Gamma Cherenkov-transition radiation of high energy electrons and methods for the measurement of the refractive index of MeV photons using total internal and external reflections (Invited talk)

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Abstract. It is given a review of the theoretical works showing that gamma ray Cherenkov-transition radiation (GCTR) is produced in some materials the refractive index of which in gamma region is greater than 1 according to the famous results of [1]. Since there are some published doubts [2] on the theoretical results of [1], and taking into account the difficulties of the experiment [1], in order to confirm or decline the results [1] it is proposed to study experimentally GCTR and an experimental method for the measurement of refractive index of MeV photons.

1. Introduction or formulation of the problem

In figure 1 it is shown qualitatively the behaviour of the dependence of the refractive index $n(\hbar\omega)$ of substances on photon energy $\hbar\omega$. As it is seen up to the X-ray region, excluding some narrow regions at anomaly dispersion frequencies, the refraction index is greater than 1, while when $n(\hbar\omega)$ >0.1 keV $n(\hbar\omega)$ <1.
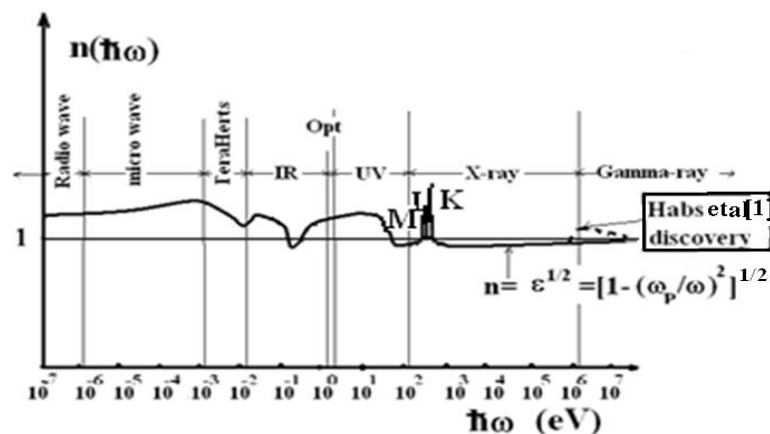

Figure 1. The dependence of n of substances upon $\hbar\omega$.

Usually, one presents the complex values of $\varepsilon(\omega)$ and $n(\omega)$ in the forms: $\varepsilon(\omega) = n^2(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) = 1 + \chi'(\omega) + i\chi''(\omega)$ and $n(\omega) = 1 + \delta(\omega) + i\Delta(\omega)$ where χ' and χ'' are the real and imaginary parts of the susceptibility, and $\chi'(\omega)$ and $\delta(\omega)$ are connected with the refraction, while $\chi''(\omega)$ and $\Delta(\omega)$ describe the absorption of photons with absorption length $L_{abs} = 1/\mu(\omega) = c/(\chi''(\omega)\omega)$ where μ is the linear absorption coefficient.

Now according to sensational experimental and theoretical results of [1] for silicon and gold $n(\hbar\omega) > 1$, or $\delta > 0$ for $\hbar\omega \geq 1$ MeV. This has many important consequences and applications (see [3–6]) for nuclear physics, gamma optics, etc. Gold lenses, similar to Snigirev X-ray lenses, have been prepared in order to focus gamma ray beams. However, no new experimental results and answer to [2] has been published. The difficult experiment [1] has been carried out at Institut Laue-Langevin on the facility GAMS5 measuring the nanoradian relative diffraction angle for three lines of Cl[36] target bombarded by thermal neutrons when a Si wedge is inserted between the two crystals of a double Bragg diffraction spectrometer. The measured relative diffraction angles allowed to determine $\delta = (+1.83\pm1.57)\times10^{-10}$, $(+1.48\pm0.13)\times10^{-9}$ and $(+1.11\pm0.30)\times10^{-9}$, for 786, 1165 and 1951 keV photons, respectively, while the δ measured for lower energies are negative according to the formula $n(\omega) = [1 - (\omega_P/\omega)^2]^{1/2}$.

The authors of [1] using the cross section of various processes, mainly of the QED nonlinear process of Delbruck scattering calculated with the help of a complicated non-perturbative QED theory obtained good agreement between the experimental and theoretical results on δ. However, A comment [2] on the work [1] has been published in which it has been shown that even using the Delbruck scattering cross sections given in [1] the contribution of Delbruck effect into δ is 10^{-6}–10^{-5} times less than it was estimated in [1]. Since there is no answer to [2] and no new experimental and theoretical results one can conclude that there is a problem in XXI century physics which it is better to solve experimentally.

On the other hand, it is well known that when a particle passes through a substance with $n(\omega) > 1$ it can produce Cherenkov radiation (CR) in radio-optical region (see [7, 8]) and X-ray region (XCR) near K-, L- edges (see [9,10]). Using the results of [1] on $n(\hbar\omega) > 1$ for photons with $\hbar\omega \geq 786$ keV in the work [11a] it has been developed the theory of GCTR produced in a plate of material using the more confident experimental data of [1] in the region 786–2000 keV. In [11b] the theory of GCTR produced in a plate of material using the theoretical data of [1] in a wider region 786-10000 keV has been given, while the problems of the formation length and GCTR produced at a single interface and in stack of plate have been considered in [11c]. Looking for other methods of manifestation of the fact that in gamma ray region the refractive index can be greater than 1 in [11d] it is considered a simpler and direct method for the measurement of $n(\hbar\omega)$ using total internal (TIR) and external refection (TER) of MeV photons. In this work it is given a review of the works [11].

2. Theory of GCTR based on Ginzburg-Frank-Garibian Theory of TR
2.2a. GCTR produced at an interface between two media and its formation length
Let us first consider the formation length in medium of GCTR. According to [12, 13] the formation length of XTR is a complex magnitude and is given by the expression

$$Z_{Med}(\theta) = \frac{2\pi V}{\omega(\gamma^{-2} + \theta^2 - \chi)} \,. \tag{1}$$

From (1) one obtains for the formation length, L_{Med}^{Form}

$$L_{Med}^{Form} = \left| Z_{Med} \right| = \frac{2\pi V}{\omega} \frac{1}{\sqrt{(\gamma^{-2} + \theta^2 - \chi')^2 + \chi''^2}} \,, \tag{2}$$

from which the formation length in vacuum ($\chi' = \chi'' = 0$) is equal to $L_{Vac}^{Form} = 2\pi V / \omega /(\gamma^{-2} + \theta^2)$ with a maximum at $\theta = 0$ equal to $L_{GCTRvac} = \lambda\gamma^2$ as for other types of radiation. As it follows from (8) when GCTR takes place and the condition $\gamma^{-2} + \theta^2 - \chi_1' = 0$ is satisfied $L_{Med}^{Form} = 2\pi L_{abs}$.

The spectral-angular distribution of the number of photons of GCTR produced at an interface of two media (Si-vacuum) is derived substituting the values of χ' and χ'' into the formulae (1.59) [13] for XTR. In small angle and $\gamma >> 1$ approximation one obtains

$$\frac{d^2 N_{GCTR}}{d(\hbar\omega)d\theta} = \frac{2\alpha\theta^3}{\pi\hbar\omega} \frac{(\chi'^2 + \chi''^2)}{(\theta^2 + \gamma^{-2})^2[(\theta^2 + \gamma^{-2} - \chi')^2 + \chi''^2]}. \tag{3}$$

2.2b. GCTR produced in a plate
Inserting the values of the complex χ into the XTR spectral-angular distribution (formula (2.21) of [13] or formula (3.27) of [9] for a plate one derives the following formula for the spectral-angular distribution of GCTR

$$\frac{d^2 N_{GCTR}}{d\hbar\omega d\theta} = \frac{2\alpha\theta^3}{\pi\hbar\omega} \frac{(\chi'^2 + \chi''^2)}{(\theta^2 + \gamma^{-2})^2[(\theta^2 + \gamma^{-2} - \chi')^2 + \chi''^2]} \times$$
$$\left\{ \left[1 - \exp\left(-\frac{L_{rad}}{2L_{abs}}\right)\right]^2 + 4\exp\left(-\frac{L_{rad}}{2L_{abs}}\right)\sin^2\frac{\hbar\omega L_{rad}(\theta^2 + \gamma^{-2} - \chi')}{4\hbar c} \right\}, \tag{4}$$

where L_{rad} is the thickness of the plate-radiator.

2.2c. GCTR produced in a stack of plates
The necessary expressions obtained with the help of corresponding formulae (3.15) of [13] as well as numerical results are given in [11c] and will not be discussed here. Before proceeding it is important to note the following: In contrast to XTR, due to the facts that for GCTR $L_{rad} < L_{abs} \approx$ a few cm, and the requirement that the particle velocity must be changed insignificantly, therefore, it is necessary to have $L_{rad} \approx 0.1 X_0$ (X_0 is the radiation length of the radiator material), GCTR produced in a stack of plates as well as at one interface have no practical values since the GCTR yield from a stack of plates can not exceed the GCTR yield from a plate and it is not reasonable to consider GCTR production at one interface because one can not replace the infinite medium of a single interface with a plate thinner than L_{abs}. Therefore, further it will be considered only GCTR produced in a plate with $L_{rad} \leq 1$ cm given by (4) which will be integrated (numerically, because analytically it is very complicated or impossible) over θ, $\hbar\omega$ and over both θ and $\hbar\omega$ to obtain $dN_{GCTR} / d(\hbar\omega)$, $dN_{GCTR} / d\theta$ and N_{GCTR}, respectively.

3. Numerical results on GCTR produced in a plate
The below given numerical results described in details in [11b], have been obtained for electron energy E = 20 GeV if there is no especial remark, and only for Si radiators with thickness $L_{rad} = 0.1 X_0 = 1$ cm using the values of $\chi' = 2\delta$ (from [1]) and $\chi'' = \mu c / \omega$ (from various tables for μ).

3.1. The spectral distribution of the number of GCTR photons
$dN_{GCTR} / d(\hbar\omega)$ is shown in figure 2 (curve "1"). The curve "2" of figure 2 shows the spectral distribution of the background bremsstrahlung calculated taking into account the longitudinal density or Ter-Mikaelian
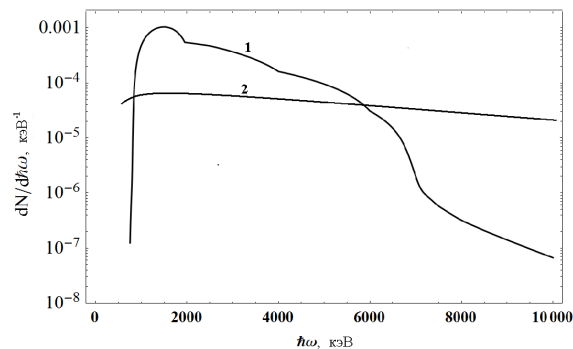
Figure 2. The spectral distributions of GCTR and bremsstrahlung.

effect (see [12]). As it follows from figure 1 GCTR intensity exceeds the background bremsstrahlung intensity in a wide interval from 800 keV up to 6000 keV with S/N>>2. The total number of GCTR and background photons obtained by numerical integration of the curve "1" and "2" of figure 2 over ħω are N_{GCTR}=1.34 and N_{Br} = 0.1.

3.2. The angular distribution of GCTR

$dN_{GCTR}/d\theta$ obtained integrating (4) over $\hbar\omega$ in the interval 800–10000 keV is shown in figure 3.
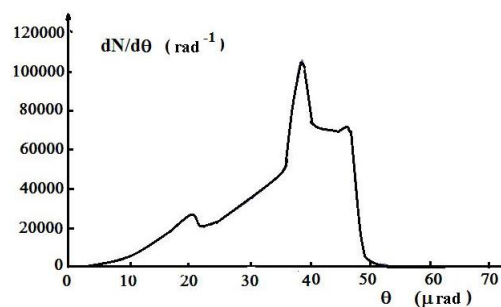

Figure 3. The angular distribution of GCTR.

From figure 3 it follows that GCTR is emitted mainly in the angular interval from $30\,\mu$ rad up to $50\,\mu$ rad.

3.3. The dependence of GCTR on the electron energy

The energy dependence of the total number of GCTR (solid curve) and bremsstrahlung (dashed curve) photons is shown in figure 4. As it seen in figure 4 the number of the GCTR photons as in case of XTR exceed the background and begins to increase sharply beginning from a "threshold" energy about 12 GeV. Then with the increase of E a "saturation" achieves with $N_{GCTR} \approx 3.4$. As it is has been shown still in [12, 14] the number of the XTR photons in transition radiation detectors (TRD) from a stack of plates without and with taking into account the absorption is equal to N_{XTR} ~11 and ~2 and exceeds the background bremsstrahlung only in a relatively narrow region $\Delta(\hbar\omega) \approx (2\text{–}30)$ keV, and this allows to construct TRD detectors, which found wide application for particle identification. Therefore one can use GCTR also for the same purpose.
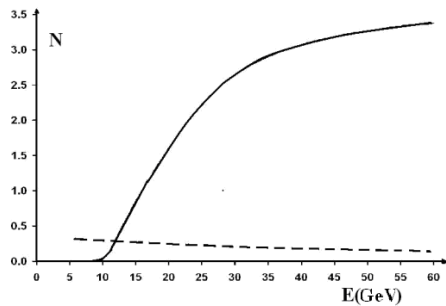
Figure 4. The dependence of the number of the GCTR (solid line) and background (dashed line) photons.

3.4. The dependence of the GCTR yield on the radiator thickness

Figure 5 shows how N_{GCTR} depends on L_{rad}. Around $L_{rad} = 1$ cm N_{GCTR} is proportional to L_{rad}, and the electron energy losses are still small. Therefore, it is reasonable to take $L_{rad} \sim (2\text{-}10)$ % X_0, and as the calculations (see [11]) show the use of a stack of plates instead of a plate has no advantage.
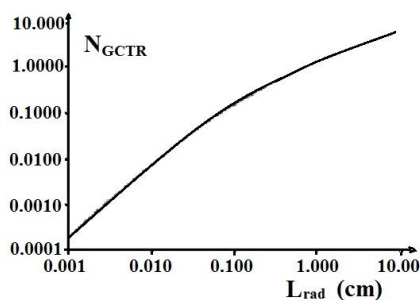

Figure 5. The dependence of N_{GCTR} on L_{rad}.

4. Experimental possibilities

One can use an experimental arrangement similar to those used at SLAC [15] and CERN SPS [16] and [17] for the study of various types of radiation of 1–200 GeV electrons. Without discussing the proposed arrangement let us note that the SLAC beam has the difficulty that it must contain 1 electron per pulse, while the SPS beam being much longer, allows to detect a few GCTR photons per pulse. The analysis of the spectra measured by 25 GeV electrons in $\hbar\omega = (1\text{–}10)$ MeV region did not reveal any exceeding over the calculated spectra of bremsstrahlung of various theories [15]. Probably, one cannot use this fact to decline the results of [1] may be because no measurements on Si have been performed.

5. Measurement of the reflection index of MeV photons using total internal and external reflection

It is well known that if $n(\omega) > 1$ (optical) it can take place TIR at the boundary medium-vacuum with $\theta_{cr}^{TIR} = \arcsin(1/n)$. Similarly, if $n(\omega) < 1$ (X-rays) it can take place TER at the boundary vacuum – medium with $\theta_{cr}^{TER} = (2\delta)^{1/2}$. If $\delta \sim 10^{-9}$, $\theta_{cr}^{TER} = 45$ µrad, which is not very small, µrad, as the angles in [1]. In many works of the past (see, [18]) it has been proposed, but not realized, to use TER for various purposes, in particular, for the "monochromatization" of MeV photon beams. However, such MeV experiments become realistic only recently connected with the advance of the method of inverse Compton scattering intense photon sources [19]. Suitable parameters are available at present at the facility HiGS of Duke university, which provides 10^8–10^{11} ph/s in a ~10 % BW, and soon at the ELI facility in Romania, which will provide ~10^{13} ph/s in a ~1 % BW.
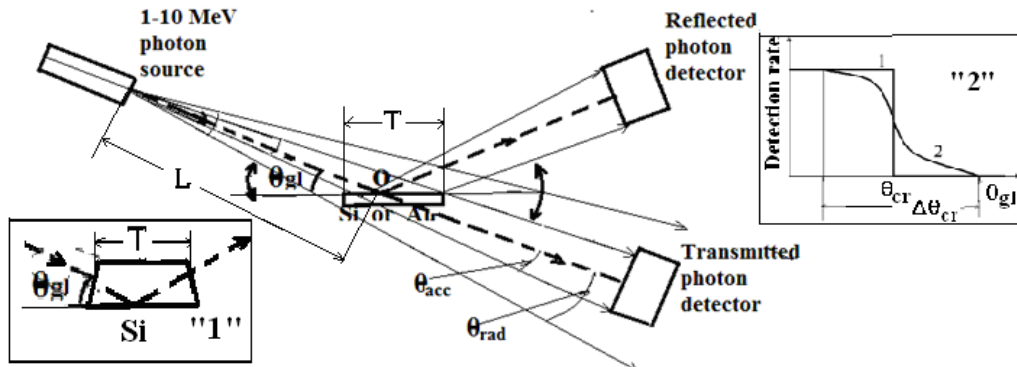
Figure 6. An arrangement for the measurement of $n(\omega)$ of MeV photons.

Figure 6 in which the main Si reflector will be replaced by that of the insert "1" for $n(\omega)<1$ and $n(\omega)>1$, respectively, explains the method of the measurements. First let us assume that $n(\omega)>1$. Increasing the glancing angle θ_{gl} by rotating the Si targets one can expect the well known dependences of the rate of TER (or TIR) photons detection rate of the reflected (transmitted) photon detector upon θ_{gl} (see the insert '2' of figure 6), from which δ is determined from the measured $\theta_{cr}^{TER}=(2\delta)^{1/2}$ with an error $\pm \Delta\theta_{gl}/2$. To carry such measurements it is necessary to have (1–10) MeV photon beams with 1) angular spread much less than the above $\theta_{cr}^{TER}=45\,\mu$rad and 2) sufficient intensity. The first condition is satisfied by the fact that only a small fraction $F\approx(\theta_{acc}/\theta_{rad})^2$ of the beam of the photon source with angular spread $\theta_{rad}\approx1/\gamma$ emitted under angles smaller than $\theta_{acc}=T\theta_{gl}/2L$ (T is the sizes of the reflector, L is the distance between the source and reflector), strikes the Si reflector. Taking T=3 cm, L=10 m, one obtains for HiGS ($\gamma=2.4\times10^3$), F=10^{-8}.
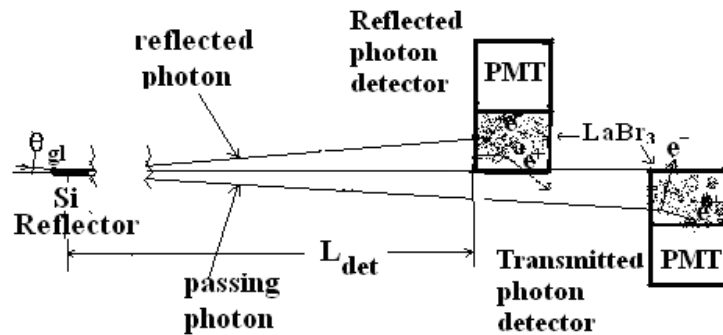

Figure 7. The proposed set-up for the reflected and transmitted photon detectors.

Therefore, for HiGS one expects to detect ~1–10^{3}TER ph/s, and the experiment is feasible. However, in reality one must take into account that the HiGS beam is available after 2 collimators at a distance of $L_{det}\approx60$ m from the source as well as the difficulties of photon detection by the reflected and transmitted photon detectors connected with the smallness of the reflection angles (see [11d]). For this purpose the set-up of figure 7 is proposed in which the photons are detected separately at $L_{det}\approx60$ m using 2 LaBr$_3$, scintillators. These problems can be solved better with the help of PET

liquid xenon time projection chamber (LXeTPC) [20] or proposed LXeTPC on the basis of graphene [21].

6. Conclusion

At present it is possible to study the predicted GCTR using $E \geq 20$ GeV electrons at SLAC or of CERN and to measure directly $n(\omega)$ of various materials in MeV region using TER or TIR. These experiments can confirm or decline the discovery of [1].

References

[1]   Habs D, Gunther M M, Jentschel M and Urban W 2012 *Phys. Rev. Lett.* **108** 184802
[2]   Donohue J T 2013 *Phys. Rev. Lett.* **110**, 129501
[3]   Habs D, Gunther M M, Jentschel M and Thirolf P G 2012 ArXiv:1201.4466
[4]   Jentschel M 2013 *Gamma Ray Optics* http://www.inn.jir
[5]   Habs D 2013 *Delbruck scattering, non-perturbative QED and possible applications*
         http://www.eli-np.ro
[6]   *CERN Courier* 2012 July 18
[7]   Jelley M 1958 *Cherenkov Radiation and Its Applications* (Pergamon Press, London)
[8]   Zrelov V P 1968 *Izluchenie Vavilova-Cherenkova i Ego Primenenie*
         v Fizike Visokikh Energii (Moscow, Atomizdat)
[9]   Bazilev V A and Zhevago N K 1987 *Izluchenie Bistrikh Chastits v Veshchestve i vo*
         *Vneshnikh Polyakh (*Moscow, Nauka)
[10] Ispirian K A 2005 *X-Ray Cherenkov Radiation*, NATO Sc. Ser. II, Math. Phys Chem.
         199 217
[11] Aginian M A, Ispirian K A and Ispiryan M, To be published in a) 2013 *European Physics*
         Letters; b) 2014 *Izvestya of NAS of Armenia;* c) 2014 Proc of this Conf.; d) 2014 Proc of this
         Conf.
[12] Ter-Mikaelian M L, *High Energy Electromagnetic Processes in Cond. Media,* Wiley Intersc.,
         N Y, 1972
[13] Garibian G M, Yan Shi 1983 *Rentgenovskoe Perekhodnoe Izluchenie,* Publishing
         home of Academy of Sciences of Armenia (Yerevan, Armenia)
[14] Ispirian K A 1965 Dissertation, Yerevan State University
[15] Klein S 1999 *Rev. Mod. Phys.* **71**, 1501
[16] Hansen H D, Uggerhoy U I, Biino C *et al* 2004 *Phys. Rev. D* **69** 032001
[17] Mazzolari A *et al* 2014 To be published in Proc of this Conf.
[18] Kumakhov M A 1986 *Radiation of Channeled Particles in Crystals*, Energoatomizdat,
         Moscow,
[19] Wu Y K, *Overview of present and future Compton photon sources,* PAC 2012,
         TUXB03
[20] Lewellen T K 2008 *Phys. Med. Biol* **53** R287-R317
[21] Ispirian K A and Ispiryan R K 2011 *Nuovo Cimento* **34** 37