Coherent X-ray Cherenkov radiation produced by microbunched beams

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Abstract. Analytical and numerical results on the coherent X-ray Cherenkov radiation (CXCR) produced by microbunched beams in the region near the K-, L-edges of materials are obtained. The results show that CXCR can serve as a suitable mechanism for production intense beams of photons in the "water window" region as well as for studying the important microbunching process at FLASH TESLA, LCLS and other FELs.

1. Introduction

At the end of the long undulators of free electron lasers (FELs) [1-4] the electron beams undergo longitudinal periodical density modulation or microbunching (MB) with period equal to the wavelength of the FEL SASE radiation, $\lambda_r = 2\pi c/\omega_r \approx L_{und}/2\gamma^2(1+K^2/2)$, where L_{und} and K are the period and parameter of the undulator, $\gamma = E/mc^2 = [1-(V/c)^2]^{1/2}$ is the electron relatyivistic factor. Using the unique parameters of the SASE beams many important experiments can be carried out, among which it is worthwhile to mention the experiments requiring water window photons with λ = (2.3–4.4) nm having relatively large and short absorption lengths for water and other elements, respectively. Recently an experiment devoted to the first observation of atomic inner-shell laser photon production at 1.46 nm has been carried out at LCLS [5]. The beams of [5] are more stable, reproducible with narrower spectral and angular distributions than the SASE beams, and these properties are necessary for some experiments.

At present the FELs' microbunched beams, the MB of which is spoiled after some distance, are sent to dumps without any applications. Only rarely these microbunched electron beams (MBEB) with energies less than 1 GeV have been used (see [1]) for production of soft coherent transition radiation (CTR) for the measurement of the parameters of MB according to the proposal of [6]. To our knowledge the MB of MBEB with energies higher than 1 GeV has not been studied, though a method based on coherent X-ray transition radiation (CXTR) has been proposed [7] and even some preparatory works [8, 9] have been carried out. Recently, it has been theoretically studied other types of coherent X-ray radiation of MBEB: coherent bremsstrahlung and resonance transition radiation in [10]; coherent X-ray diffraction radiation in [11]; coherent X-ray crystalline undulator radiation in [12]; coherent X-ray channeling radiation in [13]; coherent X-ray parametric radiation in [14, 15]. A short review of theory of these processes is given in [16].

In [17] it has been shown that in narrow regions close to K-, L- edges for some materials the refraction index $n(\omega)$ or the dielectric constant $\varepsilon(\omega)=n^2(\omega)$ are greater than 1, and therefore X-ray Cherenkov radiation (XCR) can be produced by charged particles. XCR has been experimentally studied in [18 – 21]. Reviews on XCR are given in [22, 23].

Taking into account the growing demand for intense beams of water window photons for biology and medicine, and that at present there is a possibility to study coherent X-ray Cherenkov radiation (CXCR) produced by MBEB at DESY FLASH TESLA [2,3] and SLAC LCLS [4] FELs this work is devoted to the theoretical study of CXCR.

2. The Calculation of CXCR

It is usual to present the complex values of $\varepsilon(\omega)$ and $n(\omega)$ in the forms: $\varepsilon(\omega)=n^2(\omega)=\varepsilon'(\omega)+i\varepsilon''(\omega)=1+\chi'(\omega)+i\chi''(\omega)$ and $n(\omega)=1+\delta(\omega)+i\Delta(\omega)$ where χ' and χ'' are the real and imaginary parts of the susceptibility, and absorption length $L_{abs}=1/\mu(\omega)=c/(\omega\chi''(\omega))$ where μ is the linear absorption coefficient. In order to calculate CXCR first it has been calculated χ' and χ'' by the methods described in [22]. Then the theory of CXCR produced by MBEB is developed by the method described in [6, 16] using the well-known formula

$$\frac{d^2N_{CXCR}}{d\omega d\theta}\cong N_b^2F(\omega,\theta)\frac{d^2N_{XCR}}{d\omega d\theta}, \tag{1}$$

where N_b is the number of electrons in a macrobunch, the form factor $F(\omega,\theta)$ of the MBEB are given in the works [6, 16]. By inserting the values into the XTR formula (1.59) of [24] one obtains the following formula for the spectral-angular distribution of CXCR per bunch produced at the exit interface of a plate (CXCR produced at the entrance is totally absorbed):

$$\frac{d^2N_{CXCR}}{d\hbar\omega d\theta}\cong N_b^2b_1^2\frac{2\theta^3}{137\pi\hbar\omega}\frac{(\chi'^2+\chi''^2)}{(\theta^2+\gamma^{-2})^2[(\theta^2+\gamma^{-2}-\chi')^2+\chi''^2]}\exp\left(-(k\sigma_r\theta)^2\right)\times \\ \times\exp\left(-\left(\frac{\omega}{V}-k_r\right)^2\sigma_z^2\right), \tag{2}$$

where b_1 is the modulation depth of MB [6, 16], $k=\omega/c$, $k_r=\omega_r/c$, $\sigma_{r,z}$ are the radial and longitudinal standard deviations of the Gaussian macrobunch. It is necessary to choose the parameters of the undulator in such a way that provides the equality of $\hbar\omega_r$ to the energy of the K- or L-edge of the radiator material. Formula (2) is our basic one, and as it has been mentioned above it is necessary to carry out integrations of (2) over $\hbar\omega$ and θ.

The angular distribution of CXCR photon number is obtained integrating (2) over ω in an narrow interval around ω_r following the works [6, 16]:

$$\frac{dN_{CXCR}}{d\theta}\cong N_b^2b_1^2\frac{2\theta^3V}{137\sqrt{\pi}\sigma_z\omega_r}\frac{(\chi'^2+\chi''^2)|_{\omega=\omega_r}}{(\theta^2+\gamma^{-2})^2[(\theta^2+\gamma^{-2}-\chi'|_{\omega=\omega_r})^2+\chi''^2|_{\omega=\omega_r}]}\exp\left(-(k_r\sigma_r\theta)^2\right). \tag{3}$$

The spectral distribution of CXCR obtained by integration of (2) over angles is given by the expression

$$\frac{dN_{CXCR}}{d\omega}\cong N_b^2b_1^2\frac{2(\chi'^2+\chi''^2)}{137\pi\omega}I(\omega,\theta_0)\exp\left(-\left(\frac{\omega}{V}-k_r\right)^2\sigma_z^2\right), \tag{4}$$

where

$$I(\omega,\vartheta_0)=\int_0^{\vartheta_0}\frac{\theta^3}{(\theta^2+\gamma^{-2})^2[(\theta^2+\gamma^{-2}-\chi')^2+\chi''^2]}\exp\left(-(k\sigma_r\theta)^2\right)d\theta \tag{5}$$

and $\theta_{diff} << \theta_0 << 1$, the diffraction angle $\theta_{dif} = c/\omega\sigma_r$.

The calculation of the integral $I(\omega,\theta_0)$ is done by means of its representation in the form of the sum of the four integrals to be calculated analytically:

$$I(\omega,\theta_0)=(aI_1+bI_2+cI_3+dI_4)/2 \tag{6}$$

where after the notation $\tilde{\theta}=\theta^2$:

$$I_1=\int_0^{\theta_0^2}\frac{e^{-\tilde{\theta}(k\sigma_r)^2}d\tilde{\theta}}{\tilde{\theta}+\gamma^{-2}}=e^{(k\sigma_r/\gamma)^2}\left\{\Gamma\left(0,k^2\sigma_r^2/\gamma^2\right)-\Gamma\left(0,k^2\sigma_r^2\left(\gamma^{-2}+\theta_0^2\right)\right)\right\}, \tag{7}$$

$$I_2=\int_0^{\theta_0^2}\frac{e^{-\tilde{\theta}(k\sigma_r)^2}d\tilde{\theta}}{\left(\tilde{\theta}+\gamma^{-2}\right)^2}=\gamma^2-\frac{e^{(k\sigma_r\theta_0)^2}}{\gamma^{-2}+\theta_0^2}-(k\sigma_r)^2 I_1, \tag{8}$$

$$I_{3,4}=\int_0^{\theta_0^2}\frac{e^{-\tilde{\theta}(k\sigma_r)2}d\tilde{\theta}}{\tilde{\theta}+\gamma^{-2}-\chi'\pm j\chi''}= \\ =e^{\left(\gamma^{-2}-\chi'\pm j\chi''\right)(k\sigma_r)^2}\left\{\Gamma\left(0,-(k\sigma_r)^2\left(\gamma^{-2}-\chi'\pm j\chi''\right)\right)-\Gamma\left(0,-(k\sigma_r)^2\left(\gamma^{-2}-\chi'\pm j\chi''+\theta_0^2\right)\right)\right\}, \tag{9}$$

$$a=\frac{\chi'^2+\chi''^2-2\gamma^{-2}\chi'}{\left(\chi'^2+\chi''^2\right)^2}\ ,\ b=-\frac{1}{\gamma^2}\frac{1}{\chi'^2+\chi''^2}, \qquad 10)$$

$$c,d=\frac{\chi''\mp j\left(\gamma^{-2}-\chi'\right)}{2\left(\chi'\mp j\chi''\right)^2\chi''}, \tag{11}$$

Here $\Gamma(a,z)=\int_z^\infty t^{a-1}e^{-t}dt$ is incomplete Gamma function.

For $\theta_0 << 1$ one obtains:

$$I(\omega,\theta_0)=\frac{(\theta_0\gamma)^4}{4q}-\frac{\alpha(\theta_0\gamma)^6}{6q^2}, \tag{12}$$

where

$$\alpha=\gamma^{-4}\left(4+\left(\frac{k\sigma_r}{\gamma}\right)^2\right)-2\gamma^{-2}\chi'\left(3+\left(\frac{k\sigma_r}{\gamma}\right)^2\right)+\left(\chi'^2+\chi''^2\right)\left(2+\left(\frac{k\sigma_r}{\gamma}\right)^2\right), \tag{13}$$

$$q=\left(\gamma^{-2}-\chi'\right)^2+\chi''^2.$$

The total number of CXCR photons obtained by integration of (3) over θ, or of (4) over ω is equal to

$$N_{CXCR}=N_b^2 b_1^2\frac{2V(\chi'^2+\chi''^2)|_{\omega=\omega_r}}{137\sigma_z\sqrt{\pi}\omega_r}I(\omega_r,\theta_0). \tag{14}$$

The properties of the angular and spectral distributions as well as of the total number of the CXCR photons will be discussed in the next section after numerical calculations.

3. Numerical Results

For numerical calculations it has been taken for carbon, C, ρ_C =2.2 g/cm^3 and $\hbar\omega_r = 284.2$ eV and for titanium, Ti, ρ_{Ti}= 4.54 g/cm^3 and $\hbar\omega_r = 453.8$ eV with dependences $\chi'(\hbar\omega)$ and $\chi''(\hbar\omega)$ close to [21] and to [20c] for C and Ti, respectively.

The below given numerical results obtained with the help of (3) – (14) are for the FLASH2 with electron energy E = 1.2 GeV [2, 3, 25] and at XFELs SACLA [26] and EuroFEL [27] with E = 8 and 10 GeV, respectively. In all case the parameters of the FELs are tuned to provide the necessary $\hbar\omega_r$. Unfortunately, according to [25] $\hbar\omega_r$= 453.8 eV is out of the achievable microbunching region. In all the cases it is taken $\sigma_z = 15\mu\text{m}$ and $\sigma_r = 13\ \mu\text{m}$.

3a. The Angular Distributions of CXCR

Figure 1a and b show the angular distributions of CXCR for C and Ti. It is necessary to note that in figure 1, as well as in figures 2 and 3 the the values on the ordinate axis are divided on γ^4. As it is seen in figure 1

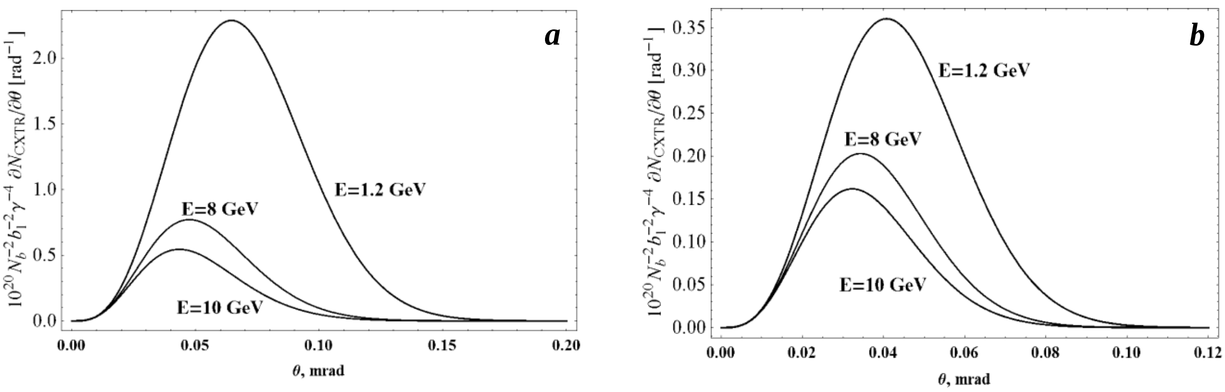

Figure 1. Angular distributions of CXCR photon number for C ***a***) and Ti ***b***).

just as the other types of radiations of microbunched beams [19] the CXCR photons are emitted mainly under angles much smaller than θ =1/γ, the angles of radiation of relativistic particles. The dependence of the angle of maximal intensity on energy is very weak, while the maximal intensity of CXCR increases strongly with the increase of the particle energy and approximately is proportional to $\sim \gamma^4$.

3b. The Spectral Distributions of CXCR

Figures 2a and b show the spectral distributions of CXCR for C and Ti.

As expected the spectral distributions of CXCR are very narrow as the well known spectral distributions of PXR and are concentrated around $\hbar\omega_r$. From figure 2a and b it is seen that the absolute width of CXCR for Ti is broader than that for C. However, all the curves have similar relative widths approximately equal to $\Delta\omega/\omega < 0.00012 = 0.012\%$ (FWHM). The numerical integration of the corresponding curves of figure 1 and figure 2 gives total number of CXCR photons equal to those given in figure 3.

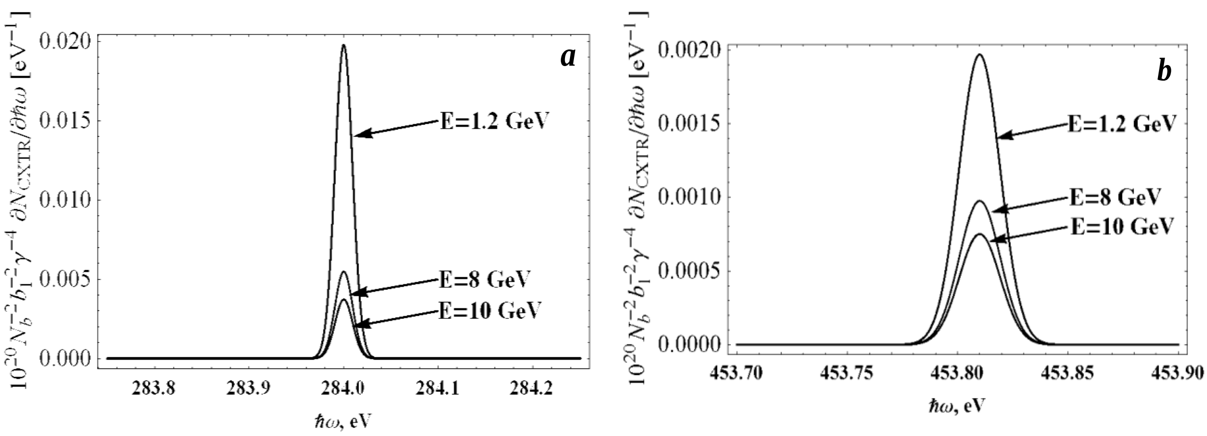

Figure 2. The spectral distributions of CXCR for C ***a***) and for Ti ***b***).

As expected the spectral distributions of CXCR are very narrow as the well known spectral distributions of PXR and are concentrated around $\hbar\omega_r$. From figure 2a and b it is seen that the absolute width of CXCR for Ti is broader than that for C. However, all the curves have similar relative widths approximately equal to $\Delta\omega/\omega < 0.00012 = 0.012\%$ (FWHM). The numerical integration of the corresponding curves of figure 1 and figure 2 gives total number of CXCR photons equal to those given in figure 3.

3c. The Energy Dependence of CXCR

Figures 3a and b show the dependence of the total number of CXCR photons upon electron energy for C and Ti, respectively.

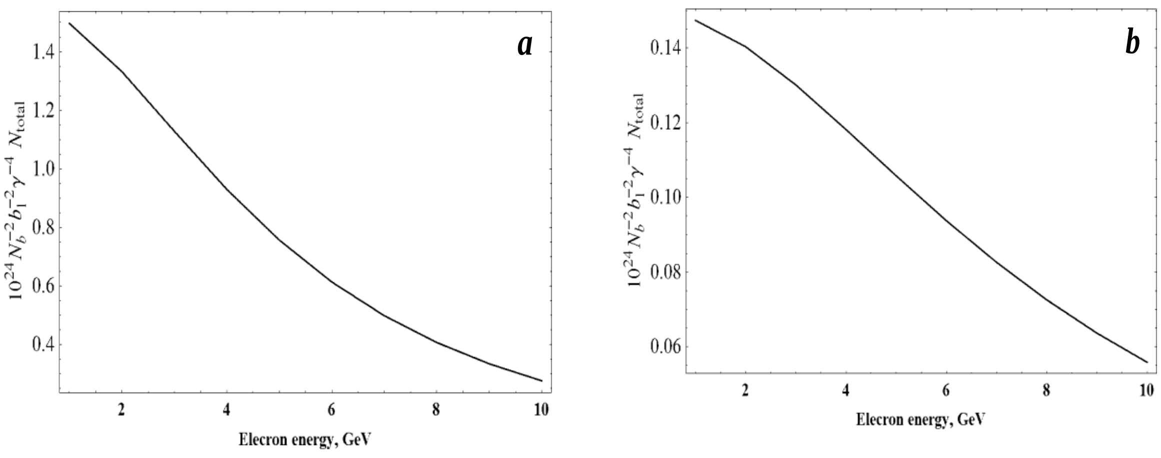

Figure 3. Total number of photons vs electron energies for C ***a)*** and for Ti ***b***).

From figure 3 it is seen that the intensity of CXCR increases with the increase of particle energy slightly weakly than $\sim\gamma^4$. As it follows from figures 2 and 3 theoretically the CXCR yields from carbon radiators is higher than that from titanium radiator. However, it is necessary to take into account the experimental difficulties connected with the C radiators (see [19, 21])

4. Discussion and Conclusions

It is of interest to compare the main characteristics (the total number and width) of CXCR with those of incoherent XCR produced by not microbunched electrons and of SASE produced by the same charge q = 1 nC or N_b = 6.24x10^9 electrons per pulse at E= 1.2 GeV. According to [3, 25] for SASE photon beam of FLASH2 it is expected to have N_{SASE}=10^{11} – 10^{13} with $\Delta\omega/\omega_r \approx (0.5-2)\%$ (FWHM). Let us take the mean expected values: N_{SASE}= 10^{12}, $\Delta\omega/\omega_r \approx 1\%$ and $\Delta\omega \approx 2.8$ eV. On the other hand, for a carbon, C, radiator according to the Table 2.1 of [21] the expected total number of the incoherent XCR photons is equal to $N_{XCR} = N_b x K_{XCR}$ = 3x10^6 with $\Delta\omega/\omega \approx 0.0035 = 0.35\%$ and $\Delta\omega \approx 1$ eV (here K= 4.8x10^{-4} is the number of XCR photons per electron). According to the above obtained results for CXCR one expects N_{CXCR} (b_1= 1) = 1.66x10^9, $\Delta\omega/\omega_r = 0.00012 = 0.012\%$ and $\Delta\omega \approx 0.033$ eV. From these values it follows that assuming b_1= 1, the CXCR beam will have ~ 600 times less intensity and 80 times narrower width than the SASE beam as well as 550 times higher intensity, and 30 times narrower width than the XCR. For real value of b_1<1 the CXCR intensity will be reduced proportional to b_1^2, however, just as in the case of [5] the CXCR width will remain by the same times narrower than SASE and XCR, which is very important for many experiments.

Taking into account these advantages and disadvantages it is worthy to use the microbunched beam after the FLASH2 undulator for production CXCR beam. The situation becomes much better for higher energy electrons and Ti radiator, since the CXCR intensity is approximately proportional to γ^4. On the other hand, in our knowledge, the real value of b_1 has not been measured, and with the help of CXCR it can be easily measured, if by another simple method, say measuring the current, one knows the value of N_b. Besides this application as it has been shown above the second application of CXCR is the production of additional monochromatic photon beams with parameters comparable with those of PXR. Let us also note that that just as in the case of [5] CXCR photon beams have stable $\hbar\omega_r$..

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