

Electron inertia effect on floating plasma potential

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Abstract. The steady one-dimensional planar plasma sheath problem, originally considered by Tonks and Langmuir, is revisited. Two-fluid equations for cold ions and isothermal electrons, including terms for particle generation and electron inertia, have been numerically integrated together with Poisson equation. The inclusion of electron inertia in the model allows us to obtain the value of the plasma floating potential as resulting from an electron density discontinuity at the walls, where the electrons attain sound velocity and the electric potential is continuous. Results from numerical computation are presented in terms of plots for densities, electric potential and particles velocities. Comparison with results from literature, corresponding to electron Maxwell-Boltzmann distribution (neglecting electron inertia), are also shown.

The steady one-dimensional planar plasma sheath problem, originally considered by Tonks and Langmuir [1], is a classic problem in plasma physics which is still object of study and discussions [2-22]. One of the universal assumptions in approaching the problem is to neglect electron motion and consider their distribution to be simply Maxwellian. By one side this simplify the equations, but when the floating plasma electric potential has to be evaluated, an unsatisfactory hypotheses for the electron flux to the walls has to be introduced, estimating it with the one side randomic flow associated to the electron Maxwellian distribution itself. Here, we want to present a model of the problem which allows properly taking into account electron inertia and estimating the plasma floating potential in a consistent way.

Let us consider an infinite partially ionized plasma, with isothermal electrons and cold single ionized ions, embedded between two perfectly absorbing parallel walls. Since electrons have a greater mobility than ions, the walls should become negatively charged, in such a way that ions are attracted to them. The system should tend to a steady state with equal electron and ion fluxes (vanishing electric current density) with the walls at a characteristic value of electric potential, named floating potential. In order to maintain the steady state, some mechanism for electron and ion production is necessary, this will be ascribed to electron collisions with neutrals (assumed uniformly distributed and infinitely massive) and will be taken into account through a particle generation term proportional to some power γ of the electron density (typically $\gamma = 0, 1$ or 2) like $G(x) = \nu n_0 [n_e(x)/n_0]^\gamma$, where $n_e(x)$ represents the electron density, $n_0 = n_e(0)$ and ν may be regarded as the ionization rate divided by the electron density at the center of the configuration ($x=0$), ion-electron collisions will be neglected. The problem



can be reduced to consider continuity and momentum conservation equations for electrons and ions together with Poisson equation.

In the literature, following Tonks and Langmuir approach, the plasma is usually divided in two regions, one in which the plasma is almost neutral and ion electron pairs are produced, commonly called pre-sheath, and the sheath, close to the walls, where ion motion is supersonic (according to Bohm criterion [2]) and electron and ion densities differ appreciably. This approach generate serious troubles when a matching between the two regions has to be done [3], but actually it is not necessary when the equations are solved numerically, as shown by Self [4], who numerically solved the whole problem with Maxwellian electron and free falling ions produced locally by electron collisions with neutrals and starting from rest. Here also a numerical treatment will be adopted and in order to compare the present approach to that of Self, we adopt a similar normalization for the different variables entering the problem.

Using Gaussian units, n_0 for the electron density at the middle of the configuration, T_e and $-e$ for the electronic constant temperature and charge, $m_{e,i}$ for the electron/ion mass and k for the Boltzmann constant it is possible to define the following dimensionless dependent variables: densities $\tilde{n}_{e,i}(y) = \frac{n_{e,i}(y)}{n_0}$; velocities $\tilde{u}_{e,i}(y) = \sqrt{\frac{m_e}{kT_e}} u_{e,i}(y)$; particles flux densities (assumed to be equal for both species) $\Gamma(y) = \tilde{n}_{e,i}(y)\tilde{u}_{e,i}(y)$ and electric potential $\phi(y) = -\frac{eV(y)}{kT_e}$ (positive defined), where the dimensionless independent variable $y = \frac{x}{L}$ has been defined in terms of $L = \frac{1}{v} \sqrt{\frac{kT_e}{m_i}}$ a typical discharge width. With such notation the corresponding two-fluid one-dimensional equations for continuity, momentum conservation and electric potential can be conveniently written [5, 6]:

$$\frac{d\Gamma}{dy} = \sqrt{\frac{2m_e}{m_i}} \tilde{n}_e^\gamma, \quad (1)$$

$$\frac{m_i}{m_e} \frac{d}{dy} \left(\frac{\Gamma^2}{\tilde{n}_i} \right) = \tilde{n}_i \frac{d\phi}{dy}, \quad (2)$$

$$\frac{d}{dy} \left(\frac{\Gamma^2}{\tilde{n}_e} \right) + \frac{d\tilde{n}_e}{dy} = -\tilde{n}_e \frac{d\phi}{dy}, \quad (3)$$

$$\frac{d^2\phi}{dy^2} = -\frac{2}{\alpha^2} (\tilde{n}_e - \tilde{n}_i). \quad (4)$$

Ion temperature has been neglected and a new constant $\alpha = \sqrt{\frac{v^2 m_i}{4\pi n_0 e^2}} = \frac{\sqrt{2}\lambda_D}{L}$, where $\lambda_D = \sqrt{\frac{kT_e}{4\pi n_0 e^2}}$ is the usual Debye length, has been introduced. In order to start the set of equations it is necessary a charge density unbalance at the middle of the configuration ($y=0$). Assuming $\tilde{n}_i(0) = 1 + \varepsilon$, it is possible to relate α to ε by imposing the vanishing of the first derivative of \tilde{n}_i at $y=0$ in equation (2), which leads to $\varepsilon=2\alpha^2$.

Equations (1-4) have been numerically integrated using the subroutine NDSolve of *Wolfram Mathematica* software [23], for a wide range of α^2 , $\gamma=0, 1$ and 2 , and m_i corresponding to the representative cases of hydrogen (H) or mercury (Hg). The integration is performed with boundary conditions imposed at the center and assuming symmetry about it. In all cases the integration has to be aborted when electrons reach the sonic isothermal velocity, i.e., $\tilde{u}_e \rightarrow 1$, since in that limit $\frac{d\tilde{n}_e}{dy}$ in equation (3) diverges. We ascribe to such singular behavior the attainment of plasma floating potential

at the wall, which depends on the chosen values of α and $\frac{m_i}{m_e}$. In the following, numerical results are presented for several cases and compared with Self results.

In figure 1, dimensionless electric potential profiles are plotted as functions of y for H, three representative values for α (0.1, 0.01 and 0.001) and $\gamma=1$. In figure 2, dimensionless electric potential profiles are plotted as functions of y for H, $\alpha=0.01$ and $\gamma=0, 1$ and 2. Filled curves represent results from the present model, while dashed ones indicate the corresponding solutions given by Self (for free falling ions and which do not suffer from any singularity in the range of integration). Our curves have to be stopped at the corresponding plasma floating potential owing to the attainment of sonic velocity by the electrons. The little displacement of the curves is due to the fact that in the present model ions lack any spread in their velocities.

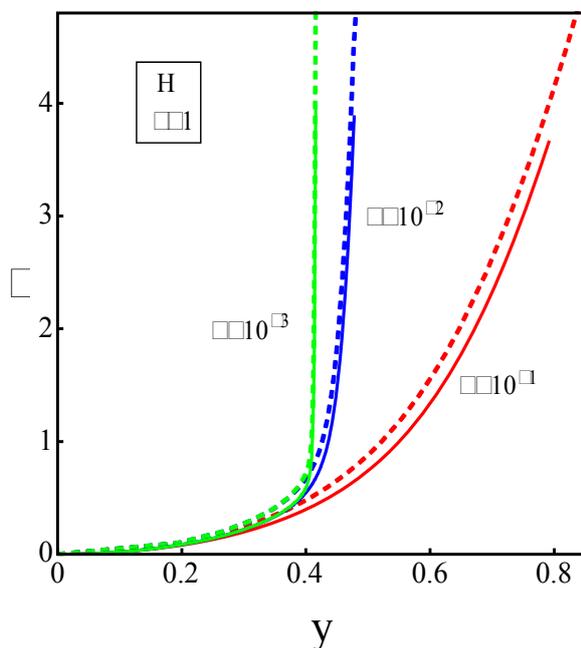


Figure 1: Potential profiles with characteristic values for α (0.1, 0.01 and 0.001) with $\gamma=1$ for atomic hydrogen (H). Filled curves correspond to the model described by the present work, while dashed curves indicate solutions given by Self.

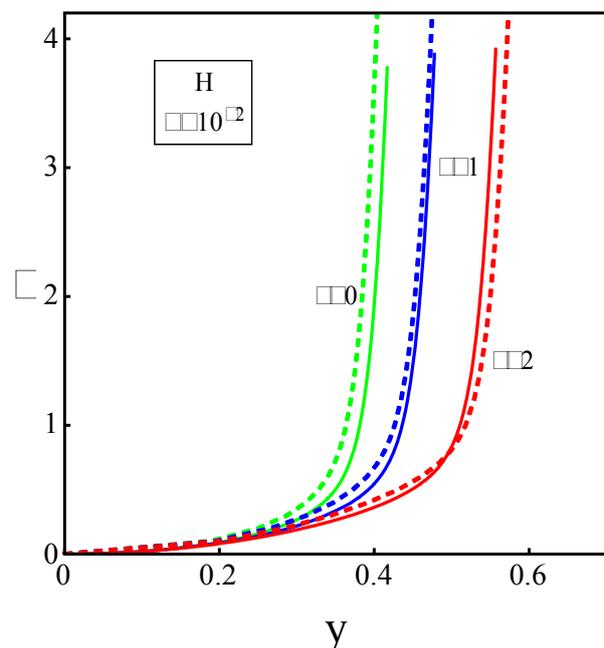


Figure 2: Potential profiles with characteristic values for γ (0, 1 and 2) with $\alpha = 0.01$ for atomic hydrogen (H). Filled curves correspond to the model described by the present work, while dashed curves indicate solutions given by Self.

It is interesting to note that Self solutions are independent of the ionic to electronic mass ratio for a given α , since he neglected electron inertia and used a convenient normalization of the spatial coordinate, so that the ionic mass disappeared explicitly from the characteristic equation of his model and the wall position (which does depend on the ionic to electronic mass ratio) is deduced *a posteriori*, equating the ion flux to the one side random flux of the Maxwellian electrons.

In order to better show the differences between Self results and ours, in table 1 we compare values of the electric floating potential and wall position corresponding to H and Hg for some typical values for α and different γ . η_w and s_w correspond to Self values for wall dimensionless potential and dimensionless wall position, while ϕ_w and y_w correspond to ours. As it can be seen, differences are not really significant, but the inclusion of electron inertia allows us to define the floating wall potential in consistency with the model equations.

Table 1: Comparison for dimensionless plasma wall potentials and dimensionless wall position obtained by Self (s_w, η_w) and by the present work (y_w, ϕ_w) for H and Hg and different values of γ and α .

| $\gamma=0$ | | | | | | | | | |
|------------|----------|-------|----------|-------|----------|-------|----------|-------|--|
| α | H | | | | Hg | | | | |
| | η_w | s_w | ϕ_w | y_w | η_w | s_w | ϕ_w | y_w | |
| 0.1 | 2.98 | 0.618 | 3.29 | 0.665 | 5.47 | 0.723 | 5.81 | 0.769 | |
| 0.01 | 3.41 | 0.398 | 3.78 | 0.417 | 6.03 | 0.414 | 6.39 | 0.432 | |
| 0.001 | 3.53 | 0.353 | 3.91 | 0.364 | 6.18 | 0.355 | 6.56 | 0.366 | |
| $\gamma=1$ | | | | | | | | | |
| α | H | | | | Hg | | | | |
| | η_w | s_w | ϕ_w | y_w | η_w | s_w | ϕ_w | y_w | |
| 0.1 | 3.29 | 0.748 | 3.65 | 0.791 | 5.94 | 0.893 | 6.30 | 0.925 | |
| 0.01 | 3.50 | 0.469 | 3.87 | 0.478 | 6.16 | 0.487 | 6.53 | 0.494 | |
| 0.001 | 3.55 | 0.415 | 3.93 | 0.416 | 6.20 | 0.417 | 6.58 | 0.418 | |
| $\gamma=2$ | | | | | | | | | |
| α | H | | | | Hg | | | | |
| | η_w | s_w | ϕ_w | y_w | η_w | s_w | ϕ_w | y_w | |
| 0.1 | 3.43 | 0.910 | 3.79 | 0.932 | 6.08 | 1.075 | 6.42 | 1.082 | |
| 0.01 | 3.54 | 0.568 | 3.91 | 0.557 | 6.19 | 0.586 | 6.56 | 0.574 | |
| 0.001 | 3.56 | 0.504 | 3.93 | 0.485 | 6.21 | 0.505 | 6.58 | 0.487 | |

In figure 3, a typical result for ion and electron densities as function of position, corresponding to H, $\alpha=0.01$ and $\gamma=1$, is shown. As it can be seen, ion and electron densities differ considerably at the wall position. Inside the figure, an enlargement of the detail for electron density close to the wall is shown. For comparison, when the electron density of our model is plotted together with that corresponding to a Maxwell-Boltzmann distribution, they practically overlap up to very close to the wall. The difference increases for larger values of α and diminishes for smaller α . The abrupt fall of the electron density of our model is due to the fact that electron velocity is reaching the sonic one at the wall.

In figure 4 we show, for H and the same parameters as in figure 3, the ion and electron dimensionless velocity profiles. The ion velocity is multiplied by a factor $\sqrt{m_i/m_e}$ so that unity corresponds to ion sound velocity as defined in literature for cold ions. Comparing with figure 3, it is possible to appreciate that when the ionic velocity is close to unity, ion and electron density are still very close, while they are well separated when the ions reach the wall and their dimensionless velocity is about 2.7 their sonic one. The electron velocity profile clearly shows how the electrons reach their thermal velocity at the wall.

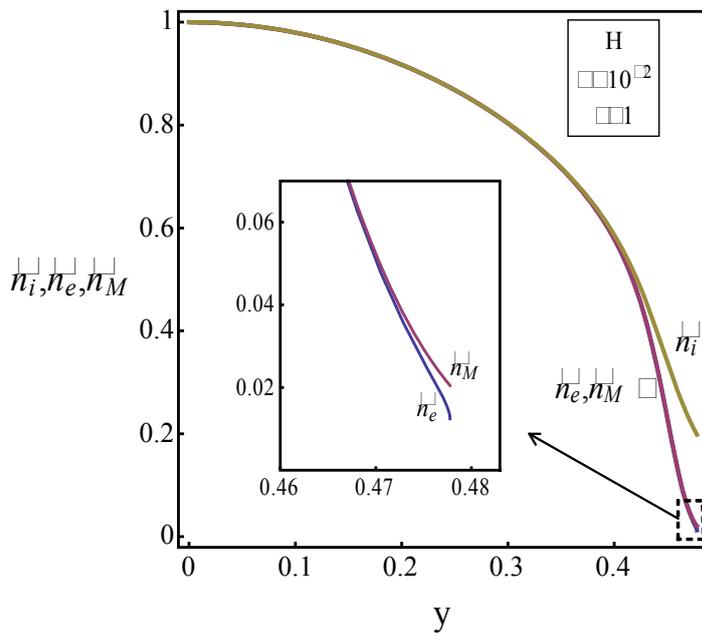


Figure 3: Dimensionless density profiles for electrons (\tilde{n}_e) and ions (\tilde{n}_i) together with electron density due to Maxwellian distribution (\tilde{n}_M). In order to show precisely the difference between our electron density and the commonly assumed Maxwell-Boltzmann distribution, we enlarged in the same figure a small region near the wall, where $\frac{d\tilde{n}_e}{dy} \rightarrow -\infty$.

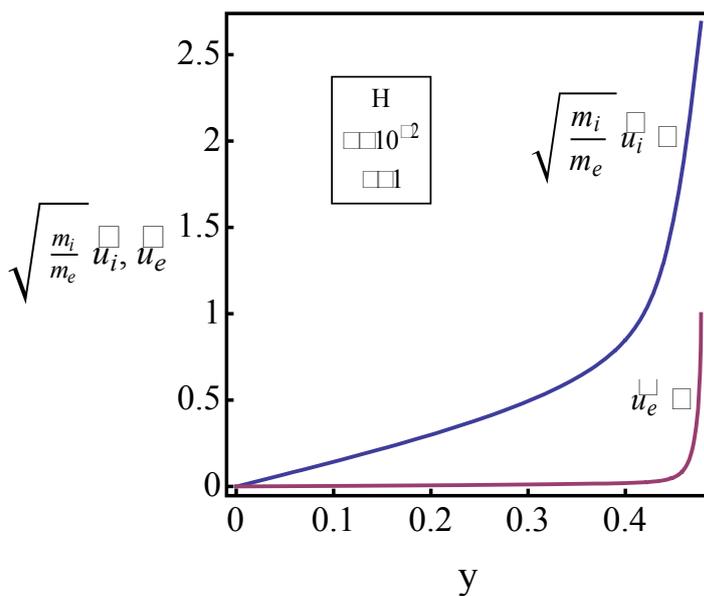


Figure 4: Ion (upper curve) and electron (lower curve) velocities, normalized to their own sound velocities, from the center to the wall for H and the same parameters as in figure 3. As \tilde{u}_e approaches 1, the integration has to be aborted as electron density derivative, in the electron momentum equation, diverges.

As it can be appreciated, the inclusion of electron inertia in the treatment of the classic plasma sheath problem introduces little changes to the floating plasma wall potential when compared with results from models that simply assume Maxwell-Boltzmann distribution for electrons and neglect their velocity. The differences are essentially due to an abrupt fall in electron density close to the end of the discharge, while the velocity with which electrons impinge on the wall is the same (insofar as electron temperature is maintained constant). In our model, electrons attain their sonic speed at the wall, and then a shock should build up there, where electron density and velocity approach discontinuities while the electric potential is continuous. Such description may be a physically attractive alternative to the usual assumption of equating the ion flux to the one side random electron flux.

Acknowledgments

The authors would like to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for financial support under Grants 2008/55778-8 and 2009/11835-0.

References

- [1] Tonks L and Langmuir I 1929 *Phys. Rev.* **34** 876
- [2] Bohm D 1949 *The Characteristics of Electrical Discharges in Magnetic Fields*, ed A Guthrie and R K Wakerling (New York, McGraw-Hill) pp 77-86
- [3] Caruso A and Cavaliere A 1962 *Nuovo Cimento* **26** 1389
- [4] Self S A 1963 *Phys. Fluids* **6** 1762
- [5] Bittencourt J A 2004 *Fundamentals of Plasma Physics* (New York, Springer) pp 193-212, 279-91
- [6] Kino G S and Shaw E K 1966 *Phys. Fluids* **9** 587
- [7] Allen J E 2009 *Plasma Sources Sci. Technol.* **18** 014004
- [8] Friedman H W and Levi E 1967 *Phys. Fluids* **10** 1499
- [9] Riemann K-U 2009 *Plasma Sources Sci. Technol.* **18** 014007
- [10] Stangeby P C and Allen J E 1970 *J. Phys. A: Gen. Phys.* **3** 304
- [11] Ingold J H 1972 *Phys. Fluids* **15** 75
- [12] Riemann K-U 1991 *J. Phys. D: Appl. Phys.* **24** 493
- [13] Harrison E R and Thompson W B 1959 *Proc. Phys. Soc.* **74** 145
- [14] Bissell R C, Johnson P C and Stangeby P C 1989 *Phys. Fluids B* **1** 1133
- [15] Godyak V A and Sternberg N 1990 *IEEE Trans. Plasma Sci.* **18** 159
- [16] Alterkop B 2004 *J. Appl. Phys.* **95** 1650
- [17] Deka U and Dwivedi C B 2010 *Braz. J. Phys.* **40** 333
- [18] Morales Crespo R, Fernández Palop J I, Ballesteros J, Hernández M A, Lucena-Polonio M V and Díaz-Cabrera J M 2010 *Plasma Sources Sci. Technol.* **19** 025012
- [19] Robertson S 2009 *Phys. Plasmas* **16** 103503
- [20] Lam S H 1964 *Phys. Fluids* **8** 73
- [21] Senda I 1995 *Phys. Plasmas* **2** 6
- [22] Hu P N and Ziering S 1966 *Phys. Fluids* **9** 2168
- [23] Mathematica 7.0, Wolfram Research Inc.