

# The Optics of Gyrotropic Crystals in the Field of Two Counter-Propagating Ultrasound Waves

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**Abstract.** We consider oblique light propagation through a layer of a gyrotropic crystal in the field of two counter-propagating ultrasound waves. The problem is solved by Ambartsumyan's layer addition modified method. The results of the reflection spectra for different values of the problem parameters are presented. The possibilities of such system applications are discussed.

## 1. Introduction

Recently the photonic crystals (PCs) which enable to tune light propagation are of great interest [1, 2]. Chiral PCs have rich optical properties [3-10]. Of particular interest are the self-organizing structural-chiral PCs (cholesteric liquid crystals (CLCs), cholesteric smectic liquid crystals, etc.) because of their rich optical properties [11]. For these crystals, the photonic band gap (PBG) exists only for the light with one circular polarization (for normal light incidence) coinciding with the sign of the medium chirality. Nevertheless, another periodic change in the chiral media parameters is possible; namely, it is possible to create one dimensional (1D) chiral PCs in other ways as well. Recently, the chiral layer media made of layers of usual gyrotropic and non-gyrotropic media or of the layers of gyrotropic media of different types [3-10] are of great interest. A gyrotropic layer placed inside an external ultrasound field becomes a 1D chiral PC [12-14]. Below we name such media as *chiral periodic media* (CPM). As it is possible to tune the parameters of the ultrasound, hence, we can obtain a chiral PC with tunable parameters. Currently, for manufacturing acousto-optical devices of information processing (modulators, deflectors, filters, processors, detectors, etc) the very crystals with both good photo-elastic and gyrotropic properties and are widely used. To such crystals, first of all, the para-tellurite ( $\text{TeO}_2$ ), tellurium (Te), quarts ( $\alpha\text{-SiO}_2$ ) etc are related.

Light diffraction on elastic waves is used in practical devices, mainly, because of the significant difference between the light speed and the sound speed. Light diffraction on acoustic waves is used to obtain optical radiation modulation.

In this work oblique light propagation through a layer of a gyrotropic crystal located in the field of two counter-propagating ultrasound waves is considered.



## 2. Theory

Let us consider light reflection/transmission through a finite layer from an isotropic-gyrotropic crystal placed in an ultrasound field. Let the medium layer occupy the space between the planes,  $z=0$  and  $z=d$  ( $d$  is the layer thickness). A plane ultrasound wave propagates along the axis,  $z$ . The ultrasound wave converts the parameters,  $\varepsilon$ ,  $\mu$  and  $\bar{\gamma}$ , into functions of the coordinate,  $z$  ( $\varepsilon$  and  $\mu$  are the layer dielectric permittivity and magnetic permeability, and  $\bar{\gamma}$  is the parameter of the natural gyrotropy of this layer). Here we assume the following dependences for these parameters' changes:

$$\begin{pmatrix} \varepsilon(z) \\ \mu(z) \\ \bar{\gamma}(z) \end{pmatrix} = \begin{pmatrix} \varepsilon \\ \mu \\ \bar{\gamma} \end{pmatrix} \left[ 1 + \begin{pmatrix} \Delta\varepsilon_1 \\ \Delta\mu_1 \\ \Delta\bar{\gamma}_1 \end{pmatrix} \sin(K_1 z) - \begin{pmatrix} \Delta\varepsilon_2 \\ \Delta\mu_2 \\ \Delta\bar{\gamma}_2 \end{pmatrix} \sin(K_2 z) \right], \quad (1)$$

where the modulation depths,  $\Delta\varepsilon_1$ ,  $\Delta\mu_1$ ,  $\Delta\bar{\gamma}_1$ , and  $\Delta\varepsilon_2$ ,  $\Delta\mu_2$  and  $\Delta\bar{\gamma}_2$  are assumed less than the unit;  $K_{1,2} = 2\pi / \Lambda_{1,2}$ , and  $\Lambda_{1,2}$  are the ultrasound wavelengths. These periodic excitations are changed both in space and time. Particularly, if the ultrasound is a traveling wave, then the periodic excitation is moving with the ultrasound speed. As the ultrasound speed is much less than the light speed, the periodic excitation caused by the ultrasound can be considered stationary and in Maxwell's equations it is possible to neglect the dependences of the media parameters upon time,

that is, while calculating the terms,  $\frac{1}{c} \frac{\partial \vec{D}}{\partial t}$  and  $\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$ , we neglect the medium parameter time derivatives, though we take their time dependences into account in the final results. As it is known, if the optical activity is absent ( $\bar{\gamma}=0$ ), this can be done if  $\Omega/\omega \ll 1$  [15]. For  $\bar{\gamma} \neq 0$ , we must also require that:

$$\left| \frac{\Omega}{\omega} \Delta\varepsilon_{1,2} \right|, \left| \frac{\Omega}{\omega} \Delta\mu_{1,2} \right|, \left| \frac{\Omega}{\omega} \Delta\bar{\gamma}_{1,2} \right| \ll \left| \frac{\omega}{c} \bar{\gamma} \right|, \quad (2)$$

to rightfully retain  $\bar{\gamma}$  in Maxwell's equation – simultaneously disregarding the time derivatives of the medium parameters. These conditions can be easily carried out. We assume that the incidence plane coincides with the plane,  $(x, z)$ , and that the wave is incident at the angle,  $\alpha$ , to the normal of the layer border (coinciding with the  $(x, z)$  plane). Let us expand the components of the electric field amplitudes of the incident, reflected and transmitted waves into the projections parallel ( $p$ -polarization) and perpendicular ( $s$ -polarization) to the incident planes:

$$\vec{E}_{i,r,t} = E_{i,r,t}^p \vec{n}_p + E_{i,r,t}^s \vec{n}_s = \begin{pmatrix} E_{i,r,t}^p \\ E_{i,r,t}^s \end{pmatrix}, \quad (3)$$

where the indices,  $i$ ,  $r$  and  $t$ , denote the incident, reflected and transmitted waves, respectively, and  $\vec{n}_p$  and  $\vec{n}_s$  are the orthonormal bases of the  $p$ - and  $s$ -polarizations.

We present the problem solution in the form:

$$\begin{bmatrix} E_r^p \\ E_r^s \end{bmatrix} = \begin{bmatrix} R_{pp} & R_{ps} \\ R_{sp} & R_{ss} \end{bmatrix} \begin{bmatrix} E_i^p \\ E_i^s \end{bmatrix}, \quad \begin{bmatrix} E_t^p \\ E_t^s \end{bmatrix} = \begin{bmatrix} T_{pp} & T_{ps} \\ T_{sp} & T_{ss} \end{bmatrix} \begin{bmatrix} E_i^p \\ E_i^s \end{bmatrix}, \quad (4)$$

where  $\hat{R}$  and  $\hat{T}$  are the 2x2 reflection and transmission matrices of the system.

We carry our calculations as follows. First, we calculate the reflection and transmission matrices for the medium sublayer of the thickness equal to the wavelength of the ultrasound wave. For this purpose we divide the thickness,  $d$ , into a great number of narrower sub-sublayers with thicknesses:  $d_1, d_2, d_3, \dots, d_N$ . If their maximal thickness is small enough, then it is possible to consider the parameters of each sub-sublayer constant. Then, according to [16, 17], the problem of determination of  $\hat{R}$  and  $\hat{T}$  for the sublayer with the thickness,  $d=\Lambda$ , is reduced to the solution of the following difference matrix equations:

$$\hat{R}_j = \hat{r}_j + \hat{t}_j \hat{R}_{j-1} (\hat{I} - \hat{r}_j \hat{R}_{j-1})^{-1} \hat{t}_j, \quad \hat{T}_j = \hat{T}_{j-1} (\hat{I} - \hat{r}_j \hat{R}_{j-1})^{-1} \hat{t}_j, \quad (5)$$

with  $\hat{R}_0 = \hat{0}$ ,  $\hat{T}_0 = \hat{I}$ . Here  $\hat{R}_j, \hat{T}_j, \hat{R}_{j-1}, \hat{T}_{j-1}$  are the reflection and transmission matrices for the media with  $j$ - and  $(j-1)$ - sub-sublayers, respectively;  $\hat{r}_j, \hat{t}_j$  are the reflection and transmission matrices of the  $j$ -th sub-sublayer;  $\hat{0}$  is the zero matrix;  $\hat{I}$  is the unit matrix. The tilde denotes the corresponding reflection and transmission matrices of the reversely traveling light. Then, to calculate the reflection or transmission of the whole system, we again apply the difference matrix equations (5), but now  $\hat{R}_j, \hat{T}_j, \hat{R}_{j-1}, \hat{T}_{j-1}$  are the reflection and transmission matrices for the system, consisting of  $j$ - and  $(j-1)$ - sublayers, respectively; and  $\hat{r}_j, \hat{t}_j$  are the reflection and transmission matrices for the  $j$ -th sublayer (with the thickness,  $d=\Lambda$ ).

Thus, the problem is reduced to the calculation of the reflection and transmission matrices for a homogeneous gyrotropic layer. The analytic solution of this problem is known [18]. We proceed from the following equations for a homogeneous isotropic gyrotropic crystal:

$$\vec{D} = \varepsilon \vec{E} - \frac{\bar{\gamma}}{c} \frac{\partial \vec{H}}{\partial t}, \quad \vec{B} = \mu \vec{H} + \frac{\bar{\gamma}}{c} \frac{\partial \vec{E}}{\partial t}. \quad (6)$$

Now we pass to the eigen polarizations (EPs) and eigen values (EVs) of the amplitude. It is known, EPs are the two polarizations of the incident wave that do not change when passing through the system; and the eigen values are the reflection and transmission amplitude coefficients for the incident light, corresponding the respective EPs [16-17 and 19].

The EPs and EVs deliver much information about the interaction of light with the system; therefore, their calculation is important for every optical system. It follows from the definition of EPs that they must be connected with the polarizations of the internal waves (the eigen modes, EMs) aroused in the medium (they mainly coincide with the polarizations of the EMs).

Denoting the ratio of the field complex amplitude components at the entrance of the system by  $\chi_i$  ( $\chi_i = E_i^s / E_i^p$ ), and those at the exit of the system by  $\chi_t$  ( $\chi_t = E_t^s / E_t^p$ ) and taking into

account that  $\begin{bmatrix} E_t^p \\ E_t^s \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} E_i^p \\ E_i^s \end{bmatrix}$ , we get their connection:

$$\chi_t = (T_{22}\chi_i + T_{21}) / (T_{12}\chi_i + T_{11}). \quad (7)$$

The function,  $\chi_t = f(\chi_i)$ , is called *polarization transfer function* [19] and it carries information about the transformation of the polarization ellipse, when the light passes through the system. Every optical system has two EPs obtained from the definition of the EPs:  $\chi_i = \chi_t$ . Hence, according to (4), we have for the  $\chi_1$  and  $\chi_2$ :

$$\chi_{1,2} = \frac{T_{22} - T_{11} \pm \sqrt{(T_{22} - T_{11})^2 + 4T_{12}T_{21}}}{2T_{12}}. \quad (8)$$

### 3. The results and discussion

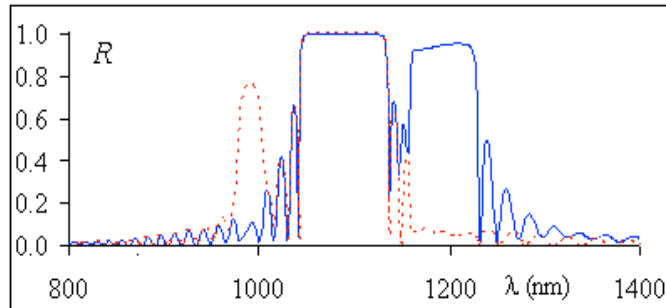
The optical properties of CPM, in the presence of one longitudinal ultrasound wave, are studied in detail in the works, [10, 13, and 14]. Therefore, we directly proceed to the study the effects caused by the presence of the two waves. The calculations are carried out for the CPM with the parameters:  $\varepsilon=2.5$ ,  $\mu=1.0$ ,  $\bar{\gamma}=0.1$ ; the layer thickness is:  $d=20 \mu\text{m}$ ;  $\alpha = 30^\circ$ .

For a clear conception of the effects when dealing with the presence of the two ultrasound waves, we consider the case of the minimum influence of the dielectric borders, that is, we assume that  $n_s = \sqrt{\varepsilon_s}$ , where  $n_s$  is the refraction index of the medium, bordering the considered layer on both sides. Then, in the first step, we consider that  $\varepsilon$ ,  $\mu$  and  $\bar{\gamma}$  do not depend on the frequency, and that their imaginary parts are very small and they do not depend on the frequency either; i.e. we do not consider the effects of optical dispersion and absorption.

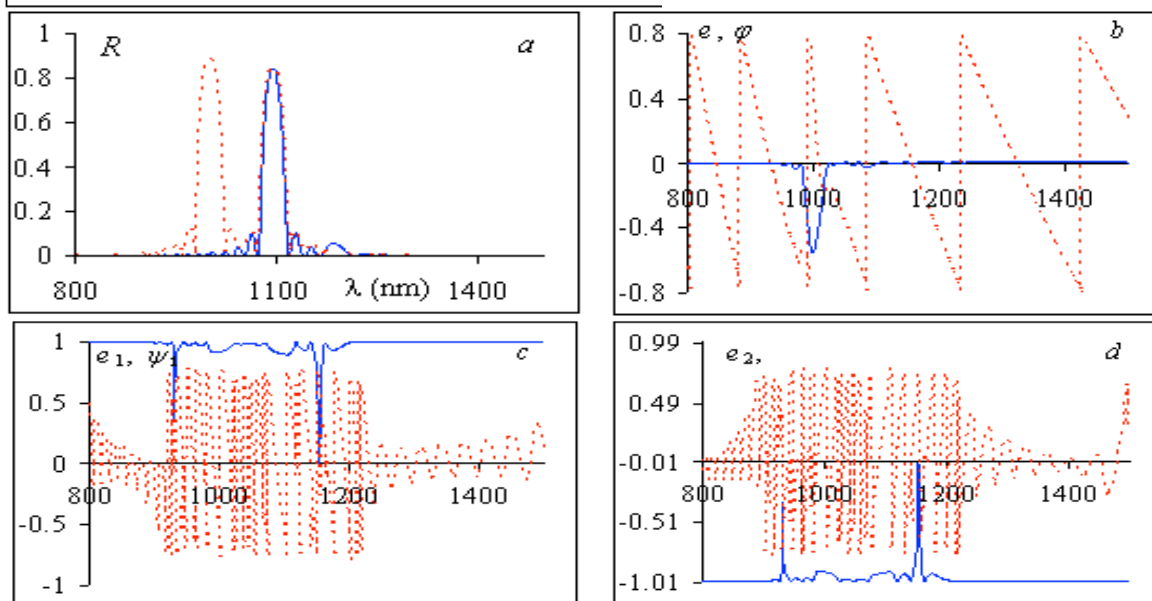
In figure 1, it is presented the reflection spectra in the presence of only one longitudinal wave. As it is seen in this picture, for the given parameters of the problem, in the first order of reflection, three PBGs are formed; one gap is not selective regarding to the polarization of the incident light,

and the others are selective (the shortwave gap is selective with regard to the polarization and is expressed weakly).

It is to be noted that the PBGs of the two types (the selective with respect to the polarization of the incident light and the non-selective one) are observed in periodical chiral PCs of very different types [3, 9, 20-22].



**Figure 1.** The dependence of the reflection coefficient,  $R$ , on the wavelength in the presence of only one ultrasound wave. The incident light has the left (the solid curve) and the right (the dashed curve) circular polarization. The problem parameters are:  $\Delta\epsilon=0.5$ ,  $\Delta\mu=0.0$ ,  $\Delta\bar{\gamma}=0.2$  and  $\Lambda=400$  nm.

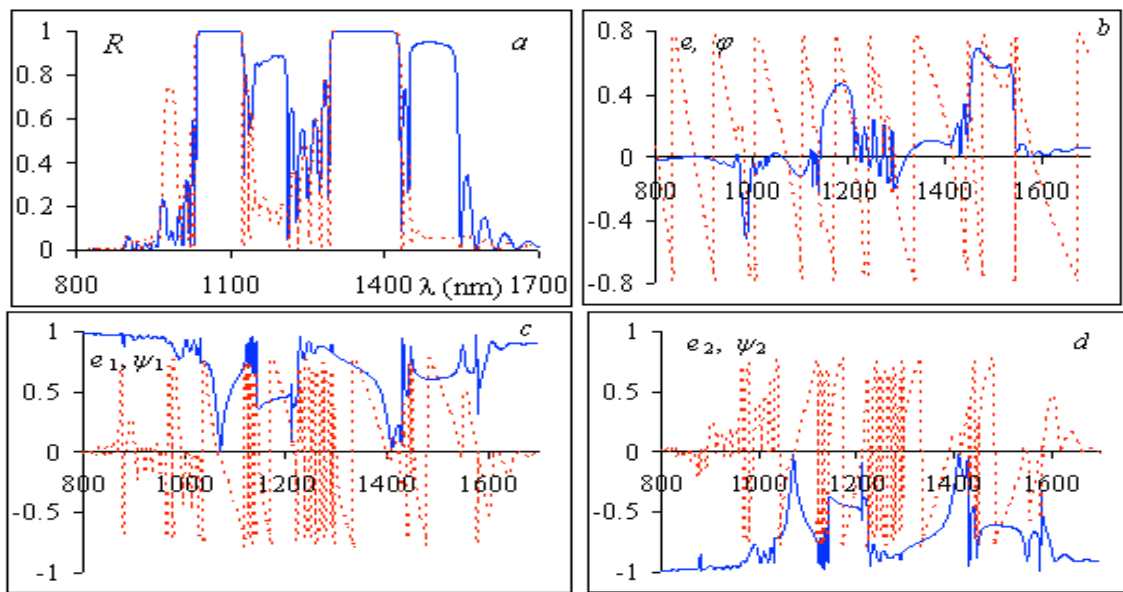


**Figure 2.** The reflection spectra (a); the spectra of: the polarization plane rotation (the dashed line) and the polarization ellipticity (the solid line) (b) and the spectra of the azimuth (the dashed line) and the ellipticity (the solid line) of the first and second EPs (c and d, respectively). The problem parameters are the following:  $\Lambda_1=\Lambda_2=0.4$   $\mu\text{m}$ ;  $\Delta\epsilon_1=0.5$ ,  $\Delta\epsilon_2=0.4$ ;  $\Delta\mu_1=\Delta\mu_2=0.0$ ;  $\Delta\bar{\gamma}_1=0.2$ ,  $\Delta\bar{\gamma}_2=0.15$ . (b)  $\Lambda_1=0.4$   $\mu\text{m}$ ,  $\Lambda_2=0.5$   $\mu\text{m}$ ;  $\Delta\epsilon_1=\Delta\epsilon_2=0.5$ ;  $\Delta\mu_1=\Delta\mu_2=0.0$ ;  $\Delta\bar{\gamma}_1=\Delta\bar{\gamma}_2=0.2$ . The other parameters are the same as in figure 1.

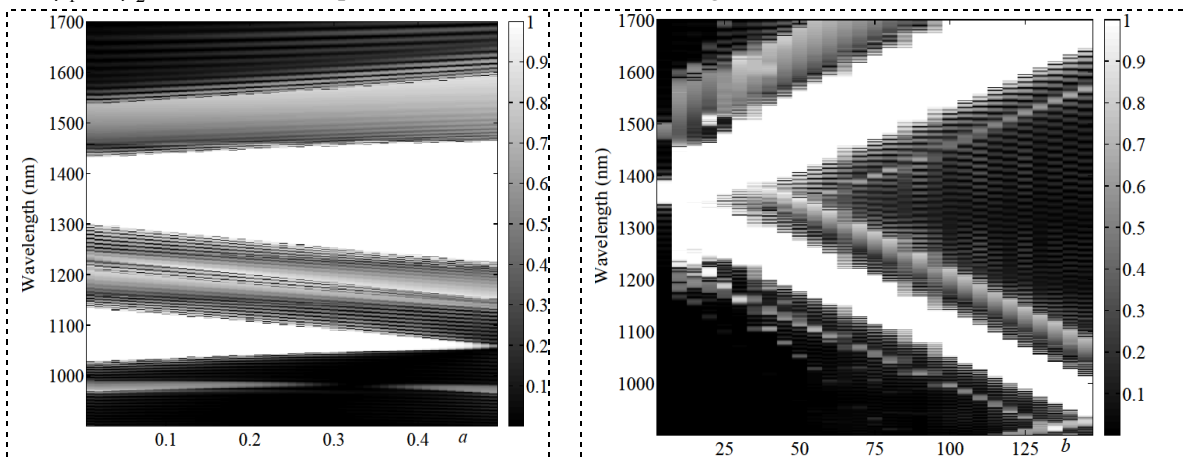
In figure 2: the reflection spectra (a); the polarization plane rotation and the polarization ellipticity spectra (b); the spectra of the azimuth and the ellipticity of the first and second EPs (c and d, respectively) for the oblique light incidence are presented in the presence of the two counter-propagating longitudinal ultrasound waves.

In figure 2 a, the incident light has left (the solid line) and right (the dashed line) circular polarizations; and in figure 2 b, it has linear polarization. As it is seen in figure 2 b, the polarization ellipticity practically equals to zero everywhere, except the PBG frequency ranges, which are selective with respect to the polarization of the incident light. Then, as it is seen from figure 2 c, d, the system EPs are non-orthogonal and quasi-circular. Here it is supposed that the wavelength of the ultrasound waves coincide and they differ only by their modulation depth.

In figure 3 there are the same spectra as in figure 2, but are presented for the presence of the two counter-propagating longitudinal ultrasound waves, whose modulation depths coincide and the wavelengths differ. In figure 3 a, the incident light has left (the solid line) and right (the dashed line) circular polarizations, too.



**Figure 3.** The same spectra as in figure 2, in the presence of two counter-propagating longitudinal ultrasound waves, whose modulation depths coincide and the wavelengths differ. The problem parameters are the following:  $\Lambda_1=0.4 \mu\text{m}$ ,  $\Lambda_2=0.5 \mu\text{m}$ ;  $\Delta\epsilon_1=\Delta\epsilon_2=0.5$ ,  $\Delta\mu_1=\Delta\mu_2=0.0$ ;  $\Delta\bar{\gamma}_1=\Delta\bar{\gamma}_2=0.2$ . The other parameters are the same as in figure 1.



**Figure 4.** The evolution of the reflection spectra, when the parameter,  $a$ , is changing. The problem parameters are as follows:  $\Delta\mu=0.0$ ;  $\Lambda_1=500 \text{ nm}$ ,  $\Lambda_2=400 \text{ nm}$ ;  $d=50000 \text{ nm}$ . The other parameters are the same as in figure 1.

**Figure 5.** The evolution of the reflection spectra, when the parameter,  $b$ , changes. The problem parameters are as follows:  $\Delta\mu=0.0$ ;  $\Delta\epsilon_1=0.55$ ,  $\Delta\epsilon_2=0.4$ ;  $\Delta\bar{\gamma}_1=0.15$ ,  $\Delta\bar{\gamma}_2=0.1$ ;  $d=50000 \text{ nm}$ . The other parameters are the same as in figure 1.

And now, presenting the parameters:  $\Delta\epsilon_1$ ,  $\Delta\epsilon_2$ ,  $\Delta\bar{\gamma}_1$  and  $\Delta\bar{\gamma}_2$ , in the form:  $\Delta\epsilon_{1,2} = 0.5 \pm a$  and  $\Delta\bar{\gamma}_{1,2} = 0.1 \pm a/10$ , we study the evolution of the reflection spectra with the alterations of the parameter,  $a$ , i.e. with the alteration of the modulation depths of the counter-propagating ultrasound waves. In figure 4, the evolution of the reflection spectra with alterations of the parameter,  $a$ , is presented. The incident light has linear polarization. The bright regions present the stronger reflection. As it is seen from the figure, the frequency locations and frequency widths of the PBGs are functions of the parameter,  $a$ .

Now we consider the situation when the wavelengths of the counter-propagating ultrasound waves are changed. We assume that these changes occur by the law:  $\Lambda_{1,2} = \Lambda_0 \pm b$ , with  $\Lambda_0 = 500$  nm. In figure 5, the evolution of the reflection spectra if the parameter,  $b$ , changes is presented. The incident light has linear polarization. As it is seen from the figure, the frequency locations, frequency widths and the number of the PBGs are functions of the parameter,  $b$ , too.

These figures show that by changing the parameters of the ultrasound waves one can tune the system reflection, that is, one can change the PBGs (the diffraction efficiency in each order; the frequency width and its location in these regions; the polarization peculiarities of these ranges).

#### 4. Conclusion

In this work we studied the peculiarities of the light diffraction in a gyrotropic layer in the presence of two counter-propagating longitudinal ultrasound waves. In this case, the diffraction pattern is much richer than in the presence of one ultrasound wave. New regions of diffraction reflection emerge within each diffraction order. The width of the regions of the diffraction reflection, their number, frequency location and frequency distance are determined by the parameters of the medium and those of the ultrasound waves; all they can be tuned.

Consequently, such systems can be used as: tunable polarization filters and mirrors; polarization mode converters; mode discriminators; multiplexers for circular polarized waves. They can also be used as sources of circular (or elliptic) polarization.

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