Influence of the Beam Divergence on Diffraction Radiation

V Shpakov[1] **and S B Dabagov**[1,2,3]

[1] RAS P.N. Lebedev Physical Institute, Moscow 119991, Russia
[2] INFN Laboratori Nazionali di Frascati, Frascati 00044, Italy
[3] NRNU MEPhI, Moscow 115409, Russia

E-mail: `shpakov@lnf.infn.it`

Abstract. Nowadays the achievements in accelerator physics allows us to operate with ever-growing energy of the particles beams. However, the increase of the beam energy requests new issues to be resolved. For instance, it becomes necessary to develop new techniques for beam diagnostics. One of the promising solutions for beam diagnostics is based on diffraction radiation (DR).

In this work similar to the methods of bunch length measurement based on transition radiation, the longitudinal size of a bunch is retrieved from the dependence of the DR intensity on the wavelength of radiation. Such dependence is expressed by so-called form-factor, which in turn depends on the beam parameters. Influence of the beam divergence on the form-factor of a beam has been examined in this work.

1. Introduction

One of the attractive DR features is the fact that it appears without direct contact, interaction, between the particle and the discontinuity, in general case, of the medium. Combined with the low radiation intensity with respect to the particle energy, it makes DR a great candidate for non-invasive beam diagnostics instrument.

DR from one particle passing through the slit system was considered in a number of papers [1–3]. However, in a real experiment we practically always deal with radiation by the bunch of particles [4, 5]. The intensity of radiation emitted by the beam depends on its coherence, on the beam form-factor. Such dependence can be used for the bunch length measurement [6]. The influence of the beam size on its form factor was previously studied both theoretically and experimentally [7, 8]. The present work is dedicated to theoretical investigation of the beam divergence influence on the beam form-factor that can be revealed from the DR spectrum.

2. DR theory

DR from one particle was studied in a number of papers [1, 2] and we do not consider this case in details. In this work we use the approach, which was applied, for example, in [9, 10]. Following those works we will consider one of the components of the DR field. Let us consider the coordinate system associated with the particle trajectory (figure 1). In such coordinate system the unit vector $\mathbf{e_z}$ is directed along the particle velocity $\mathbf{V}$, the unit vector $\mathbf{e_x}$ is parallel to the half-plane edge and $\mathbf{e_y}$ is perpendicular to the edge. In further calculations we consider only one of the components of the DR field, namely E_y. Omitting some known details the E_y component for DR from infinite half-plane can be presented in the form

$$E_{\mathrm{y}}^{DR} = \frac{e}{2\pi^2 V} \frac{e^{-h_1(\sqrt{k_x^2+\alpha^2}+ik_y)}}{(\sqrt{k_x^2+\alpha^2}+ik_y)}\,, \tag{1}$$

where V is the particle velocity, e is the particle charge, h_1 is the impact parameter half-plane, the coefficient $\alpha = \frac{k}{\gamma\beta}$, where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector, $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic parameter, $\beta = V/c$ is the normalized particle velocity, and c is the light velocity.

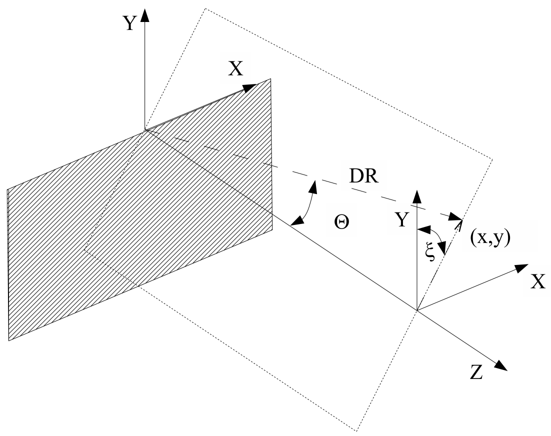

Figure 1. Scheme for DR from a infinite half-plane. The direction of the DR is defined by angles Θ and ξ. Θ is the angle between radiation vector and velocity vector, ξ is the angle between the plane of radiation and the $y0z$-plane.

As above mentioned, in this work we consider DR applied to the longitudinal beam diagnostics. For such diagnostics it is necessary to know the form-factor dependence on the wavelength of radiation. Influence of both longitudinal and transverse sizes of the beam on its form-factor is considered earlier [11, 12]. In present work we consider the beam divergence influence on its form-factor as well as the case when such influence is meaningful.

It is well known that intensity of radiation emitted by the beam (in general case any radiation, synchrotron radiation, transition radiation, etc...) depends on its coherence degree [13] and can be written as

$$I = I_0(N + N(N-1)F)\,, \tag{2}$$

where N is the number of particles in the beam, I_0 is the intensity of radiation from a single particle, and F is the beam form-factor. The coherence degree of the beam expressed by its form-factor, which, in turn, depends on the parameters of the beam and on the wavelength of radiation. Thus, having known the dependence $I(\lambda)$, we can obtain the beam parameters. Such technique is widely used today for longitudinal beam diagnostics of short bunches.

Full form-factor of the beam based on transition radiation [14] can be determined in the following way

$$F_{\text{beam}} = \left| \int_V S(\mathbf{r}) e^{-i\phi(\mathbf{r})} d\mathbf{r} \right|^2 , \tag{3}$$

where $S(\mathbf{r})$ is the normalized particles distribution inside of the beam, and $\phi(\mathbf{r})$ is the phase difference between the particles [2]. The integral is taken over the all volume occupied by the beam. However, as shown by equation (1), for DR the intensity emitted by one electron depends on the impact parameter and, as a consequence, can vary for different particles. For the far-field approximation we can consider DR as emitted by one point (figure 1). Thus, the strength of DR field can be expressed by

$$E(y) \propto E_0 e^{-\dfrac{2\pi y\sqrt{1+\gamma^2\beta^2\sin^2\Theta\sin^2\xi}}{\gamma\beta\lambda}} , \tag{4}$$

where E_0 is the constant amplitude, y is the impact parameter and λ is the wavelength of radiation. Both angles Θ and ξ define the direction to the observation point (figure 1). Hence, the form-factor of the beam with respect to DR can be presented in the form

$$F^{DR} = \left| \int_V S(\mathbf{r}) e^{-\dfrac{2\pi y\sqrt{1+\gamma^2\beta^2\sin^2\Theta\sin^2\xi}}{\gamma\beta\lambda}} e^{-i\phi(\mathbf{r})} d\mathbf{r} \right|^2 \tag{5}$$

3. Beam divergence from a form-factor for DR

The influence of the beam divergence on its form-factor were earlier considered for transition radiation [15]. In that work the influence of the beam divergence on the intensity of radiation were separated from that caused by the spatial beam distribution. In our approach below the beam divergence to be taken into account will be reveled from analysis of phase shifts.

Let assume the total particle distribution inside the beam to be defined by two independent distributions in longitudinal and transverse directions

$$S(\mathbf{r}) = S_{\text{long}}(z) S_{\text{tr}}(x, y) \tag{6}$$

In this case the phase difference for various beam particles is defined as follows (figure 1)

$$\phi(\mathbf{r}) = \frac{2\pi}{\lambda}(x \sin\Theta \sin\xi + \frac{z}{\beta}) \tag{7}$$

Taking into consideration equations (6) and (7), equation (5) for the beam form-factor becomes equal to

$$F^{DR} = \left| \int dxdydz S_{\text{long}}(z) S_{\text{tr}}(x, y) e^{-\dfrac{2\pi y\sqrt{1+\gamma^2\beta^2\sin^2\Theta\sin^2\xi}}{\gamma\beta\lambda}} e^{-\dfrac{2\pi i}{\lambda}(x\sin\Theta\sin\xi + \dfrac{z}{\beta})} \right|^2 \tag{8}$$

For relativistic beams the longitudinal size is usually much larger than its transverse size for the observer. For this reason, the beam form-factor is often defined as its longitudinal form-factor. Let us consider the beam with Gaussian distribution in longitudinal direction and neglect

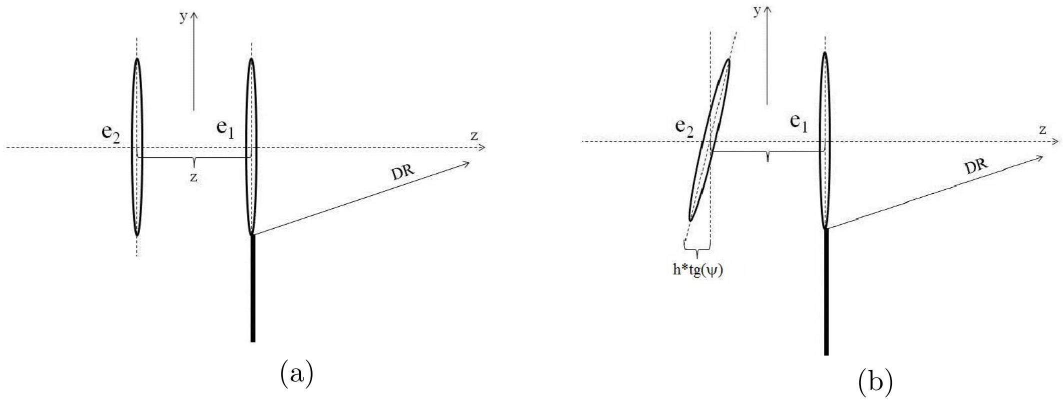

Figure 2. The additional phase difference appearance for particles with different incident angles. Here the impact parameters for both particles are equal to h. In figure b the particle e_2 has an incident angle ψ.

its transverse size. In order to take into account the beam divergence let us consider "z0y" plane as shown in figure 1.

For normally incident particles (figure 2(a)) the phase difference is defined by the distance between particles inside the bunch (see equation (7)). However, if one of the particles passes near the half-plane with non zero incident angle (figure 2(b)) additional phase difference appears. Here we assume that radiation is emitted when the particle field reaches the point of radiation. Thus, new phase difference can be defined as

$$\phi(\mathbf{r}, \psi) = \frac{2\pi}{\lambda\beta}(z + h\tan\psi), \tag{9}$$

where h is the impact parameter, which is considered to be constant for all particles inside the beam. Since in a new expression for the phase difference a new variable has appeared, we need to extend the expression for the beam form-factor (8). Let $S_{div}(\psi)$ be the angular distribution of the particles inside the beam. Thus, assuming that spatial and angular distributions are independent, another expression for the beam form-factor can be written

$$F^{DR}_{\text{div}} = \Bigg| \int dxdydzd\psi S_{\text{long}}(z) S_{\text{tr}}(x,y) S_{\text{div}}(\psi) \times$$

$$\times e^{-\dfrac{2\pi(y-h)\sqrt{1+\gamma^2\beta^2\sin^2\Theta\sin^2\xi}}{\gamma\beta\lambda}} e^{-\dfrac{2\pi i}{\lambda}(\dfrac{z}{\beta} + \dfrac{(y-h)\tan\psi}{\beta})} \Bigg|^2 \tag{10}$$

Let consider the beam angular distribution as Gaussian distribution, and ψ to be small enough, thus $\tan(\psi) \sim \psi$. Then, one can analytically derive the form-factor, in the form

$$F^{DR}_{\text{div}} \sim e^{-\frac{2\pi^2}{\lambda^2\beta^2}(\sigma_z^2 + h^2\sigma_{div}^2)}, \tag{11}$$

where σ_{z} is the standard particle deviation from the beam center in longitudinal direction, and σ_{div} is the standard particle deviation from the normal incidence. We would like to remind that in this approximation the transverse beam size has been neglected. For relativistic beam, $\beta \sim 1$, the divergence could play an essential role in the beam form-factor if the condition $\lambda \sim h\sigma_{\rm div}$ is satisfied. For $h \sim \gamma\lambda$ this condition can be transformed into $\sigma_{\rm div} \sim 1/\sqrt{2}\pi\gamma$. Let estimate the divergence contribution in a beam form-factor for the system of electron beam with $\gamma = 1000$ and divergence $\sigma_{\rm div} = 100$ μrad, passing at the half-plane, assuming that the wavelength of radiation is $\lambda = 800$ nm. The impact parameter thus equals to $h \approx \gamma\lambda = 1$ mm. The divergence contribution in a beam form-factor is expressed by the following exponent $e^{-2\pi^2\gamma^2\sigma^2_{\rm div}} \approx 0.82$. It is not negligible that means, the beam form-factor obtains an additional coefficient (less than unit) due to the beam divergence.

4. Conclusion

In presented work the role of the beam divergence in its form-factor for DR has been analyzed. It is shown how beam divergence could influence on the form-factor as well as when this influence may be significant. However, in the approximation we used the transverse beam size has been neglected. Such assumption is allowed only if the beam size is much smaller than the impact parameter. Moreover, in such approximation, even for relativistic beam its divergence has to be taken into account under the conditions $\sigma_{\rm div} \sim 1/\sqrt{2}\pi\gamma$. The influence of the transverse beam size on the beam form-factor usually can be neglected, that allows us to equate the longitudinal form-factor(which depends only on longitudinal size) with its total form-factor.

In this work the evaluation of the beam form-factor for radiations, such as synchrotron or transition, to the beam form-factor for DR taking into account the beam divergence has been considered. More accurate expression for DR beam form-factor without neglecting the transverse beam size will be separately determined in the future.

Acknowledgments

This work was supported by the Ministry of Education and Science of the Russian Federation in the frames of Competitiveness Growth Program of National Research Nuclear University MEPhI, Agreement 02.A03.21.0005.

References

[1] Ter-MikaelianM L 1972 *High-energy electromagnetic processes in condensed media* (New York: Wiley Interscience)
[2] Potylitsyn A P, Ryazanov M I, Strikhanov M N, Tishchenko A A 2010 *Diffraction radiation from relativistic particles* (Berlin: Springer)
[3] Castellano M 1997 *Nucl. Instr. Meth. A* **394** 275-280
[4] Castellano M, Chiadroni E and Cianchi A 2010 *Nucl. Instr. Meth. in Phys. Res. A* **614** 163168
[5] Karataev P et al. 2008 *Phys. Rev. ST Accel. Beams* **11** 032804
[6] Kube G, Backe H, Lauth W and Schope H 2003 *Proc. DIPAC 2003, Mainz, Germany* 40
[7] Doucas G et al. 2006 *Phys. Rev. Spec. Top. - Accel. Beams* **9** 092801
[8] Xiang D and Huang W-H 2007 *Phys. Rev. ST Accel. Beams* **10(1)** 012801
[9] Karataev P et al. 2005 *Nucl. Instr. Meth. in Phys. Res. B* **227** 158169
[10] Fiorito R B and Rule D W 2001 *Nucl. Instr. Meth. in Phys. Res. B* **173** 6782
[11] Potylitsyn A P 2005 *Nucl. Instr. Meth. in Phys. Res. B* **227** 191-197
[12] Sukhikh L G, Kube G, Potylitsyn A P and Schlott V 2011 *Proceedings of DIPAC-2011, Hamburg, Germany*
[13] Nodvick J S and Saxon D S 1954 *Phys. Rev.* **96** 180-184
[14] Schneider G et al. 1997 *Nucl. Instr. Meth. in Phys. Res. A***397** 221-231
[15] Shibata Y et al. 1994 *Phys. Rev. E* **50(2)** 1479-1484