mathrm{sc}}$ can be formulated. In solving the boundary integral equations for $\boldsymbol{u}^{\mathrm{sc}}$, the convolution integrals with time are evaluated by IRK based CQM [5] and the surface integrals over the bonding and debonding interfaces are discretized by constant elements. For the acceleration, the fast multipole method is applied to IRK based CQ-BEM [6].

The boundary integral equations for $\boldsymbol{u}^{\mathrm{sc}}$ in D^I and D^{II} are simultaneously solved using appropriate interface conditions. The interface condition on the bonding area S_h is the continuity of displacement and stress. For the debonding area S_d, three types of interface conditions, "separation", "stick", and "slip", are considered [2]. "separation" means that two surfaces of upper and lower materials are separated with no traction, while "stick" and "slip" are contact conditions under compressive normal stress state. For the "stick" condition, the surfaces of two materials move with no relative velocity. On the other hand, the "slip" condition allows a relative tangential movement with dynamic friction force.

3. Numerical examples

Numerical examples are presented for nonlinear ultrasonic wave problems of bi-material interface subjected to a small static compressive stress 0.74kPa normal to the interface and the normal incidence disturbance of longitudinal waves with 2 and 4MHz frequencies. The incident wave is a sinusoidal plane wave with three cycles and 10nm amplitude. The debonding area is a circular interface crack with radius 0.5mm. We assume that the material constants for bi-material are given in figure 1. The coefficients of static and kinematic friction are given by 0.61 and 0.47, respectively.

3.1. Vertical displacement at the center points on top and bottom debonding area

In figures 2 (a) and (b), the vertical displacements at the center points on the top and bottom debonding surfaces, subjected to 2MHz and 4MHz sinusoidal incident waves, respectively, are demonstrated as a function of time. Note that the time scales in these two figures are different. The bottom surface is forced to move according to the direct incidence of the sinusoidal wave, whereas the top surface is freely movable for a short time after deformation together with the bottom surface in contact condition, and then affected by the diffracted waves from the edge of the circular debonding area. Thus the periods of vibrations of top and bottom surfaces are different, and the dynamic behaviors of the debonding surfaces, especially the top surface, are different depending on the frequency. Note that these clapping motions of the debonding interface in three dimensional problem are very similar to the results of two dimensional simulations [3].

3.2. Vertical displacement 2.0mm above the center of debonding area

Figure 3 shows the vertical displacements of total and scattered wave fields ((a) and (b)) at the internal point located 2.0mm above the center of the debonding area, and the normalized frequency spectra of the scattered waves $\boldsymbol{u}^{sc}$ ((c) and (d)). Figures (a) and (c) are the results for the 2MHz sinusoidal incident wave, and figures (b) and (d) are for the 4MHz case. Note that the vertical axes for the vertical displacements of scattered waves are shown on the right side of the graphs. For comparison, the results for the displacements of the free fields in the case of no debonding are shown.

In both figures of (a) and (b), the time variations of vertical displacements of total waves show periodic waveforms and small aftereffects. However, the aftereffects do not continue for a long time compared with the results of two dimensional simulations [3]. In the case of 2MHz, the vertical displacement of scattered wave generated by the clapping motion shows a shorter period than the fundamental frequency of 2MHz, and hence large second and higher harmonics as well as sub-harmonics are seen in the frequency spectrum (see figure (c)). In the case of 4MHz, on the other hand, the waveform of scattered wave is a little distorted compared with the case of 2MHz, and the frequency spectrum shows relatively small amplitudes of the second and third harmonics. From these results, it can be said that the generation of nonlinear ultrasonic waves like higher harmonics and sub-harmonics due to contact conditions on the debonding interface depend largely on the frequency.

4. Conclusions

In this paper, the boundary integral equations for bi-material subjected to an incident plane wave were formulated, and the IRK based CQ-BEM was implemented in the numerical simulation. The interface conditions for debonding areas that consists three phases of "separation", "stick", and "slip", were developed for the simulation of nonlinear ultrasonic waves. Numerical results showed that the generation of nonlinear ultrasonic waves are largely dependent on the frequency as well as the contact conditions on the interface.

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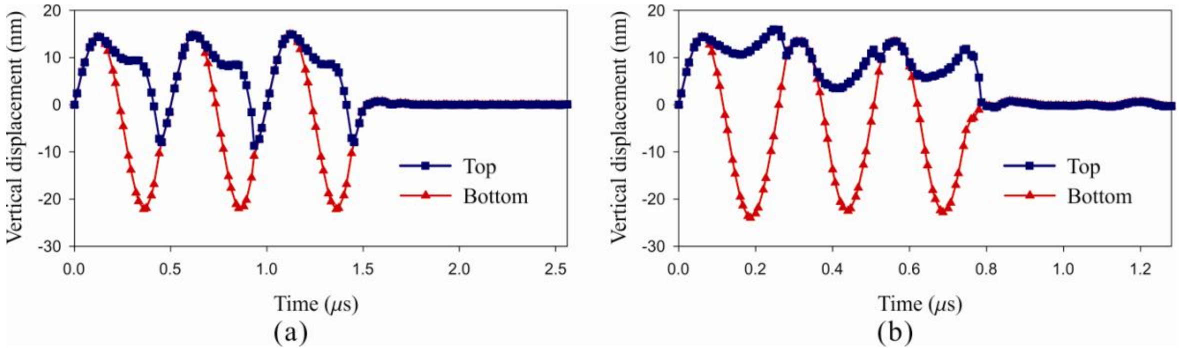

Figure 2. Time variations of vertical displacements at the center points on top and bottom debonding surfaces in the cases of normal incidence of sinusoidal longitudinal waves with (a) 2MHz and (b) 4MHz.

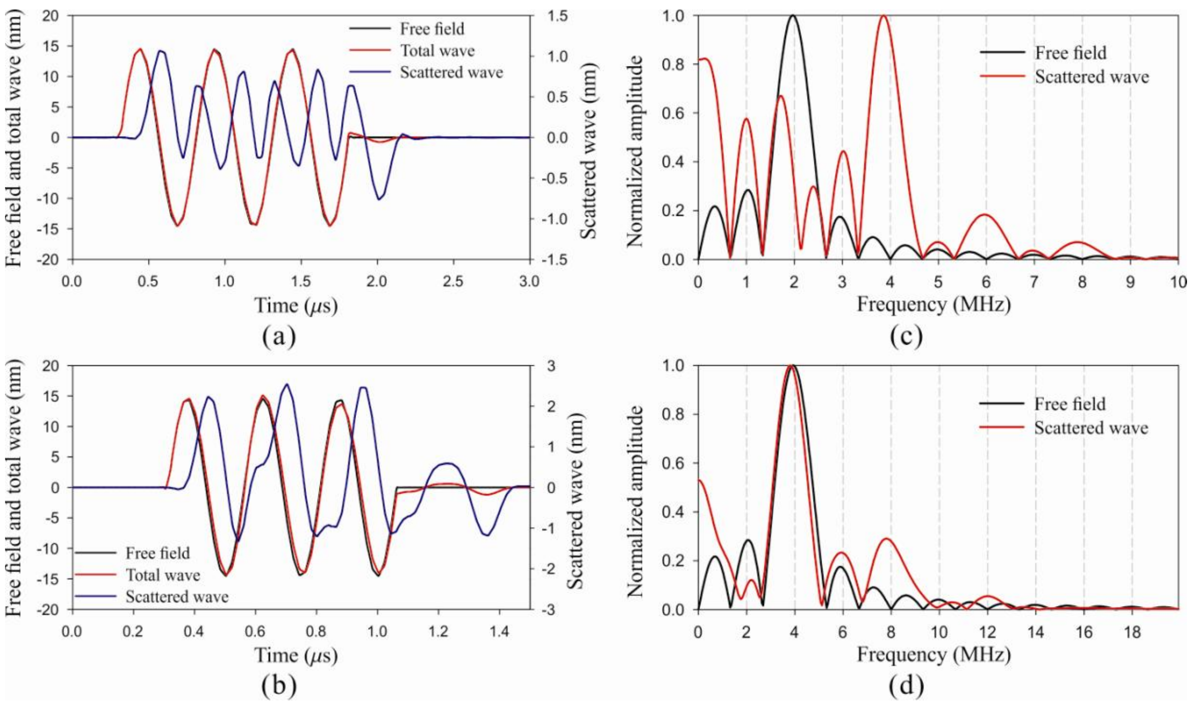

Figure 3. Vertical displacements as a function of time ((a) and (b)) and frequency spectra of scattered waves ((c) and (d)) at the point 2.0mm above the center on top debonding surfaces in the cases of normal incidence of sinusoidal longitudinal waves with 2MHz (top) and 4MHz (bottom) frequencies.