

Avoided crossings of modes of a finite target with adiabat shaping in inertial fusion implosions

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Abstract. In Inertial Confinement Fusion (ICF), a cold target material is accelerated by a hot low density plasma. In this work, the small amplitude disturbances in ablative Rayleigh-Taylor instability in the presence of entropy gradients is studied using a sharp boundary approximation, being the inverse entropy-gradient scale length the quantity $K_o = \partial \ln S_o / \partial y$. For simplicity, the target considered here is a finite incompressible slab. Due to ablation, the phenomenon of the interactions of two types of modes (Rayleigh mode and entropic mode) is shown. This manifest itself in the occurrence of rapid shifts of rate growth (bumping, avoided crossings). The entire situation is strongly reminiscent of the well-know “avoided crossings” of modes of two coupled oscillators. The growth rate of one mode (Rayleigh mode) approaches that of another one (entropic mode) which is “bumped” to a quite different rate growth, while the “bumping” mode settles at roughly at the original rate growth of the bumped mode.

1. Introduction

Anderson et al. [2], obtained a RT (Rayleigh-Taylor) dispersion for a semi-infinite incompressible slab relation, using a sharp boundary approximation, which is valid for $kL_s \sim kd \gg 1$ where d is the target thickness, and L_s is the entropy gradient scale length. However, this model does not take account the thickness slab; therefore, don't show the disappearance of the cut-off wave number due to the resonant interaction of the RT mode with the most unstable entropic mode.

The model presented here overcomes this obstacle: a more accurate prediction about the RT mode is obtained by assuming a slab with finite thickness. Our model is more complete than the Anderson's model because, determinates all the entropic modes of the slab, showing the disappearance of the cut-off wave number. For simplicity, we are considering a planar target of thickness d (small compared with the ablation radius R of the spherical shell), which is moving with a constant acceleration \vec{g} due the ablation pressure, generated by the heat flux coming from corona. Moreover, an entropy gradient $\vec{\nabla} S$, of the same direction and sense that acceleration, exist in the slab. This slab is continuously ablating with a mass ablation rate.

For simplicity, the ambient flow speed in the slab is very low, and it is generally useful to neglect it taking account the conservation of mass in the ablation front. Thus, the dynamical pressure inside the target and in the region of the ablating plasma close to the ablation front is small compared with the thermal pressure (the ablating plasma is very subsonic close to the ablation front). Also, the effect of the gravity on the ablating fluid can be neglected. In order to solve the initial value problem, we



consider single interface (ablation front) which separates an incompressible fluid (the slab region) from a non-uniform ablating plasma. In ablative RT [1], the growth rate is characterized by the Froude number, $F_r = v_a^2 / (gL_o)$, where v_a is the ablation velocity, $v_a = \dot{m} / \rho_a$, \dot{m} is the mass ablation rate, and ρ_a is the ablation density, being L_o the characteristic thickness of the ablation front.

2. Linear stability analysis and the initial problem value

Let $w_{sk}(y, s)$ denote the Laplace transform with respect to time, and the Fourier transform with respect to y of the flow velocity. So, for the y component of the perturbed flow we obtain the equation:

$$\frac{d^2 w_{sk}(y, s)}{dy^2} + \frac{K_o}{\gamma} \frac{d w_{sk}(y, s)}{dy} - k^2 \left(1 - \frac{K_o g}{\gamma s^2} \right) w_{sk}(y, s) = Q(y, s) \quad (1)$$

Where γ is the ratio of specific heats and g the slab acceleration, and:

$$Q(y, s) = -\frac{k^2 H(y, 0)}{s^2} + \frac{1}{s^2} \frac{d^2 H(y, 0)}{dy^2} + \frac{K_o}{\gamma} \frac{1}{s^2} \frac{d H(y, 0)}{dy} \quad (2)$$

$$H(y, 0) = s w(y, 0) + \frac{d w(y, 0)}{dt} \quad (3)$$

The boundaries conditions on each side are:

$$\frac{s^2}{k^2 g} \left(\frac{K_o}{\gamma} w_{sk}(y, s) + \frac{d w_{sk}(y, s)}{dy} \right) - \frac{K_o H(y, 0)}{\gamma k^2 g} - \frac{1}{k^2 g} \frac{d H(y, 0)}{dy} - w_{sk}(y, s) \Big|_{y=-d} = 0 \quad (4)$$

$$\begin{aligned} \frac{s^2}{k^2 g} \left(\frac{K_o}{\gamma} w_{sk}(y, s) + \frac{d w_{sk}(y, s)}{dy} \right) - \frac{K_o H(y, 0)}{\gamma k^2 g} - \frac{1}{k^2 g} \frac{d H(y, 0)}{dy} - w_{sk}(y, s) \\ + \frac{\beta}{r_d} w_{sk}(y, s) \Big|_{y=0} = 0 \end{aligned} \quad (5)$$

As shown in (Piriz et al.) [3] taking the blowoff density as a density in the blowoff region calculated at a distance of the order of the perturbation wave length from front the ablation front, the density jump r_d , for $Fr \gg 1$, yields $r_d = [4\beta / (5F_r)]^{2/5}$, being $\beta = kv_a^2 / g$ the normalized wave number. Moreover, $\varepsilon = K_o d / \gamma$ is the normalized inverse entropy-gradient scale length; kd the normalized thickness; $s v_a / g$ the normalized growth rate.

2.1. Results and discussion

These transforms can be inverted explicitly to express the fluid variables as integrals of Green's functions multiplied by initial data. For more simplicity, taking $\varepsilon = 0.05$ and $Fr = 6$, we obtain numerically the following dispersion relations which are poles of the Green's function (see figure 1). It is explicitly demonstrated that when the growth rates of different modes are almost identical, strong mode coupling occurs (resonant interaction). The existence of coupling between branches shifts their point of intersection into the complex region: an avoided crossing is produced.

The obtained figure shows, in finer detail, the avoided crossings between the Rayleigh Taylor mode and entropic modes. It easy to notice that the most dangerous mode is the Rayleigh mode. Moreover, the eigenmodes have an accumulation point at 0.

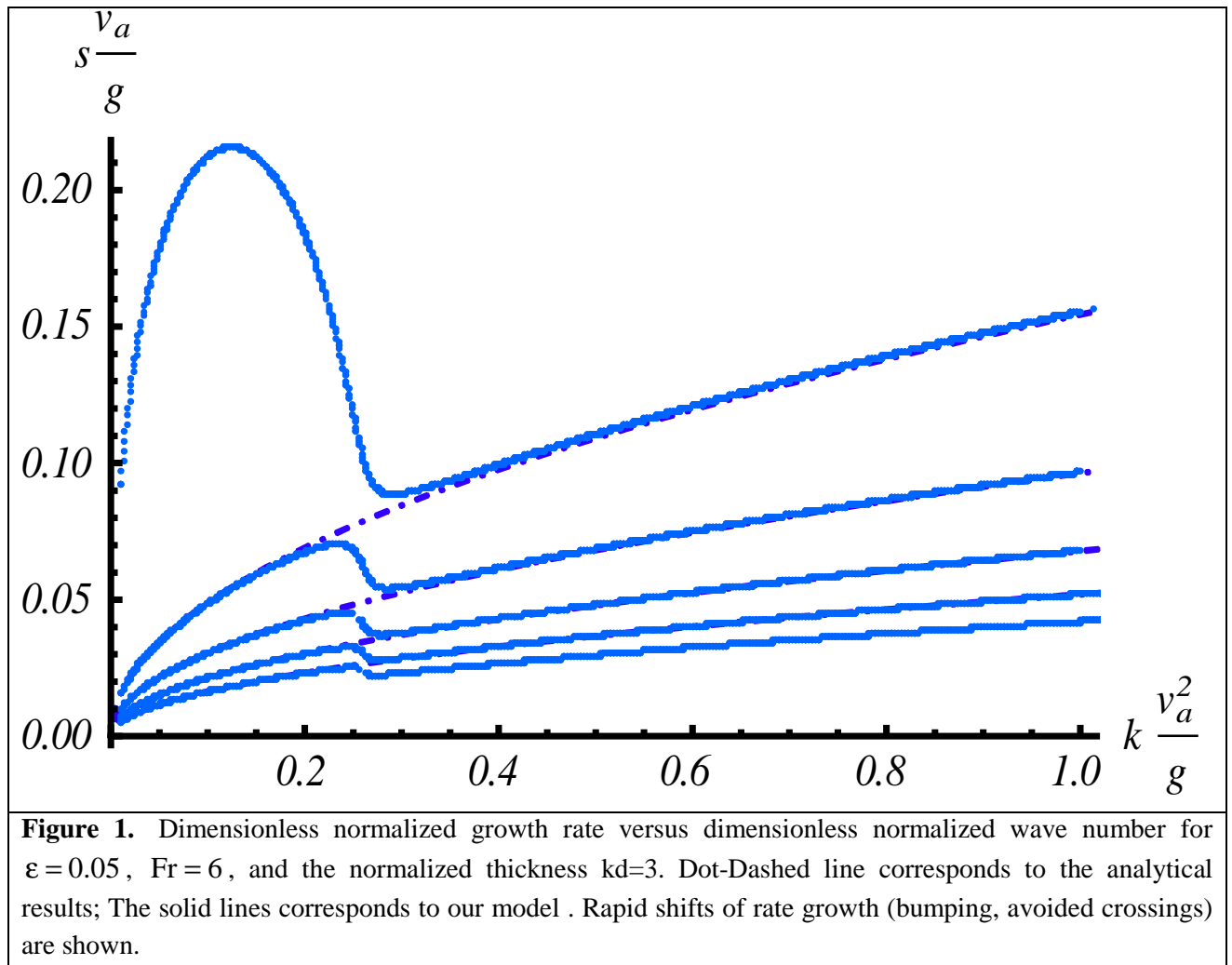
So, when the normalized wave number diminishes, the phenomenon of mode "bumping" [4-5] can be observed: the growth rate of a certain mode approaches that another one which is "bumped" to a quite different growth rate, while the "bumping" mode settles at roughly the original growth rate of the bumped mode. In this work, when the normalized wave number increases, the first entropic mode appears to be the continuation of the Rayleigh mode; the second entropic mode appears to be the continuation of the first entropic mode...

It is interesting to obtain the analytical relation dispersion for entropic modes without ablation, which is in good agreement with the numerical result shown in figure 1 (except at avoided crossings):

$$\frac{sv_a}{g} = \frac{kd \sqrt{\frac{K_o d}{\gamma} \frac{kv_a^2}{g}}}{\sqrt{(m\pi)^2 + (kd)^2 + \frac{(kd)^2}{4} \left(\frac{K_o d}{\gamma}\right)^2}}; \quad m=1,2,3... \quad (6)$$

The entropic modes form an infinity discrete set. As for entropic modes, these are quantized. In practice, one is usually interested in the first two or three fastest growing modes.

On the other hand, as $kd \rightarrow \infty$ the separation of the entropic eigenvalues goes to zero. Thus, we approach a continuous spectrum of entropic eigenvalues.



3. Conclusion

The effect of avoided crossings in ablative Rayleigh-Taylor instability in the presence of entropy gradients is analyzed. So, we observe the mode “bumping” which explain the disappearance of the cut-off wave number in a slab with finite thickness.

4. References

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