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Initial value problem approach of two regions of system solar: the continuous spectrum

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Abstract. The standard mathematical procedures in Helioseismology field are based on normal mode approach for various models of solar interior and atmosphere**.** We consider a two region model of a system solar interior and solar atmosphere. For simplicity, the two different regions are assumed quasi-isothermal, semi-infinite, without magnetic fields and separated by a boundary z=0. It was found useful to phrase of stability as initial value problem (IPV) in order to ensure the inclusion of certain continuum modes otherwise neglected**.** In addition to discrete mode (f-mode), sets of continuum modes due to a branch cuts in the complex plane, not treated explicitly in the literature, appears. It will be seen that an ambiguity of the usual normal mode method is avoided.

1. Introduction

Since Helioseismology actually depends on surface oscillations for detection, the extension with free boundary modes is an important step towards reality of a solar model. Several authors have purposed solutions of the initial value problem through the use of a Laplace Transform in time [2-4]. The principal finding of the initial value problem approach is that, in addition to the discrete eigenvalues linked to the normal modes, there exists a continuous spectrum of eigenvalues. Thus, the modal approach cannot provide a complete solution.

We use the Cartesian coordinates with the *z*-axis pointing towards the interior of the Sun and parallel to the constant gravity $\vec{g}$. As to the applicability of the Cartesian geometry in our analysis, it is valid for perturbations of a sufficiently small wave length when the effects of stellar sphericity are unimportant. This then places the centre of the Sun into $z/H \rightarrow \infty$. Our model assumes an isothermal atmosphere at $z<0$ above the solar surface and the isothermal solar interior (special case) with at z>0. This model would permit the investigation of a surface mode (f-mode), which is a free boundary oscillation of the solar surface involving both the interior and the atmosphere of the Sun The medium is considered a perfect gas. In this paper we exclude macroscopic motions and magnetic fields.

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2. Linear stability analysis and the initial problem value

We start from the standard set of hydrodynamic equations describing the dynamics processes in a fully ionized hydrogen plasma in presence of gravity with constant acceleration [1-2]:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{v} \right) = 0 \tag{1}$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \ \vec{v} \cdot \vec{\nabla} \cdot \left(\rho \vec{v} \right) = -\vec{\nabla} p - \rho \vec{g} \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} p = \gamma \frac{p}{\rho} \left(\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho \right) \tag{3}$$

Considering the displacement vector $\vec{\xi}$, where $\vec{v}_1 = \frac{\partial \vec{\xi}}{\partial t}$ linearization leads to:

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} - \vec{\nabla} \left(\gamma \ p \vec{\nabla} \cdot \vec{\xi} \right) + \rho \ \vec{\nabla} \left(\vec{g} \cdot \vec{\xi} \right) - \rho \ \vec{g} \vec{\nabla} \cdot \vec{\xi} = 0 \tag{4}$$

2.1. *Results and discussion*

We define the Laplace transform $\xi_z \left(z, \omega \right)$ in the complex - ω plane:

$$\xi_z \left(z, \omega \right) = \frac{1}{2\pi} \int_{-i\infty+\sigma}^{+i\infty+\sigma} \xi_z \left(z, \omega \right) e^{-i\omega t} d\omega \tag{5}$$

Then, we consider the initial value problem and introduce the Laplace transform. So, we obtain the following ODE:

$$\frac{d^2 \xi_z}{dz^2} + \lambda \frac{d^2 \xi_z}{dz^2} + \frac{\omega^4 - k^2 v_s^2 \omega^2 + k^2 v_s^2 N^2}{v_s^2 \omega^2} \xi_z \ = f \left(z, \omega, k, F_1, F_2 \right) \tag{6}$$

Being $N^2 = \frac{\left(\gamma - 1 \right) g^2}{v_s^2}$ the Brunt-Väisäläa frequency; $\lambda = \frac{1}{\tau H}$ for (z<0) and $\lambda = \frac{1}{H}$ (z>0). On the other hand, F_1 and F_2 are initial conditions:

$$F_1 = \xi_z \left(z, t \right) \Big|_{t=0^+} ; \ F_2 = \frac{\partial \xi_z \left(z, t \right)}{\partial z} \Bigg|_{t=0^+} \tag{7}$$

We can find the Green´s function for the former ODE subject to conditions at $|z| \to \infty$. The continuity of $G\left(z, \omega, z_o \right)$ at $z = 0$ and $z = -z_o$, and the jump conditions on $\frac{dG\left(z, \omega, z_o \right)}{dz}$ at $z = 0$ and $z = -z_o$. So we have:

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$$G(z,\omega,z_o)=\begin{cases} Be^{\frac{z(-1+\alpha)}{2H\tau}} & \mathrm{z}<\text{-}\mathrm{z}_{\mathrm{o}} \\ Ce^{\frac{z(-1+\alpha)}{2H\tau}}+De^{\frac{z(-1-\alpha)}{2H\tau}} & \text{-}\mathrm{z}_{\mathrm{o}}<z<0 \\ \mathrm{A}e^{\frac{z(-1-\beta)}{2H}} & \mathrm{z}>0 \end{cases} \tag{8}$$

On the other hand:

$$\alpha=\sqrt{1-\frac{4H^2\tau\left[g^2k^2(\gamma-1)+\tau\ \omega^2\left(\omega^2-v_{sc}^2k^2\right)\right]}{v_{sc}^2\omega^2}};\quad \beta=\sqrt{1-\frac{4H^2\left[g^2k^2(\gamma-1)+\omega^2\left(\omega^2-v_{sc}^2k^2\right)\right]}{v_{sc}^2\omega^2}} \tag{9}$$

So, the solution is:

$$\xi_z(z,\omega)=\int_0^\infty G(z,\omega,z_o)\,f(z,\omega,k,F_1,F_2)\ dz_o \tag{10}$$

The Green´s function for $\mathrm{z}>0$:

$$G(z,\omega,z_o)=\frac{-2H\tau\ \omega^2 e^{-\frac{1+\beta}{2H}z+\frac{\tau(1+\beta)z-(z+z_o)(1+\alpha)}{2H\tau}}}{2gHk^2(\tau-1)-(\alpha+\beta)\omega^2}e^{-\frac{1+\beta}{2H}z} \tag{11}$$

In accord with the boundary conditions $G(\pm\infty,s,z_o)=0$, the square root are defined so that Re(α)>0 y Re(β)>0 for Re (ω)$>\sigma$. It is evident that Green´s function has a pole (f-mode).

Expanding Green´s function about α=0, and β=0, this function does not turn out to be an even function of α and β .So, the Green´s function has branch points at $\omega^2=\gamma_b{}^2$ (associated with continuous spectra).

It is of interest to follow the migration of the pole as k is varied. For $k>k_{cr}$, $\gamma_p>\gamma_{b1}$. If k is diminished, then the γ_p pole moves toward γ_{b1}. For $k=k_{cr}$, then $\gamma_p=\gamma_{b1}$. For $k<k_{cr}$ the pole drops on to the lower sheet of β y the f-mode disappears. The disappearance of the γ_p pole (f- mode) is important. Thus, there will be some critical wave number k_{cr} such for $k>k_{cr}$ ($k<k_{cr}$) f-mode exist (do not exist).

The following figure 1 shows dispersion curve. The temperature for the solar interior is 10^4 K, and for the corona 10^6 K, being the discontinuous drop $\tau = 100$. Gravitational acceleration is assumed constant within the domains of studied harmonic perturbations and we take g= 274 m/s^2 as it is on the solar surface.

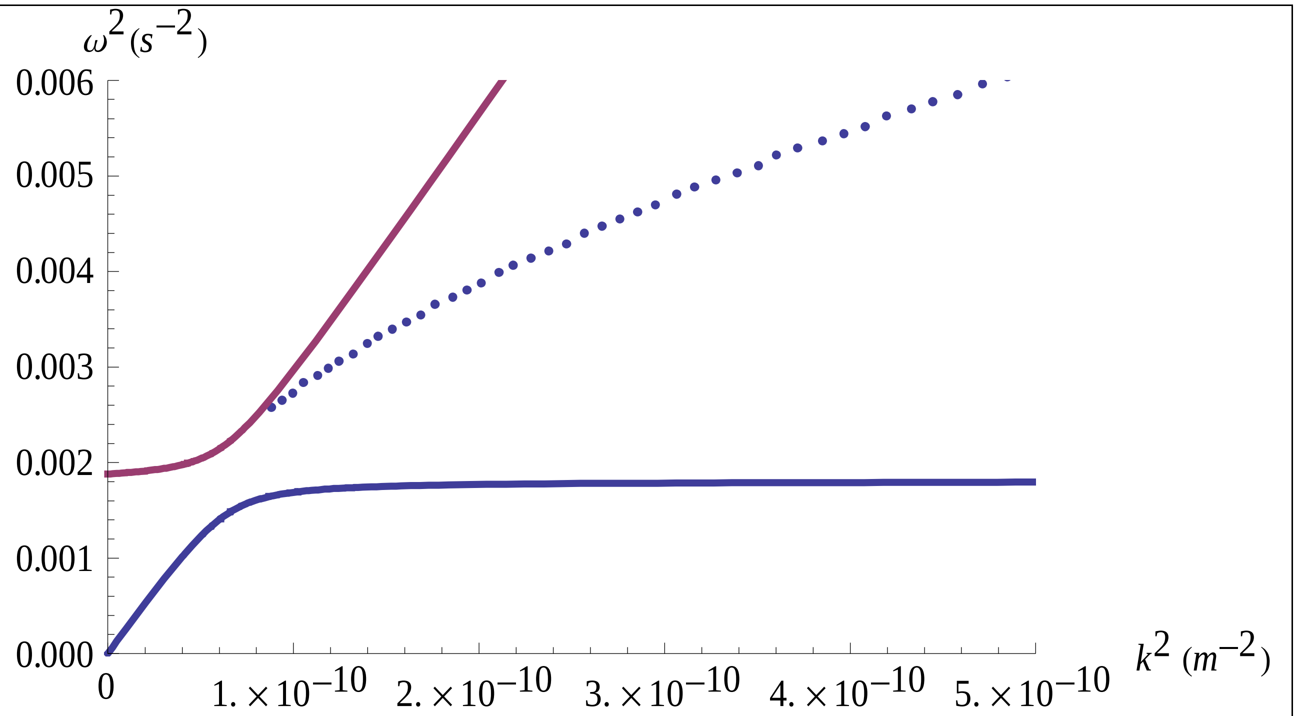

Figure 1. Dispersion curve (f-mode) for $\tau = 100$. Dots correspond to our numerical results. Solid lines corresponds to branch points (frecuency cut-off) . For $k^2 < k^2_{cr}$ f-mode does not exist.

Here, the critical wave number is $k_{cr} \approx 9 \cdot 10^{-6} m^{-1}$.

3. Conclusion

Surface mode (f-mode) is analysed in a two regions solar model using initial value problem (IPV). In the special case, where the solar interior is isothermal, there will be some critical wave number k_{cr} such that for $k > k_{cr}$ $(k < k_{cr})$ f-mode exist (do not exist). On the other hand, a set of continuum p-modes and g-modes due to branch cuts in the complex plane, not treated explicitly in the literature, explain the disappearance of the f-mode.

4. References

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