

Coherent Production of Relativistic Singlet Ps Atoms by High Energy Photons and Electrons in Aligned Crystals – Revisited

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Abstract. Cross-sections of coherent production of relativistic (moving with relativistic velocities) singlet Ps atoms in the ultrathin crystal have been calculated and estimations of counting rates are obtained to propose the experiments.

1. Introduction: a short historical survey, key questions, challenges and motivations for future studies

The first papers on production of relativistic (moving with a relativistic velocity) positronium (Ps) atoms by high-energy photons and electrons in the Coulomb field of the separate atom [1–4] were published more than 27 years ago. The further developments of the theory (beyond the Born approximation) was performed in the papers [5–8], again many years ago.

The next step in this story was connected with theory of coherent production of singlet Ps atoms by high-energy photons in a crystal [9]. The next paper [10] was dedicated to coherent production of relativistic Ps (singlet and triplet) by high energy electrons. According [9] and [10], the sharp coherent peaks may appear in photo – and electroproduction cross-sections at definite Ps energies and emission angles.

The investigations in this field have been continued in the later works [11–13]. In particular, more detailed theoretical investigations of Ps coherent type B photo- and electroproduction in the crystals was presented in Ref. [12]. In fact, besides theoretical studies of cross - sections, it was the 1st real proposal of experiment on coherent Ps photoproduction at REFER (Hiroshima University, Japan). Unfortunately, the experiment was not done at that time.

The QED process closely connected to coherent Ps photoproduction in a crystal is the coherent photoproduction of free (non-bound) electron-positron pairs. The idea of the first experiment on coherent type B e^+e^- pairs photoproduction in a crystal [14] was to approach Ps production kinematics, i.e. to detect created e^+e^- pairs under hard collimation. The experiment [14] was successful and resulted in observation of brilliant coherent effect in the yield of symmetric (equal energy) electron-positron pairs in the kinematics “approaching Ps kinematics”.

The problems connected with Ps generation and their experimental investigation were the subject of International Workshop "Hadronic atoms and positronium in the standard model" Dubna 1998. [15] and XII International Workshop on Positron and Positronium Physics (Sandbjerg, Denmark 19–21 July 2003) [16]. More than 10 years passed since these Workshops, but predicted coherent effects still have not been observed experimentally.

Why is it interesting to study these effects experimentally?



1. The new coherent effects in a crystal target at a few hundreds MeV photon/electron beams energies can be discovered, while the detection seems the worth challenge for experimentalists.

2. Having the monochromatic beam of Ps atoms, one can consider the possibility the direct measurement of singlet Ps lifetime in a vacuum. All previous measurements were concerned with lifetime measurements of Ps stopped in a matter.

3. Production of Ps atoms by the photons/electrons in the Coulomb field of an atom is one of the QED effects, still not observed experimentally. Using of the crystal target allows increase in Ps yield in comparison with amorphous target.

Here, we reexamine the experimental proposal [14] in connection with programs of current and planned experiments on crystal-assisted processes using high energy photon and electron beams at SAGA-LS (Japan), MAMI (Germany) and BTF (LNF Frascati).

2. Coherent production of relativistic singlet Ps atom by high-energy photons

Differential over emission angles cross - section of singlet Ps production by a photon with the energy ω in the Coulomb field of an atom is [2–3]:

$$\frac{d\sigma_1}{d\Omega} = \frac{Z^2 \alpha^6}{8m^2 \gamma^4} \frac{\sin^2 \Theta}{(1 - \beta^2 - 2\beta \cos \Theta)^2 (1 - \beta \cos \Theta)^2}. \quad (1)$$

Here, α is the fine structure constant, γ and β are the relativistic factor and velocity of created Ps, Θ is Ps emission angle with respect to a photon momentum, m is an electron rest mass, Z is nuclear charge of the target atom (units $\hbar=c=1$ are used). The transferred to an atom momentum squared is:

$$q^2 = (E^2 + p^2 - 2Ep \cos \Theta) = E^2 (1 - \beta^2 - 2\beta \cos \Theta). \quad (2)$$

Here, $E = \omega$ is the Ps energy. If the z axis is parallel to a photon momentum, $\mathbf{k} \parallel OZ$, then the components of the momentum transferred are

$$\mathbf{q} = \begin{cases} q_x = -p \sin \Theta \cos \phi \\ q_y = -p \sin \Theta \sin \phi \\ q_z = \omega - p \cos \Theta = E(1 - \beta \cos \Theta). \end{cases} \quad (3)$$

Here, p is the momentum of created Ps, ϕ is the angle of Ps momentum in a plane perpendicular to the photon momentum. Let us introduce the transverse component of the momentum transferred, $\mathbf{q}_\perp = \{q_x, q_y\}$, and express the differential cross-section of Ps production by high energy photon in the Coulomb field of an atom in variables of momentum transferred:

$$d\sigma_1 = \frac{Z^2 \alpha^6 E}{8\beta m^2 \gamma^4} \frac{q_\perp}{q^4 q_z^2} \frac{dq_x dq_y}{\sqrt{p^2 - q_\perp^2}}. \quad (4)$$

It is well known that in an aligned crystal a cross - section of any coherent process can be written as a sum of coherent and incoherent parts [17–18]:

$$d\sigma_{cr} = d\sigma_{coh} + d\sigma_{incoh},$$

$$d\sigma_{coh} = I(\mathbf{q}) \exp(-q^2 \overline{u^2}) d\sigma_1; \quad d\sigma_{incoh} = N \left[1 - \exp(-q^2 \overline{u^2}) \right] d\sigma_1. \quad (6)$$

The exponential $\exp(-q^2 \overline{u^2})$ is the Debye-Waller factor, which takes into account the thermal vibrations of the crystal atoms, $\overline{u^2}$ is the mean-squared deviation of the crystal atom from its

equilibrium position, $I(\mathbf{q})$ is an interferential multiplier responsible for appearance of the coherent effect and N is the number of atoms. We will consider the case when a photon initial momentum is parallel to a crystallographic axes – so-called type-B coherent photo-production, and the number of atoms in the crystallographic axes is not very large, i.e. the case of a thin crystal. In this case $I(\mathbf{q})$ should be written in a following form:

$$I(\mathbf{q}) = N_x N_y \frac{(2\pi)^2}{d_x d_y} \sum_{n,l} \delta(q_x - g_x n) \delta(q_y - g_y l) \times \frac{\sin^2(N_z q_z d_z / 2)}{\sin^2(q_z d_z / 2)} |S(\mathbf{q})|^2 \quad (7)$$

In the last equation $N_x \gg 1$, $N_y \gg 1$, N_z are the numbers of the crystal atoms in x, y, z directions respectively, which contribute to coherent process; $g_x = (2\pi/d_x)n$, $g_y = (2\pi/d_y)l$, $n, l = 1, 2, 3, \dots$ are the components of the reciprocal lattice vector \vec{q} in x and y directions, d_x, d_y, d_z are the lattice constant in x, y, z directions respectively, and $S(\mathbf{q})$ is the crystal structure factor.

In the proposed experiment [12] it is planned to measure the Ps yield within small angular cone. To compare our results with [12] (based on 1D model) we integrated $d\sigma_{cr}$ over Ps emission angles. In our case the integration over Ps emission angles is replaced by integration over the transverse transferred momentum \mathbf{q}_\perp . After substitution of Eq. (7) into Eq. (5) and integration over \mathbf{q}_\perp we arrive at

$$\begin{aligned} \sigma_{coh} = & N_x N_y \frac{(2\pi)^2}{d_x d_y} \sum_{n,l} \sigma(\mathbf{q}_\perp^{n,l}, q_z(\mathbf{q}_\perp^{n,l})) |S(\mathbf{q}_\perp^{n,l}, q_z(\mathbf{q}_\perp^{n,l}))|^2 \times \\ & \times \exp\left\{-\left[(\mathbf{q}_\perp^{n,l})^2 + q_z(\mathbf{q}_\perp^{n,l})^2\right] \frac{1}{u^2}\right\} \times \frac{\sin^2(N_z q_z(\mathbf{q}_\perp^{n,l}) d_z / 2)}{\sin^2(q_z(\mathbf{q}_\perp^{n,l}) d_z / 2)} \end{aligned} \quad (8)$$

Here $\mathbf{q}_\perp^{n,l}$ is the transverse transferred momentum with components: $q_x = g_x n$ and $q_y = g_y l$. As it follows from Eq. (3),

$$q_z(\mathbf{q}_\perp^{n,l}) = E \left\{ 1 - \beta \cos\left[\arcsin\left(\left|\mathbf{q}_\perp^{n,l}\right|/p\right)\right] \right\}.$$

If the atomic potential is the screened Coulomb potential $V(r) = \frac{Ze}{r} \exp\left(-\frac{r}{R}\right)$, than $\sigma(q_\perp, q_z)$ takes a form:

$$\sigma(q_\perp, q_z) = \frac{Z^2 \alpha^6 \beta E^4}{8m^2 \gamma^4} \frac{q_\perp}{(q^2 + R^{-2})^2} \frac{q_z^{-2}}{\sqrt{p^2 - q_\perp^2}},$$

here R is the atomic screening radius.

The summation in Eq. (5) is performed taking into account following condition: $q_x^2 + q_y^2 = q_\perp^2 \leq E^2 \beta^2 \sin^2 \Theta_{\max}$, here Θ_{\max} is the maximal emission angle of Ps with respect to the incident photon momentum. The value of Θ_{\max} is determined by experimental setup. The incoherent part of the Ps photoproduction cross - section in a crystal is defined in a similar way:

$$\sigma_{incoh} = \int_0^{2\pi} d\varphi \int_0^{\Theta_{\max}} \sin \Theta d\Theta \frac{d\sigma_{incoh}}{d\Omega}.$$

3. Coherent singlet Ps production by relativistic electrons in a crystal

The triplet and singlet Ps can be produced also by relativistic electron passing through a crystal [10]. We restrict our consideration only by singlet Ps coherent production. In this case one can derive the cross - section in a framework of the virtual photon method [10, 13]:

$$\frac{d\sigma_E^{cr}}{dE_1} = \sigma_{cr}(\omega) \cdot n(\omega), \quad (9)$$

where $n(\omega)$ is the virtual photon spectrum associated with incident relativistic electron

$$n(\omega) = \frac{\alpha}{2\pi} \frac{1}{E_1} \left\{ \left[1 + \left(\frac{E_1 - \omega}{\omega} \right)^2 \right] \ln \frac{E_1(E_1 - \omega)}{\omega^2} - 2 \left(\frac{E_1 - \omega}{\omega} \right) \right\}. \quad (10)$$

In Eq. (10), E_1 is an initial electron energy, ω is the virtual photon energy. The cross-section $\sigma_{cr}(\omega)$ has been studied in Sec. 2. Therefore, further investigation is straightforward. The cross - sections of coherent of type B production of Ps by relativistic electrons (lower part) and photon (upper part) as a function of created Ps energy are plotted in the figure 1. The energies of the incident electron are $E_1 = 855$ MeV (figure 1a) and $E_1 = 1500$ MeV (figure 1b). The target is silicon Si <100>.

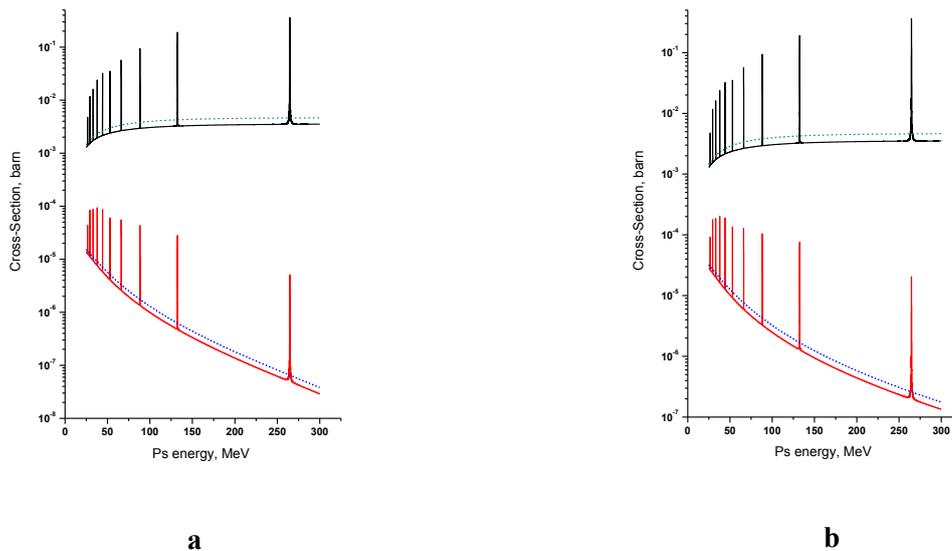


Figure 1. Comparison of Ps cross-sections: coherent photoproduction vs coherent electroproduction in a crystal

The cross-section is integrated over emission angles of Ps assuming that the maximal emission angle of Ps is $\Theta_{max} = 5$ mrad. The number of atoms in a crystal axis is 1000, the crystal temperature $T = 300$ K. Dotted lines show the cross-sections of Ps production by electrons and photons in an amorphous target of equivalent thickness, with the same number of atoms as in a crystal target.

4. New Experimental Possibilities: SPARC, SAGA-LS & MAMI

Our calculations showed the appearance of brilliant coherent beaks at definite Ps energies (up to several hundreds MeV) in photo and electroproduction of relativistic singlet Ps atoms. The difference of these two processes is as follows: if one uses the tagged photon beam, coherent maxima arise at

very definite photon energies, since the photon energy is completely converted to Ps energy. If one uses the relativistic electron beam, there arises very broad energy spectrum of created Ps atoms, with brilliant coherent maxima, since the broad equivalent photon spectrum is converted into broad Ps energy spectrum, with many brilliant coherent maximums. Since the intensity of tagged photon beam is as usual several orders of magnitude less than that of bremsstrahlung (BS), the use of BS beam (standard Schiff spectrum) seems to be more preferable to observe the coherent photoproduction of relativistic Ps atoms. Since the first proposal of experiment [12] at REFER facility (Hiroshima University, Japan), the experiments have not been performed up to day due to some reasons. We may suggest, that the experimental setup could be the same as suggested in [12], with appropriate scaling of the dimensions. In Ref [12] we performed calculations for electron energies $E = 150$ MeV and corresponding two-photon decay length of singlet Ps atoms.

New possibilities are open up at the modern operating moderate-energy electron accelerators: SPARC LNF Frascati (150 MeV), SAGA-LS Japan (255 MeV) and in Mainz at MAMI B (855 MeV) and MAMI C (1508 MeV), where the various crystal-assisted experiments (channeling radiation, parametric X-radiation, scattering and deflection by crystals) are going on. Why: type B coherent effect appears at photon energies up to several hundreds MeV. In the case of electroproduction, there appear a wide energy spectrum of created Ps atoms, but the heights of coherent peaks slightly increases (as $\ln E$) with increase of electron beam energy. Besides, an increase of the beam energy guarantees the straight – line electron trajectory parallel to a crystal axis (neglecting the channeling effect). The principal is the use the ultrathin crystal targets, since when creating the Ps atom penetrates through the bulk of the crystal and can be broken (the break-up cross-sections are known). The complete theory must take into account the simultaneous processes of Ps generation and break-up (similar to generation of X-Rays by relativistic particles in a solid target).

The important figures for experimentalists are of course the counting rates which defines necessary beam time. Let us consider the case of MAMI C (electron energy up to 1508 MeV). The electron beam has the intensity about 10^{13} electron/s. The intensity of the BS beam can be expected as $N_{BS} = 10^{10}$ photons/s, while the intensity of tagged photon beam is much lower, $N_T = 10^6$ photons/s.

The schematics of possible experimental setups could be the same as suggested in [12] for coherent production of relativistic singlet Ps by BS photon beam and in for coherent Ps production by electron beam, respectively.

Consider the BS spectrum from 1508 MeV electrons, we arrive at the Schiff spectrum of photon beam. The convolution of this BS spectrum with singlet Ps photoproduction cross-section from Section 2 will result in a specific Ps energy spectrum emitted from the crystal, which in fact is similar to Ps spectrum resulting from electroproduction (Section 3). The difference is that it is possible to produce singlet Ps with energies up to 1500 MeV using the BS photon spectrum, while electroproduction of only the soft part of singlet Ps energy spectrum is accessible. In both cases, the Ps energy spectrum has brilliant coherent peaks at the same Ps energies.

Since the maximal decay length of produced singlet Ps is expected at maximal Ps energy close to 1500 MeV, $l_\tau \cong \gamma\beta\tau \approx 51.2$ m, this value may characterize the maximal longitudinal size of experimental setup. For type-B singlet Ps production, as considered here, the crystal axis must be precisely aligned with photon/electron beam direction. There are few possible methods to align the crystal: to use the orientation dependence of channeling radiation (CR), to measure the orientation dependence of Ka-X ray emission, or to measure parametric X-Ray (PXR) yield. In fact, the last method can be used both for alignment and electron beam monitoring. Among two first ones, the CR measurement seems to be more effective, see, e.g. [19].

The expected Ps photoproduction rate in vicinity of strongest coherent peak when the incident BS photon beam of intensity $N_{BS} = 10^{10}$ photons/s will be used, can be estimated using the cross-section given, in figure 1 and the density of atomic strings on the crystal surface, of order of $1/d^2$. By integration between Ps energy $E = 263 - 266$ MeV, we easily obtain the production rate of order of 350 singlet Ps per hour from Ge crystal target, if the number of atoms which give rise to coherent

effect is $N_z = 1000$. This rate is to be contrasted with that from an amorphous target, which is approximately 16 times smaller.

In the case of electroproduction, using the cross-sections given in figure 1 our calculation gives the production rate of order of 16 ± 20 singlet Ps per hour integrated within energy region $E = 260 - 270$ MeV, from $\langle 100 \rangle$ Si crystal and much less from amorphous (misaligned) target. In addition, in an experiment on Ps coherent electroproduction in a crystal, the long-lived triplet Ps can be produced [7] with smaller cross-section.

5. Conclusions

New (besides scattering, deflection and radiation) physics can be explored in the field of interaction of high-energy photons (> 100 MeV) and relativistic electrons (positrons) with aligned crystals, i.e. new QED effects – production of bound electron-positron pairs – relativistic (moving with relativistic velocity) Ps atoms. Coherent effects in a crystal lead to enhancement of production cross-sections at definite energies of emitted Ps, i.e. allowing generation of more intense Ps beams for future Ps physics and tests of QED.

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