

Coherent Bremsstrahlung from planar channeled positrons

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Abstract. Theory of coherent bremsstrahlung from planar channeled positrons in a crystal is developed in the framework of the virtual photons method. It is shown that positron channeling results in splitting and appearance of additional structure of coherent peaks in axial coherent bremsstrahlung spectrum.

1. Introduction

It is well known that many physical processes, accompanying a passage of the high energy particles at small angles with respect to the crystal axes or planes, differ from those in an amorphous target. In fact, in this case coherent effects occur (see, e.g. [1, 2]). The coherent effect in the interaction of charged particle with a crystal arises due to periodical arrangement of the crystal atoms, when the interaction amplitudes of the particle with individual atoms are summing up in the phase. As a result of this summing, an interferential multiplier appears in the cross-section. It leads to appearance of coherent peaks when the transferred momentum coincides with one of the reciprocal lattice vectors. This happens at selected parameters (beam energy, angles of incidence into a crystal with respect to crystallographic axes or planes).

The channeling phenomenon appears when the fast charged particle passes through a crystal at small angle with respect to the crystal axis or plane, and the crystal thickness is rather large, so that the Born approximation or many-wave diffraction approach do not work. In this case, a charged particle interacts with averaged (continuous) potential of the crystal axis or plane and one can neglect the periodic part of the crystal potential.

The coherent scattering or channeling of relativistic charged particles in a crystal leads to emission of coherent bremsstrahlung (CB)[1, 2] or channeling radiation [3–5].

In fact, both channeling and coherent process can occur simultaneously – this is called a combined effect. For example, the coherent bremsstrahlung from relativistic axially channeled electrons differs from ordinary CB. In this case, the coherent peak in the energy spectrum of emitted photons is split [6]. In the case of combined effect, the CB theory should be modified.

In the present paper we investigate the combined effect in CB from planar channeled positrons. To this purpose, we consider the special geometrical conditions of the passage particle through the crystal. In our case, the positron moving between two planes of the crystal in the regime of planar channeling and simultaneously parallel to the crystallographic axis. Moreover, this axis is located between the planes and is parallel to them. These geometric conditions can be realized for example in a tungsten crystal if one select the (100) planes and $\langle 001 \rangle$ axis.

To study this combined effect we modified the method of virtual photons, taking into account the channelling of the radiating particles.



2. Cross-section of the photon scattering by a channeled positron

In the reference frame of the channeled positron passing through the crystal the electric field of an atom is perpendicular to its magnetic field. According to the Lorentz transformations, the fields are compressed in the longitudinal direction of the particle motion. Therefore, in a moving reference frame the electromagnetic field of the crystal is similar to the fields of a plane wave (if channeled positron relativistic factor $\gamma > 1$). Therefore, the electromagnetic field of the crystal can be replaced by a flux of virtual photons. Further, a process of bremsstrahlung can be described in terms of the virtual photons scattering by a channeled positron [1].

It is convenient to consider the scattering of photons by channeled positron in a coordinate system moving with the longitudinal velocity of channeled positron. In the moving reference frame one can use the non relativistic approach to the scattering process.

The cross-section of the photon scattering by a channeled positron was found in [7] :

$$d\sigma = \frac{\alpha^2 \hbar^2}{m^2 c^2} \frac{\hbar \omega_2}{\hbar \omega_1} \left| \sum_{pol} \sum_n \left\{ \frac{(\mathbf{e}_2 \mathbf{R}_{fn})(\mathbf{e}_1 \mathbf{A}_{ni})}{(E_i + \hbar \omega_1) - E_n} + \frac{(\mathbf{e}_1 \mathbf{A}_{fn})(\mathbf{e}_2 \mathbf{R}_{ni})}{E_i - E_n - \hbar \omega} \right\} \right|^2 \times \delta[(\mathbf{k}_{\parallel i} + \mathbf{k}_{\parallel}) - (\mathbf{k}_{\parallel f} + \mathbf{k}_{\parallel})] \delta[(E_i + \hbar \omega_1) - (E_f + \hbar \omega)] d^2 \mathbf{k}_{\parallel f} d\Omega d\hbar \omega_2. \quad (1)$$

Here \sum_{pol} is the sum over incident and scattered photon polarization vectors \mathbf{e}_1 and \mathbf{e}_2 and

$$M_{fi} = \sum_n \left\{ \frac{(\mathbf{e}_2 \mathbf{R}_{fn})(\mathbf{e}_1 \mathbf{A}_{ni})}{(E_i + \hbar \omega_1) - E_n} + \frac{(\mathbf{e}_1 \mathbf{A}_{fn})(\mathbf{e}_2 \mathbf{R}_{ni})}{E_i - E_n - \hbar \omega} \right\}, \quad (2)$$

is the process matrix element, with

$$\mathbf{A}_{kl} = -\frac{e}{mc} \int \Psi_k^*(\mathbf{r}) \hat{\mathbf{p}} \sqrt{\frac{2\pi\hbar}{\omega_1}} \exp[-i\mathbf{k}_1 \mathbf{r}] \Psi_l(\mathbf{r}) d\mathbf{r}, \quad (3)$$

being the matrix element of the photon absorption and

$$\mathbf{R}_{kl} = -\frac{e}{m} \int \Psi_k^*(\mathbf{r}) \hat{\mathbf{p}} \sqrt{\frac{2\pi\hbar}{\omega_2}} \exp[-i\mathbf{k}_2 \mathbf{r}] \Psi_l(\mathbf{r}) d\mathbf{r}, \quad (4)$$

being the matrix element of photon emission from the channeled positron.

In equations (13), \mathbf{k}_1 is the photon wave vector, \mathbf{e}_1 is the polarization vector and ω_1 is the frequency of the incident photon while \mathbf{k}_2 , \mathbf{e}_2 , ω_2 are the wave vector, polarization vector and frequency of the scattered photon, respectively, $\hat{\mathbf{p}}$ is a positron momentum operator and $\Psi_l(\mathbf{r})$ is a channeled positron wave function.

If the incident photon moves along the OZ axis then its wave vector is:

$$\mathbf{k} = \{0, 0, k\}, \quad k = \omega/c,$$

while the wave vector of scattered photon can be written as:

$$\mathbf{k}' = \{k' \sin \Theta \cos \Phi, k' \sin \Theta \sin \Phi, k' \cos \Theta\}, \quad k' = \omega'/c.$$

Here, Θ and Φ are the photon scattering angles. The scattering angles are given in cylindrical coordinates.

The channeled positron wave function has a form

$$\Psi_{i(f)}(\mathbf{r}) = \varphi_{i(f)}(x) \exp(i\mathbf{k}_{\parallel i(f)} \mathbf{r}_{\parallel}),$$

here the exponential $\exp(i\mathbf{k}_{\parallel}\mathbf{r}_{\parallel})$ describes the positron free motion along the crystal plane with \mathbf{k}_{\parallel} being its longitudinal wave vector while $\varphi(x)$ is a channeled positron transverse wave function. We chose the continuous planar potential for positrons in the form [4]:

$$V(x) = \frac{4V_0}{d^2} x^2.$$

The transverse wave functions corresponding to this potential are well known [3, 8].

The energies of the incident and scattered photons are related by the formula [7]:

$$\begin{aligned} \hbar\omega_2 = \hbar\omega_1 \cos \Theta - mc^2 + \\ + mc^2 \sqrt{\left(1 - \frac{(\hbar\omega_1)^2 + 4mc^2(\varepsilon_{\perp i} + \hbar\omega_1 - \varepsilon_{\perp f}) + \hbar\omega_1(\hbar\omega_1 \cos 2\Theta - 4mc^2 \sin \Theta)}{2m^2 c^4}\right)} \operatorname{ctg}^2 \Theta. \end{aligned} \quad (5)$$

3. The cross section of coherent bremsstrahlung from a planar channeled positron

When the particle is moving parallel to the axis of the crystal, it is enough to consider account its interaction with single crystallographic axis. The electrostatic potential of the crystal axis is a sum of the Coulomb potentials of the individual atoms forming the axis and is equal to:

$$V(\vec{r}) = \sum_{i=1}^N V_1(|\vec{r} - \vec{r}_i|), \quad V_1(r) = \frac{Ze}{r} \exp\left(-\frac{r}{R}\right). \quad (6)$$

Here V_1 is the potential of a single atom, R is the screening radius, N is the number of atoms in the axis, Z is atomic number.

The spectrum of virtual photons of crystal axis as it seen in the rest frame of the positron moving parallel to the axis is given by [9]:

$$\begin{aligned} n(\omega)d\omega = \frac{Z^2 e^2}{\pi^2} N \times \\ \left\{ \left[L - B \exp\left[-\left(\frac{\hbar\omega}{\gamma\hbar c} \bar{u}\right)^2\right] \right] + \frac{2\pi}{d} \sum_{g_n} B \exp\left[-\left(\frac{\hbar\omega}{\gamma\hbar c} \bar{u}\right)^2\right] \delta(k_1 - g_n) |S|^2 \right\} \frac{d\omega}{\omega}, \end{aligned} \quad (7)$$

$$L = \pi \ln \left[\frac{a\lambda^{-2}}{(\hbar\omega/\gamma\hbar c)^2 + R^{-2}} \right], \quad a \approx 1,$$

$$B(x) = \pi \left\{ -(1+x)e^x Ei(-x) - 1 \right\}, \quad x = \left[\left(\frac{\hbar\omega}{\gamma\hbar c} \right)^2 + R^{-2} \right] \bar{u}^2.$$

Here d is the lattice constant, ω is the frequency of virtual photon, $\lambda = mc/\hbar$ is the Compton wave length of electron, \bar{u}^2 is the mean-square displacement of the crystal atom from equilibrium position, $Ei(-x)$ is the exponential integral function, $g_n = 2\pi/d$ is the 1D reciprocal lattice vector, $|S|$ is the structure factor of a crystal axis, γ is the positron relativistic factor. To compare, the spectrum of virtual photons of a separate atom is:

$$n_{\text{am}}(\omega)d\omega = \frac{Z^2 e^2}{\pi^2} L \frac{d\omega}{\omega}. \quad (8)$$

If we neglect the terms of order of $(\hbar\omega/\hbar c)$ in the functions L and B in equations. (7,8) than we obtain the Ter-Mikaelian result for virtual photons spectrum [1].

The photons scattering cross-section by a channeled positron (1) should be transformed into the laboratory coordinate system. For this purpose we use the formula (5) and the Lorentz transformation for the photon energy (the Doppler shift):

$$\hbar\omega_r = \gamma \hbar\omega'(1 - \beta \cos \Theta'). \quad (8)$$

Here we denote $\beta = v/c$, and v is the longitudinal velocity of channeled positron in the laboratory coordinate system, $\hbar\omega_r$ is the energy of emitted photon.

In accordance with the virtual photons method, the cross-section of CB by channeled positron is:

$$\frac{\sigma^{CR}(\omega_r)}{d\omega_r} = \int_{\omega_{MIN}}^{\omega_{MAX}} \frac{\sigma(\omega_r, \omega)}{d\omega_r d\omega} n(\omega) d\omega, \quad (9)$$

and the cross-section of positron bremsstrahlung in amorphous target is

$$\frac{\sigma^{AM}(\omega_r)}{d\omega_r} = N \int_{\omega_{MIN}}^{\omega_{MAX}} \frac{\sigma_1(\omega_r, \omega)}{d\omega_r d\omega} n_{am}(\omega) d\omega, \quad (10)$$

here $\sigma(\omega_r, \omega)$ is cross-section of the virtual photon scattering by the channeled positron in the laboratory reference frame and $\sigma_1(\omega_r, \omega)$ is cross-section of the virtual photon scattering by the free positron (positron) in the laboratory reference frame. The limits of integration are determined by the condition $-1 \leq \cos \Theta \leq 1$:

$$\omega_{MIN} = \frac{\hbar\omega + \gamma(1 + \beta)(\varepsilon_{\perp f} - \varepsilon_{\perp i})}{\gamma(1 + \beta)\hbar}, \quad \omega_{MAX} = \frac{\hbar\omega + \gamma(1 - \beta)(\varepsilon_{\perp f} - \varepsilon_{\perp i})}{\gamma(1 - \beta)\hbar}.$$

Here $\varepsilon_{\perp i(f)}$ is the transverse energy of initial (final) state of channeled positron.

The obtained cross-section for CB from channeled positron should be averaged over all possible transition of channeled

$$\frac{\sigma^{CR}(\omega_r)}{d\omega_r} = \frac{1}{N} \sum_{fi} \frac{\sigma_{fi}^{CR}(\omega_r)}{d\omega_r}. \quad (11)$$

Here N is a number of transitions.

The results of calculation of the CB cross-section are shown in the figure 1: figure 1 a – contribution to the CB cross-section only from the channeled positrons (summed over all transitions of positron), figure 1 b – ordinary CB cross-section from free (not channeled) positrons in a crystal (calculated according to Ter-Mikaelian book [1]) and figure 1 c – total CB cross-section from free and channeled positrons. The target is (100) tungsten crystal, and positron emits CB interacting with the $\langle 001 \rangle$ axis.

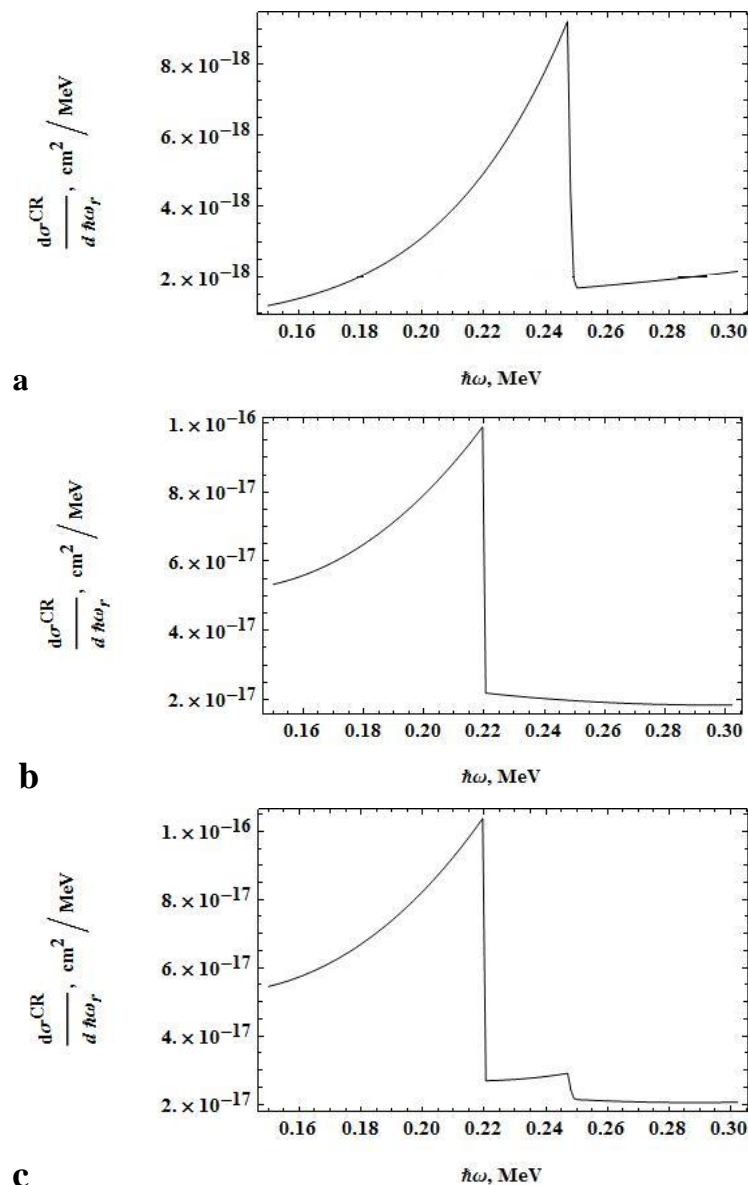


Figure 1. The cross-section of CB from positrons.

4. Conclusions

The results of calculation show that channeled positrons give small contribution to the total CB from positrons in a crystal. This correction is ten times less than ordinary radiation. As a result, one has appearance the additional structure of the coherent peak.

The combined effect in CB from relativistic axially channeled electrons was experimentally detected in [6].

In the present paper we use simple approximation for the continuous crystal potential. In a future we plan to use more exact transverse wave function (see, e.g. [10]).

The similar combined effect for coherent e^+e^- pair photoproduction in a field of the crystal axis was considered in [11]. In that case a positron was created in a bound (channeled) state with a crystal axis.

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