

Coherent effects in the transition radiation of electron bunches on acoustic superlattices

V V Parazian and A A Saharian

Institute of Applied Problems in Physics, 25 Nersessian Street, 0014 Yerevan, Armenia

E-mail: vparazian@gmail.com, saharian@ysu.am

Abstract. We investigate the spectral-angular distribution of the forward transition radiation from an electron bunch interacting with an acoustic superlattice generated in a finite thickness plate. In the quasiclassical approximation, a formula for the radiation intensity is provided for a general distribution function of electrons in the bunch. We investigate coherence effects in the case of modulated electron distribution. The features of the radiation intensity are described in dependence of the ratio of the acoustic wavelength to the period of the modulation of the bunch. We demonstrate the appearance of the peaks in the radiation intensity for special values of this ratio.

1. Introduction

Transition radiation is produced when a relativistic particle traverses an inhomogeneous medium. Such radiation has a number of remarkable properties and at present it has found many important applications. In particular, transition radiation can be fruitfully exploited for energy detection or mass identification of high energy particles. The detectors based on the transition radiation have been used and are currently being used in a wide range of accelerator based experiments and in astroparticle and cosmic ray experiments.

The intensity of the transition radiation can be increased considerably by using the interference effects in periodic structures (for a review see [1]-[5]). The corresponding emission, called resonant transition radiation, results from constructive interferences between the waves emitted by each element of the periodic structure. In experiments, incident electrons cross stacks of thin foils and the transition radiation is formed at the interfaces between the material and vacuum. For the generation of high frequency transition radiation periodical structures with a small period are needed and the fabrication of this kind of radiator is a difficult task. In addition, irregularities in the spacing between the foils can rapidly destroy the constructive interference. In Ref. [6] it was suggested to generate the periodic radiator for the X-ray transition radiation by using acoustic waves. The parameters of this type of radiator are easily controlled by tuning the wavelength and the amplitude of the acoustic wave. The radiation from a charged particle for a semi-infinite laminated medium has been recently considered in [7]. The optical transition radiation in an ultrasonic superlattice excited in a finite thickness plate is considered in [8] under normal incidence of a relativistic electron and in [9] the results are generalized for the oblique incidence. It is shown that the acoustic waves generate new resonance peaks in the spectral and angular distributions. The heights and the location of these peaks can be controlled by choosing the parameters of the acoustic wave. In the papers given above the transition radiation from



a single particle is considered. From the point of view of the experimental observation it is an important step to investigate the radiation from particle beams. Here we consider the forward transition radiation generated by a monoenergetic electron bunch in a dielectric plate in presence of ultrasonic waves.

2. Electromagnetic field and the radiation intensity

Consider the transition radiation emerging in the forward direction when a monoenergetic bunch of N particles moves at constant velocity $\mathbf{v} = v\mathbf{n}_z$ along the z -axis, enters normally into a plate whose surfaces coincide with planes $z = -l$ and $z = 0$. We assume that longitudinal ultrasonic vibrations are excited in the plate along the normal to its surface (along the axis z), that form a superlattice. The dielectric permittivity inside the plate we shall take in the form

$$\varepsilon(z) = \varepsilon_0 + \Delta\varepsilon \cos(k_s z + \omega_s t + \varphi), \quad (1)$$

for $-l \leq z \leq 0$. In (1), ω_s , k_s are the cyclic frequency and the wave number of the ultrasound, φ is the initial phase. Under the condition $\nu_s l/v \ll 1$, with $\nu_s = \omega_s/(2\pi)$, during the transit time of the electron the dielectric constant in the superlattice is not notably changed. For relativistic electrons and for the plate thickness $l \lesssim 1$ cm this leads to the constraint $\nu_s \ll 10^{11}$ Hz. In the discussion below we shall assume that the plate is immersed in a homogeneous medium with dielectric permittivity ε_1 .

Here we are interested in the radiation with frequencies ω satisfying the condition $\omega \gg k_s c$. The presence of small parameter $k_s c/\omega$ allows one to use the quasi-classical approximation for the evaluation of the radiation field. The current density will be taken in the form

$$\mathbf{j} = ev \sum_{j=1}^N \delta(x - X_j) \delta(y - Y_j) \delta(z - Z_j(t)) \mathbf{n}_z. \quad (2)$$

In the Lorentz gauge, the vector potential of the electromagnetic field for j -th particle can be written as $\mathbf{A}_j = A^{(j)} \mathbf{n}_z$. This condition determines the radiation polarization. The magnetic field intensity is perpendicular to the plane containing $\mathbf{n}_z$ and the photon wave vector.

For the j -th particle in the bunch one has $Z_j(t) = -l + v(t - t_0^{(j)})$. Assuming that $|\Delta\varepsilon| \ll 1$, the Fourier transform of the vector-potential in the region $z > 0$ is presented as

$$\begin{aligned} A(\mathbf{k}_\perp, \omega, z) &= \frac{ie e^{ik_3^{(1)} z}}{2\pi^2 c k_3^{(1)}} \sum_{j=1}^N e^{-i\omega Z_j/v - ik_x X_j - ik_y Y_j} \left[\frac{e^{i\phi^{(j)} - il(\omega/v - k_3^{(1)})} - 1}{2i(\omega/v - k_3^{(1)})} + e^{ia_1 \sin \varphi^{(j)}} \sqrt{\frac{k_3^{(1)} \varepsilon_1}{k_3^{(0)} \varepsilon_0}} \right. \\ &\quad \times \left. \sum_{m=-\infty}^{+\infty} J_m(a_1) e^{-im\varphi^{(j)} - il(\omega/v - k_{3m}^{(0)})/2} \frac{\sin(l(\omega/v - k_{3m}^{(0)})/2)}{\omega/v - k_{3m}^{(0)}} \right], \end{aligned} \quad (3)$$

where and in what follows $\mathbf{k}_\perp = (k_1, k_2)$, $Z_j = Z_j(0)$, $J_m(x)$ is the Bessel function of the first kind, $k_{3m}^{(0)} = k_3^{(0)} + mk_s$, and

$$\begin{aligned} k_3^{(i)} &= \sqrt{\omega^2 \varepsilon_i / c^2 - k_\perp^2}, \quad i = 0, 1, \quad \varphi^{(j)} = \varphi - (Z_j + l)\omega_s/v, \\ \phi^{(j)} &= (k_3^{(0)} - k_3^{(1)})l + 2a_1 \sin(k_s l/2) \cos(\varphi^{(j)} - k_s l/2), \quad a_1 = \omega^2 \Delta\varepsilon / (2c^2 k_s k_3^{(0)}). \end{aligned} \quad (4)$$

The spectral-angular density of the radiation intensity during the electron transit time in the forward direction ($z \gg l$), averaged over the phase φ , is given by the formula

$$I_N(\omega, \theta, \phi) = 2\pi \varepsilon_1^{3/2} \frac{\omega^4}{c^3} \sin^3 \theta \cos^2 \theta \int_0^{2\pi} d\varphi |A(\mathbf{k}_\perp, \omega, z)|^2, \quad (5)$$

where θ is the angle between the z -axis and the wave vector of the radiation,

$$k_x = (\omega/c)\sqrt{\varepsilon_1} \sin \theta \cos \phi, \quad k_y = (\omega/c)\sqrt{\varepsilon_1} \sin \theta \sin \phi, \quad (6)$$

with ϕ being the azimuthal angle for the radiation direction. By taking into account the expression (3), we get

$$\begin{aligned} I_N(\omega, \theta, \phi) = & \frac{e^2 \beta_1^2 \sin^3 \theta}{2\pi^2 c \sqrt{\varepsilon_1}} \sum_{j,j'=1}^N e^{-2iq_{jj'}\omega/\omega_s - ik_x X_{jj'} - ik_y Y_{jj'}} \left\{ U \sum_{m,m'=-\infty}^{+\infty} B_m(\theta, \omega) \right. \\ & \times \left[U B_{m'}(\theta, \omega) J_{m-m'}(2a_1 \sin q_{jj'}) e^{i(m'-m)(k_s l - \pi)/2} e^{i(m'+m)q_{jj'}} \right. \\ & + \frac{J_{m-m'}(a_1)}{1 - \beta_1 \cos \theta} J_{m'}(b_1) e^{2im'q_{jj'}} \sin[(k_3^{(0)} - \omega/v)l/2 + (m' - m)k_s l/2 + m'\pi/2] \\ & \left. - U \sum_{m=-\infty}^{+\infty} \frac{W_m B_m^2(\theta, \omega)}{1 - \beta_1 \cos \theta} + \frac{1 + J_0(2b_1 \sin q_{jj'}) - 2 \cos[(k_3^{(0)} - \omega/v)l] J_0(b_1)}{4(1 - \beta_1 \cos \theta)^2} \right\}, \quad (7) \end{aligned}$$

where $\beta_i = v\sqrt{\varepsilon_i}/c$, $U_{jj'} = U_j - U_{j'}$, $U = X, Y$, $q_{jj'} = (Z_j - Z_{j'})\omega_s/2v$, $b_1 = 2a_1 \sin(k_s l/2)$, and

$$B_m(\theta, \omega) = J_m(a_1) \frac{\sin(l\omega W_m/2v)}{W_m}, \quad W_m = W + mv \frac{k_s}{\omega}, \quad W = 1 - \beta_1 \sqrt{\frac{\varepsilon_0}{\varepsilon_1} - \sin^2 \theta}. \quad (8)$$

Now one has $k_3^{(0)} = (\omega/c)\sqrt{\varepsilon_0 - \varepsilon_1 \sin^2 \theta}$,

$$a_1 = \frac{\omega \Delta \varepsilon}{2ck_s \sqrt{\varepsilon_0 - \varepsilon_1 \sin^2 \theta}}, \quad U = \left(\frac{\varepsilon_1 \cos \theta / \varepsilon_0}{\sqrt{\varepsilon_0 / \varepsilon_1 - \sin^2 \theta}} \right)^{1/2}. \quad (9)$$

For $N = 1$ the expression (7) reduces to the one derived in [8].

3. Averaging over the coordinates of particles

For the radiation intensity averaged over the coordinates of particles in the bunch we get

$$I_N(\omega, \theta, \phi) = N I_1(\omega, \theta) + N(N-1) I_c(\omega, \theta, \phi), \quad (10)$$

where the contribution of the coherent effects is given by the second term on the right-hand side with

$$I_c(\omega, \theta, \phi) = \frac{1}{2} \int d^3 R_j \int d^3 R_{j'} I_{jj'}(\omega, \theta, \phi) f(R_j) f(R_{j'}), \quad j' \neq j. \quad (11)$$

(Note that the formula (10) is valid for various types of radiation processes, including the transition radiation, the diffraction radiation and the synchrotron radiation (see [10]).) We assume that the distribution function is normalized in accordance with $\int d^3 R_j f(R_j) = 1$. In (11), $I_{jj'}(\omega, \theta)$ is given by the expression (7) if we remove the summation sign and $R_j = (X_j, Y_j, Z_j)$. Assuming that $f(R_j) = f_x(X_j) f_y(Y_j) f_z(Z_j)$, with the functions $f_u(U_j)$ separately normalized to 1, and introducing the notation $F_u(\tau) = \left| \int_{-\infty}^{+\infty} ds f_u(s) e^{i\tau s} \right|^2$, with $u = x, y, z$, we find

$$\begin{aligned} I_c(\omega, \theta, \phi) = & \frac{e^2 \beta_1^2 \sin^3 \theta}{2\pi^2 c \sqrt{\varepsilon_1}} F_x(k_x) F_y(k_y) \left\{ 2U \sum_{m,m'=-\infty}^{+\infty} B_m(\theta, \omega) \left[U B_{m'}(\theta, \omega) F_{mm'}(\omega, \theta) \right. \right. \\ & \times \cos \left(lk_s \frac{m - m'}{2} \right) - \frac{J_{m'}(a_1) J_{m'+m}(b_1)}{1 - \beta_1 \cos \theta} \sin \left(\frac{l\omega}{2v} W_{m'} + \pi \frac{m' - m}{2} \right) F_{m+m'}(\omega) \\ & \left. - U \sum_{m=-\infty}^{+\infty} \frac{W_m B_m^2(\theta, \omega)}{1 - \beta_1 \cos \theta} F_z(\omega/v) + \frac{[1 - 2 \cos(l\omega W/v) J_0(b_1)] F_z(\omega/v) + 2F(\omega, \theta)}{4(1 - \beta_1 \cos \theta)^2} \right\}. \quad (12) \end{aligned}$$

In (12) the following notations are introduced:

$$\begin{aligned} F_{mm'}(\omega, \theta) &= \int_0^\infty d\tau G(\tau) J_{m'-m} \left(2a_1 \sin \left(\frac{\tau \omega_s}{2v} \right) \right) \cos \left(\left(\omega_s \frac{m' + m}{2} - \omega \right) \frac{\tau}{v} - \pi \frac{m - m'}{2} \right), \\ G(\tau) &= \int_{-\infty}^{+\infty} du f_z(u - \tau) f_z(u), \quad F_m(\omega) = \int_0^\infty d\tau G(\tau) \cos((2m\omega_s - \omega) \tau / v), \\ F(\omega, \theta) &= \int_0^\infty d\tau G(\tau) \cos(\omega \tau / v) J_0(2b_1 \sin(\tau \omega_s / (2v))). \end{aligned} \quad (13)$$

Note that we can write $F_z(u)$ in terms of the function $G(\tau)$: $F_z(u) = 2 \int_0^\infty d\tau G(\tau) \cos(u\tau)$.

Note that the dominant contribution in the integrals for the form-factors in (13) comes from the region of the integration $\tau \lesssim L_z$, where L_z is the longitudinal size of the bunch. Now one has $\tau \omega_s / v \lesssim 2\pi (L_z / \lambda_s) (v_s / v)$, where λ_s and v_s are the wavelength and the velocity for the acoustic wave. If $L_z \lesssim \lambda_s$ one has $\tau \omega_s / v \ll 1$ for relativistic electrons. In this case the form-factor $F_{mm'}(\omega, \theta)$ is small for $m \neq m'$ and the main contribution in the corresponding part in (12) comes from the term $m = m'$ with $F_{mm}(\omega, \theta) \approx F_{m/2}(\omega)$. We also have $F(\omega, \theta) \approx F_z(\omega/v)/2$. If in addition, m is not too large, so $m\omega_s \ll \omega$, we see that $F_m(\omega) \approx F_z(\omega/v)/2$. In this case the summation over m' in the second term in the square brackets of (12) is done explicitly by using the addition theorem for the Bessel function and one finds

$$I_c(\omega, \theta, \phi) \approx F_x(k_x) F_y(k_y) F_z(\omega/v) I_1(\omega, \theta, \phi). \quad (14)$$

Hence, under the conditions $L_z \lesssim \lambda_s$ and $m\omega_s \ll \omega$ the influence of the bunch structure is separated in the form of the form-factor $F_x(k_x) F_y(k_y) F_z(\omega/v)$.

The general formula (12) is further simplified for large values of the plate thickness. Introducing the quantity $I_c^\infty(\omega, \theta) = \lim_{l \rightarrow \infty} I_c(\omega, \theta) / l$, from (12) we get

$$I_c^\infty(\omega, \theta, \phi) = F_x(k_x) F_y(k_y) \frac{e^2 \beta_1 \omega}{2\pi c^2} \frac{\varepsilon_1 \sin^3 \theta \cos \theta}{\varepsilon_0 \sqrt{\varepsilon_0 / \varepsilon_1 - \sin^2 \theta}} \sum_{m=-\infty}^{+\infty} C_m(\omega) J_m^2(a_1) \delta(W_m), \quad (15)$$

with $\delta(x)$ being the Dirac delta function and $C_m(\omega) = F_{mm}(\omega, \theta)$. This expression for the radiation intensity is valid under the condition $l \gg 2\pi v / \nu$, where $\nu = \omega / (2\pi)$. Note that one has $\nu \gg \nu_s$ and this condition does not contradict the condition $l \ll v / \nu_s$ (see the paragraph after formula (1)) we have assumed before. In (15), the term with $m = 0$, which we denote by $I_{c,0}^\infty(\omega, \theta, \phi)$, describes the radiation which propagates along the direction $\theta_0 = \arcsin(\sqrt{\varepsilon_0 / \varepsilon_1} \sin \theta_{Ch})$, $\theta_{Ch} = \arccos(1/\beta_0)$ is the Cherenkov angle in the medium with permittivity ε_0 . This term corresponds to the Cherenkov radiation in the plate which propagates in the region $z > 0$ after the refraction at $z = 0$. Integrating over θ we find

$$I_{c,0}^\infty(\omega, \phi) = \int_0^\pi d\theta I_{c,0}^\infty(\omega, \theta, \phi) = \frac{e^2 \omega}{\pi c^2} F_x(k_x) F_y(k_y) C_0(\omega) (1 - 1/\beta_0^2) J_0^2(a_1), \quad (16)$$

where now $a_1 = \omega v \Delta \varepsilon / (2c^2 k_s)$ and in the expressions (6) one has $\theta = \theta_0$.

Now we consider the terms with $m \neq 0$. For a given m , the frequency for the radiation propagating along the direction θ is given by the relation $\omega = m v k_s / W$, where $m > 0$ for $W > 0$ and $m < 0$ for $W < 0$. After the integration over ω , for $I_{c,m \neq 0}^\infty(\theta, \phi) = \int d\omega I_c^\infty(\omega, \theta, \phi)$ we find

$$I_{c,m \neq 0}^\infty(\theta, \phi) = \frac{e^2 \beta_1^3 k_s^2 \sin^3 \theta \cos \theta}{2\pi \varepsilon_0 \sqrt{\varepsilon_0 / \varepsilon_1 - \sin^2 \theta} |W|^3} \sum_{mW > 0} F_x(k_x) F_y(k_y) C_m(m v k_s / W) m^2 J_m^2(a_1), \quad (17)$$

where in the expressions (6) the substitution $\omega = m v k_s / W$ should be made.

4. Example of a microbunched beam

As an example of a microbunched beam we consider a beam with Gaussian distribution functions in x and y and with modulated Gaussian distribution in the longitudinal direction:

$$f_u(\tau) = \frac{e^{-\tau^2/(2\sigma_u^2)}}{\sqrt{2\pi}\sigma_u}, \quad u = x, y, \quad f_z(z) = \frac{e^{-z^2/(2\sigma_z^2)}}{\sqrt{2\pi}\sigma_z} \sum_{n=-\infty}^{\infty} d_n e^{ink_r z}, \quad (18)$$

where k_r is beam modulation wave number and $d_{-n} = d_n^*$, $d_0 = 1$. For these functions one has

$$F_u(k_u) = e^{-\sigma_u^2 k_u^2}, \quad u = x, y, \quad F_z(u) = \left| \sum_{n=-\infty}^{\infty} d_n e^{-\sigma_z^2 (nk_r + u)^2/2} \right|^2. \quad (19)$$

For the function $G(\tau)$ one gets the expression

$$G(\tau) = \frac{e^{-\tau^2/(4\sigma_z^2)}}{2\sqrt{\pi}\sigma_z} h(k_r\sigma_z, \tau/(2\sigma_z)), \quad h(x, u) = \sum_{n, n'=-\infty}^{\infty} d_n d_{n'} e^{-(n+n')^2 x^2/4} \cos[(n-n')xu]. \quad (20)$$

With the use of (20), the expression for the function $F_m(\omega)$ takes the form:

$$F_m(\omega) = \frac{1}{2} e^{-(2m\omega_s - \omega)^2 \frac{\sigma_z^2}{v^2}} \sum_{n, n'=-\infty}^{\infty} d_n d_{n'} e^{-(n^2 + n'^2) k_r^2 \frac{\sigma_z^2}{2}} \cos \left[(n-n') (2m\omega_s - \omega) k_r \frac{\sigma_z^2}{v} \right]. \quad (21)$$

In the simple case $d_n = 0$, $n = \pm 2, \pm 3, \dots$, defining $d_1 = d_b e^{i\beta}$, the distribution function is given by

$$f_z(z) = \frac{e^{-z^2/(2\sigma_z^2)}}{\sqrt{2\pi}\sigma_z} [1 + 2d_b \cos(k_r z + \beta)]. \quad (22)$$

For the function $h(x, u)$ we find

$$h(x, u) = 1 + 2d_b^2 \cos(2xu) + 4d_b e^{-x^2/4} \cos \beta \cos(xu) + 2d_b^2 e^{-x^2} \cos(2\beta). \quad (23)$$

For the numerical example we consider large values of the plate thickness. For the angular density of the number of quanta radiated at a given frequency $\omega_m = mvk_s/W$, per unit trajectory of the particle, one has

$$\mathcal{N}_{N,m}(\theta, \phi) = N\mathcal{N}_{1,m}(\theta, \phi) + N(N-1)\mathcal{N}_{c,m}(\theta, \phi). \quad (24)$$

For large values of the plate thickness, for the coherent part $\mathcal{N}_{c,m}^\infty(\theta, \phi) = \lim_{l \rightarrow \infty} \mathcal{N}_{c,m}(\theta, \phi)/l$, from (17) one gets

$$\mathcal{N}_{c,m}^\infty(\theta, \phi) = \frac{e^2}{\hbar c} \frac{\varepsilon_1 \beta_1^2 k_s \sin^3 \theta \cos \theta}{2\pi \varepsilon_0 \sqrt{\varepsilon_0 - \varepsilon_1} \sin^2 \theta W^2} F_x(k_x) F_y(k_y) C_m(\omega_m) |m| J_m^2(a_1). \quad (25)$$

In figure 1 we have plotted the quantity (25) for $m = -1$ as a function of $\sigma_z k_s$ and θ for the electron energy 17.5 MeV and for the longitudinal distribution function (22) with $d_b = 0.5$, $\beta = 0$, $\sigma_z = 3 \times 10^{-2}$ cm. We assumed that the plate is made from fused quartz and $\varepsilon_1 = 1$. For the velocity of the acoustic wave one has $v_s = 5.6 \times 10^5$ cm/s and we have taken $\Delta\varepsilon = 0.02$. For the left plot $\sigma_z k_r = 5$ and for the right one $\sigma_z k_r = 10$. For the transverse part of the distribution function we have taken $F_x(k_x) F_y(k_y) = 1$. With increasing $\sigma_z k_s$ the separation between the peaks and the height of the right peak increase. In addition, the right peak exhibits microstructure. We have numerically checked that the contributions of the higher harmonics $m = -2, -3, \dots$ are small compared to the one for $m = -1$.

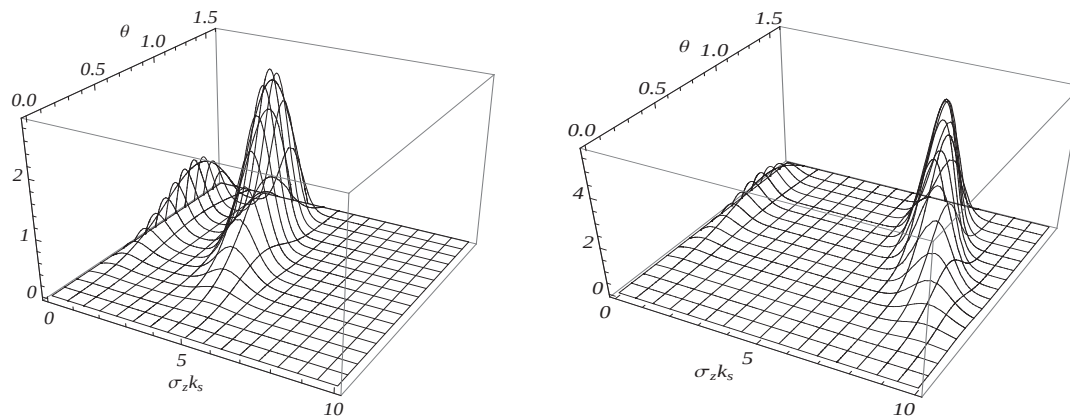


Figure 1. Coherent part in the number of the radiated quanta, $10^8 \mathcal{N}_{c,m}^\infty$ (see (25)) at a given harmonic corresponding to $m = -1$.

5. Conclusion

In the present paper we have considered the coherence effects in the transition radiation of a microbunched electron beam on an acoustic superlattice excited in a finite thickness dielectric plate. Assuming that the amplitude of the modulation for dielectric permittivity is small, the spectral-angular density of the radiation intensity in the forward direction is given by formula (7). After averaging over the coordinates of particles in the bunch, it is presented as (10), where for the contribution of coherent effects one has the expression (12). Under the conditions $L_z \lesssim \lambda_s$ and $m\omega_s \ll \omega$, the influence of the bunch structure on the radiation intensity is separated in the form of the form-factor (see (14)). For a thick plate, the general formula is further simplified to (15). In this formula, the $m = 0$ term corresponds to the Cherenkov radiation in the plate which propagates in the region $z > 0$ after the refraction at the plate boundary. As an application of the general expression for the transition radiation intensity, in section 4 we have considered a beam with Gaussian distribution functions in transverse directions and with modulated Gaussian distribution in the longitudinal direction. The spectral-angular distribution of the radiation intensity can be controlled by tuning the parameters of two periodic structures: the beam modulation wave number k_r and the acoustic wave number k_s .

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