

Landau Damping in Space Plasmas with Two Electron Temperature Non-Maxwellian Distribution Functions

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Abstract. Space plasmas generally possess distribution functions that exhibit high or super thermal energy tails in velocity space which can have different temperatures, a dense cold population and a hot population. Moreover, in laboratory plasmas when a laser or electron beam is passed through a dense plasma, hot low density electron populations can be generated. Presence of such low density electron distributions can act to increase the magnitude of the wave damping rate. In this paper we employ non-Maxwellian distribution function such as the generalized (r, q) distribution function with two electron temperatures to study the Landau damping of electrostatic waves. The results show that the Landau damping increases significantly when the percentage of high energy particles increases and with the increase of the high energy tail and less pronounced shoulders in the profile of the distribution function.

1. Introduction

Electron velocity distribution functions (VDFs) with pronounced superthermal tails and shoulders are frequently observed in space plasmas [1-5], which are often modeled by κ -distributions or generalized (r, q) distribution functions [3,6-8]. Observations of electron VDFs from solar wind shown significant deviations from Maxwellian distribution function and; a dense thermal core and a hot superthermal population ‘halo’ can be distinguished in slow solar wind [9-12]; whereas in a fast solar wind a core and two hot superthermal populations ‘halo’ and ‘strahl’ are generally observed [13-15]. Such electron VDFs cannot be modelled by simple one electron population κ -distributions.

In laboratory plasmas a small population of electrons possessing much higher energies than the original laser beam can be produced [16-18]. Simulation results of electron or laser beam propagation in dense plasmas often show electron distributions that are characterized by power-law tails of hot electrons superposed on an approximately Maxwellian bulk distribution [19, 20]. The presence of such low density electron distributions can act to increase the wave damping rate. In this paper, we studied Landau damping with generalized (r, q) distribution function consisting of two populations, a hot population and a cold bulk population. The underlined theory can be used to understand the physical picture in the laboratory and space plasmas.

The generalized (r, q) distribution functions is the sum of a fractional ‘F’ hot and a cold bulk electron distributions, which is written as



$$f_{rq}(v) = \frac{(1-F)C}{\pi} \left(\frac{a}{DT_c} \right)^{3/2} \left[1 + \frac{1}{q-1} \left(\frac{a}{D^2 T_c} v^2 \right)^{r+1} \right]^{-q} + \frac{(F)C}{\pi} \left(\frac{a}{DT_h} \right)^{3/2} \left[1 + \frac{1}{q-1} \left(\frac{a}{D^2 T_h} v^2 \right)^{r+1} \right]^{-q} \quad (1)$$

where,

$$C = \frac{3(q-1)^{-3/2(1+r)} \Gamma(q)}{\Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(1 + \frac{3}{2(1+r)}\right)} \quad (2)$$

$$D = \frac{3(q-1)^{-1/(1+r)} \Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(\frac{3}{2(1+r)}\right)}{\Gamma\left(q - \frac{5}{2(1+r)}\right) \Gamma\left(\frac{5}{2(1+r)}\right)} \quad (3)$$

Here, Γ is the usual Gamma function; T_c and T_h are the cold and hot electron temperatures and $a = m/2k_B$, where m is the electron mass and k_B is the Boltzmann constant. The total electron density is the sum of the hot and cold electron densities. In the limit $r = 0$ and $q = \kappa + 1$, the equation (1) reduces to κ -kappa distribution function and in the limiting case when $r = 0$ and $q \rightarrow \infty$, the equation (1) reduces to the well known Maxwellian distribution function. We note that $q > 1$ and $q(1+r) > 5/2$ are the conditions which arise from the normalization and definition of temperature for the distribution function given in equation (1).

Figures 1 and 2 show the profiles of generalized (r, q) distribution function for 1% hot population at 10 eV added to cold dense population at 1 eV for different values of r and q . Figure 1 is plotted for different values of spectral index q when $r = 1$ (upper panel) and $r = 2$ (lower panel). From the upper panel when $r = 1$, we note that as the value of q increases the high energy tail decreases. From the lower panel when $r = 2$, we note that as the value of q increases the high energy tail decreases similar to the upper panel and shoulders in the profile of the distribution function become more prominent with the increase of r . Figure 2 is plotted for different values of spectral index r when $q = 2$ (upper panel) and $q = 5$ (lower panel). For $q = 2$ in the upper panel, we note that as r increases the shoulders in the profile of distribution function tend to increase and the high energy tail decreases. For $q = 5$ in the lower panel, we note that as r increases the shoulders in the profile of distribution function tend to increase similar to the upper panel and the high energy tail decrease as compared to the upper panel. Therefore, from the Figures 1 and 2 we can note that when q increases the high energy tail decreases and when r increases shoulders in the profile of distribution becomes more prominent.

2. Dispersion Relation

We follow the general formulism of kinetic theory to derive the dispersion relation for the electrostatic waves with complex frequency $\omega = \omega_r + i\omega_i$ and real wave number k . This procedure yields the propagation and damping characteristics of electron plasma waves in the limiting case when $k\lambda_D \ll 1$, where

$$\lambda_D = \left(\frac{T_\alpha}{4\pi N e^2} \right) = \left(\frac{(q-1)^{1/(1+r)} \Gamma\left(q - \frac{5}{2(1+r)}\right) \Gamma\left(\frac{5}{2(1+r)}\right)}{3 \Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(\frac{3}{2(1+r)}\right)} \right)^{1/2} \left(\frac{v_{th\alpha}}{\omega_p} \right) \quad (4)$$

is the electron Debye length, v_{th} is the thermal velocity, ω_p is the electron plasma frequency, T is the electron temperature, $\alpha = c, h$ (for cold or hot population) and N is the total number density. The general dispersion relation of electrostatic waves is

$$1 - \frac{\omega_p^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial f / \partial v}{(v - \omega/k)} dv = 0 \quad (5)$$

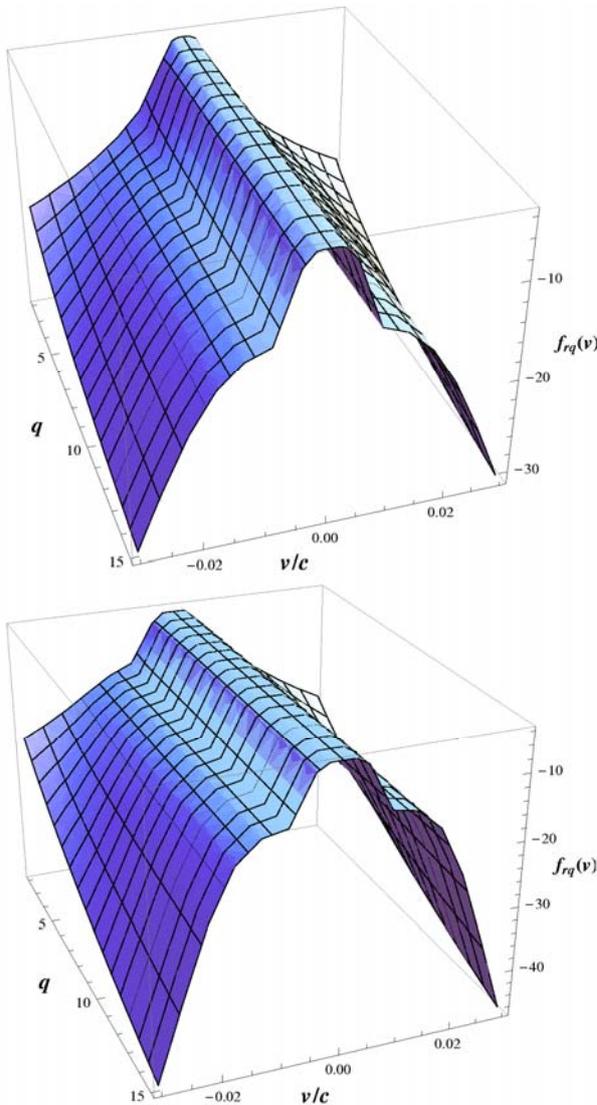


Figure 1. Profiles of generalized (r, q) distribution function for different values of q when $r = 1$ (upper panel), $r = 2$ (lower panel), and $F = 0.01$, $T_c = 1$ eV and $T_h = 10$ eV.

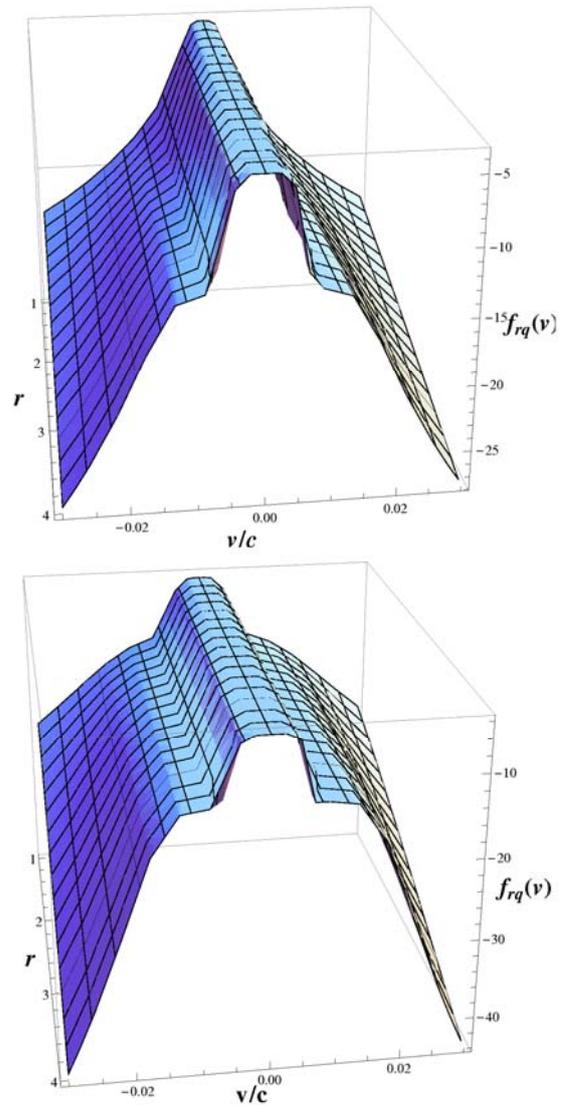


Figure 2. Profiles of generalized (r, q) distribution function for different values of r when $q = 2$ (upper panel), $q = 5$ (lower panel), and $F = 0.01$, $T_c = 1$ eV and $T_h = 10$ eV.

Using equation (1) in above equation, we get the general dispersion relation as

$$1 + \frac{2 \omega_p^2}{k^2} \left[\frac{(1-F)}{v_{thc}^2} \{H + \xi_c Z_1^{(r,q)}(\xi_c)\} + \frac{F}{v_{thh}^2} \{H + \xi_h Z_1^{(r,q)}(\xi_h)\} \right] = 0 \quad (6)$$

where

$$H = \frac{3 (q-1)^{-1/(1+r)} \Gamma\left(q - \frac{1}{2(1+r)}\right) \Gamma\left(1 + \frac{1}{2(1+r)}\right)}{2 \Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(1 + \frac{3}{2(1+r)}\right)} \quad (7)$$

and

$$Z_1^{(r,q)}(\xi_\alpha) = \frac{3 (q-1)^{-3/2(1+r)} \Gamma(q)}{4 \Gamma(q - 3/2(1+r)) \Gamma(1 + 3/2(1+r))} \int_{-\infty}^{\infty} \frac{ds}{(s_\alpha - \xi_\alpha)} \left(1 + \frac{1}{q-1} s_{c,h}^{2(r+1)}\right)^{-q} \quad (8)$$

is the generalized plasma dispersion function [6]. Here $\xi_\alpha = \omega / kv_{th\alpha}$, $s_\alpha = v / v_{th\alpha}$ and again $\alpha = c, h$.

For the Langmuir waves propagating in unmagnetized plasma it is justified to consider the ions as immobile giving uniform background that simply maintains the charge neutrality. Therefore, neglecting the ion terms in equation (6) and using the appropriate limiting form $\xi_\alpha \gg 1$ of the generalized plasma dispersion function (8), the dispersion relation for hot and cold species can be written as

$$D_r(\Omega) + i D_i(\Omega) = 0 \quad (9)$$

where the real and imaginary terms are

$$D_r(\Omega) = 1 - \frac{1}{\Omega^2} [(1-F) + F] - \frac{2 K D k^2 \lambda_D^2}{\Omega^4} \left[(1-F) + \frac{T_h}{T_c} F \right] \quad (10)$$

and

$$D_i(\Omega) = \frac{2 \pi \Omega C}{D^{3/2} k^2 \lambda_D^2} \left[\begin{aligned} & (1-F) \left(1 + \frac{1}{q-1} \left\{ \frac{\Omega^2}{D k^2 \lambda_D^2} \right\}^{r+1}\right)^{-q} \\ & + F \left(\frac{T_c}{T_h}\right)^{3/2} \left(1 + \frac{1}{q-1} \left\{ \frac{\Omega^2}{D k^2 \lambda_D^2} \frac{T_c}{T_h} \right\}^{r+1}\right)^{-q} \end{aligned} \right] \quad (11)$$

respectively. Here $\Omega = \omega / \omega_p$ and

$$K = \frac{3 (q-1)^{1/(1+r)} \Gamma\left(q - \frac{5}{2(1+r)}\right) \Gamma\left(1 + \frac{5}{2(1+r)}\right)}{10 \Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(1 + \frac{3}{2(1+r)}\right)} \quad (12)$$

The solution of $D_r(\Omega_r) = 0$ gives the real frequency Ω_r of the wave and the solution is then given as (for positive root)

$$\Omega_r \simeq \sqrt{\frac{1}{2} + \frac{1}{2} \left(1 + 8 D K k^2 \lambda_D^2 \left(1 - F + \frac{T_h}{T_c} F\right)\right)^{1/2}} \quad (13)$$

And the damping rate of the wave is determined by

$$D_i \simeq - \frac{D_i}{\partial D_r / \partial \Omega} \Big|_{\Omega_r} \quad (14)$$

Using equation (13), in evaluating the above equation, gives

$$\Omega_i = - \frac{\pi C}{D^{3/2}} \frac{\Omega^8}{k^3 \lambda_D^3} \left(\Omega^4 + 4 K D \Omega^2 k^2 \lambda_D^2 \left(1 - F + \frac{T_h}{T_c} F \right) \right)^{-1} \left[(1 - F) \left(1 + \frac{1}{q-1} \left\{ \frac{\Omega^2}{D k^2 \lambda_D^2} \right\}^{r+1} \right)^{-q} + F \left(\frac{T_c}{T_h} \right)^{3/2} \left(1 + \frac{1}{q-1} \left\{ \frac{\Omega^2}{D k^2 \lambda_D^2} \frac{T_c}{T_h} \right\}^{r+1} \right)^{-q} \right] \quad (15)$$

In the limit $8 K D k^2 \lambda_D^2 (1 - F + F(T_c/T_h)) \ll 1$, $\Omega_r = \sqrt{1 + 2 K D k^2 \lambda_D^2 (1 - F + F(T_c/T_h))}$ and the equation (15) can be further reduced to

$$\Omega_i = - \frac{\pi C}{D^{3/2}} \frac{1}{k^3 \lambda_D^3} \left[(1 - F) \left(1 + \frac{1}{q-1} \left\{ \frac{1}{D k^2 \lambda_D^2} + 2 K (1 - F + F(T_h/T_c)) \right\}^{r+1} \right)^{-q} + F \left(\frac{T_c}{T_h} \right)^{3/2} \left(1 + \frac{1}{q-1} \left\{ \frac{1}{D k^2 \lambda_D^2} + 2 K (1 - F + F(T_h/T_c)) \left(\frac{T_c}{T} \right) \right\}^{r+1} \right)^{-q} \right] \quad (16)$$

3. Numerical Solution

The numerical solution of equation (16) is shown in Figures 3 to 5 for different values of r and q. Figure 3 depicts the magnitude of damping rates for 1% hot population at 10 eV added to cold dense population at 1 eV in the limit $k \lambda_D \ll 1$ in the appropriate limiting form $\xi_\alpha \gg 1$ of the generalized plasma dispersion function (8) for different values of q when r is fixed. From Figure 3 upper panel, we can see that the damping rate significantly increases when q decreases from 15 to 2 when r = 1. This is due to the fact that as the high energy tail increases in the profile of distribution function, the damping rate increases for the low values of q which can be seen in Figure 1 (upper panel). From the lower panel of Figure 3, we can note that the damping rate increases when q decreases from 15 to 2 when r = 2 similar to the damping rate as shown in upper panel but the comparison of upper and lower panels shows that the damping rate decreases as r increases. This is due to the fact that as r increase, the shoulder increases in the profile of the distribution function and high energy tail decreases which correspond to the Figure 1 (lower panel). Thus the damping rate decreases when either r or q increases.

Figure 4 depicts the magnitude of damping rates for 1% hot population at 10 eV added to cold dense population at 1 eV in the limit $k \lambda_D \ll 1$ in the appropriate limiting form $\xi_\alpha \gg 1$ of the generalized plasma dispersion function (8) but for different values of r when q is fixed. From Figure 4 (upper panel), we can see that the damping rate significantly decreases when r increases from 1 to 4 when q = 2. This is due to the fact that as the shoulders increase in the profile of distribution function the high energy tail decreases which can be seen in Figure 2 (upper panel) and hence the damping rate decreases when r increases. From the lower panel of Figure 4, we can note that the damping rate decreases when r increases from 1 to 4 when q = 5 similar to the damping rate as shown in upper panel. But the comparison of upper and lower panels shows that the damping rate decreases

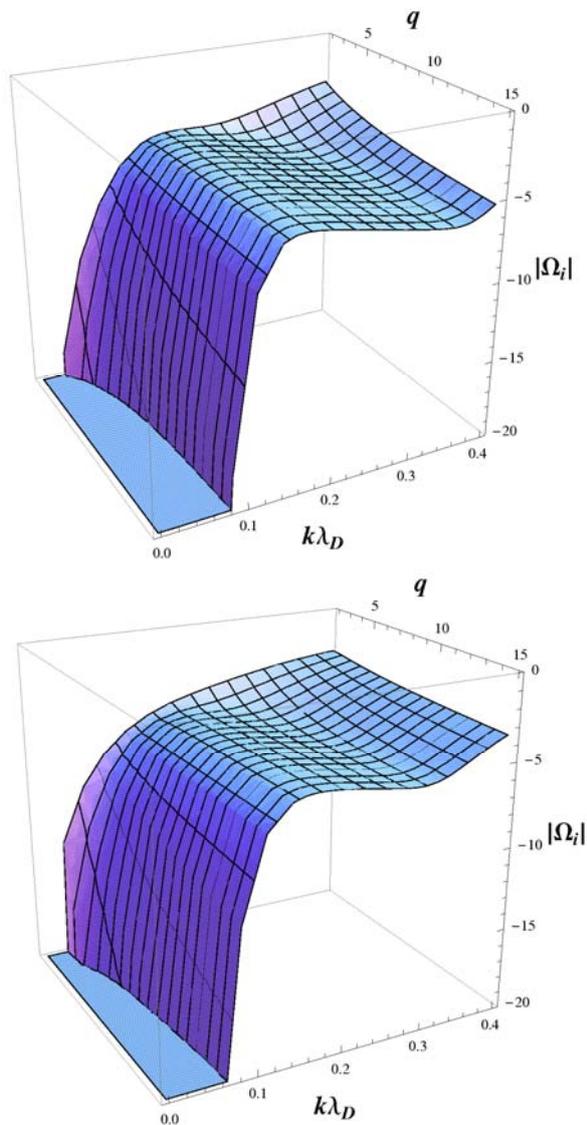


Figure 3. Magnitude of Landau damping for different values of q when $r = 1$ (upper panel), $r = 2$ (lower panel), and for $F = 0.01$, $T_c = 1$ eV and $T_h = 10$ eV.

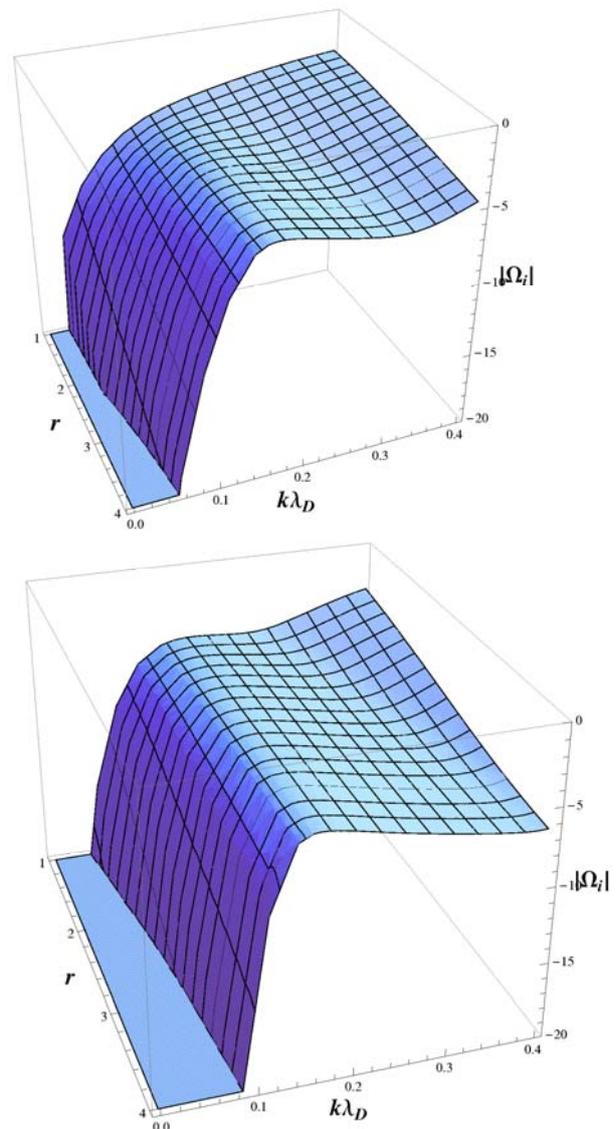


Figure 4. Magnitude of Landau damping for different values of r when $q = 2$ (upper panel), $q = 5$ (lower panel), and for $F = 0.01$, $T_c = 1$ eV and $T_h = 10$ eV.

significantly as q increases. This is due to the fact that as q increases, the high energy tail decreases in the profile of the distribution function which correspond to the Figure 2 (lower panel). For fixed value of $k \lambda_D$, the comparison of Figure 3 and 4 shows that the damping rate significantly increases when either the value of r or q decreases in the range when $k \lambda_D < 0.15$.

Figure 5 shows the magnitude of damping rates for the larger fraction of the hot population (10%) at 10 eV added to cold dense population at 1 eV in the same limiting cases as for Figures 3 and 4 for different values of r and q . From Figure 5 upper panel, we can note that the damping rate enhances significantly when the percentage of hot population increases as compared to the lower percentage of hot population Figure 3 (upper panel) for $r = 1$. Similarly from Figure 5 lower panel, we can note that the damping rate enhances significantly when the percentage of hot population increases as compared to the lower percentage of hot population Figure 3 (lower panel) for $q = 2$. Therefore, for the larger

fraction of the hot population (10%), the Landau damping enhances significantly for all values of r and q and in the range $0.0 < k \lambda_D < 0.4$.

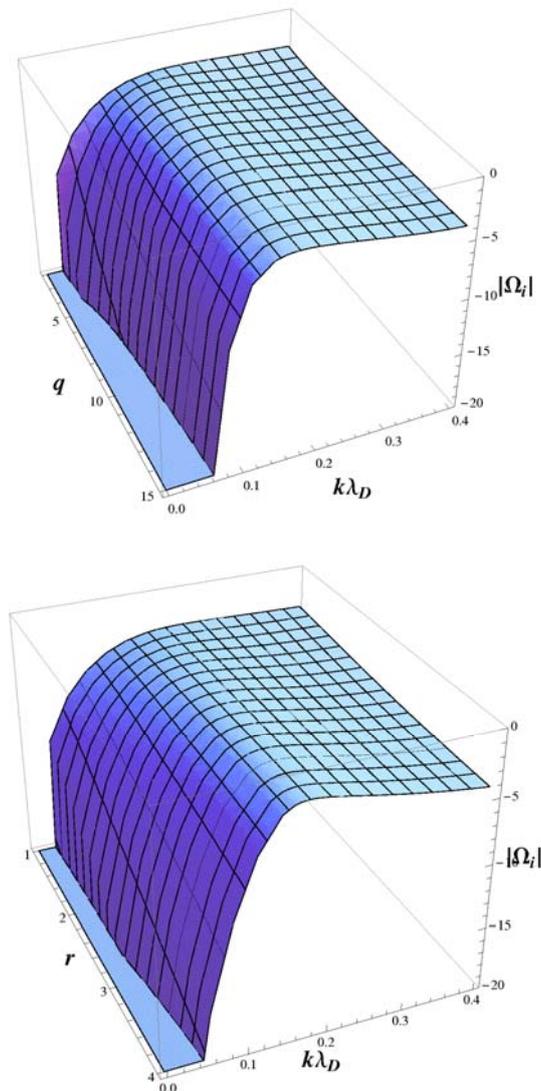


Figure 5. Magnitude of Landau damping when the hot population is 10% of the bulk population, i.e. $F = 0.1$, for different values of q when $r = 1$ (upper panel), and for different values of r when $q = 2$ (lower panel). Other parameters are the same as that of Figures 3 and 4.

4. Conclusion

In this paper, the effect of enhanced Landau damping of electrostatic waves is studied in the presence of low density superthermal electron distribution when added to dense bulk population of cold plasmas. We followed the standard kinetic approach to evaluate the damping rate of Langmuir waves modelled generalized (r, q) distribution function. We have shown that Landau damping increases as the q decreases from 15 to 2 or r decreases from 4 to 1 when we take hot population as the 1% of the bulk cold population. The same trend has also been observed when we consider 10% hot population. But when we consider the increased hot

population, the Landau damping significantly increases as compared to the case when we take lower percentage of hot population.

5. References

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