Mechanism of nonlinear oscillitons evolved from simple solitary waves

John Z. G. Ma[1] **and A. Hirose**[2]

[1]Space Science Branch, Canadian Space Agency, 6767, Route de l'Aéroport, Longueuil (Saint-Hubert), Québec, J3Y 8Y9, Canada
[2]Plasma Physics Laboratory, University of Saskatchewan, 116 Science Place, Saskatoon, Saskatchewan, S7N 5E2, Canada

E-mail: [1]johnzg.ma@asc-csa.gc.ca and [2]akira.hirose@usask.ca

Abstract. Besides three conventional, nonlinear simple waves (namely, sinusoidal, sawtooth, and spicky / bipolar) measured in space plasmas, numerous extraordinary nonlinear oscillitons (i.e., amplitude-modulated solitons) have been captured by satellites in last decades. By employing a self-similar, two-fluid (electron-ion) model, we focus on the evolution from single-frequency solitary waves to multi-frequency oscillitons. It is shown that the structures can be triggered by either the charge non-neutrality auto-modulationally, or the electron inertia mutual-modulationally.

1. Introduction

For more than 70 years, a salient nonlinear waveform has been captured in solar-terrestrial space by numerous experiments: a low-frequeny (LF) envelop, while the amplitude of which is modulated violently by higher-frequency (HF) oscillations. In extraterrestrial spaces, detections of such LF-HF "symbiotic" packets began from the 1980s when Voyager 1 & 2 spacecrafts in the upstream of Jupiter's bow shock[1]. In the vicinity of Io, the Galileo spacecraft recorded similar waveforms which were later supposed to be connected with ion-cyclotron (IC) waves[2]. In the Martian upstream region, Mars Global Surveyor confirmed that such structures also exist in the bow shock where the HF signature is exactly at the local proton IC frequency while the LF component is at about seven gyro-periods[3]. In geospace, many high-resolution satellites introduced various nonlinear HF-LF wavefield diagnoses in different plasma regions, such as Freja[4-6], Polar[7, 8], FAST[9, 10], Viking[11, 12], and Cluster[11-13]. Even low-altitude rockets measured this kind of waveforms in auroral regions[14].

Nomenclaturally, this kind of extraordinary nonlinear envelops was described as "oscillitons" (i.e., "oscillations"+"solitons")[15]. In view of nonlinear electromagnetic whistler modes, many authors accepted that oscillitons are excited in multi-ion plasmas in propagations parallel to, and supposedly also, oblique to a static, background magnetic field[16-26]. Impressively, Kourakis & Shukla[27] developed a generic methodological formulation for oscillitons by taking into account either the mutual modulation effect (due to the interaction between HF and LF modes) or the auto-modulation one (due to the self-interaction of a carrier mode). As a matter of fact, in studying large-amplitude, electromagnetic, nonlinear waves in electron-positron plasmas[28], Verheest et al.[29] perceived a sensitive quality, that was unrecognized previously, of the whistler


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oscillitons through numerical computations: At not too large amplitudes, oscillitons are very similar to single-frequency solitary envelops obtained by solving the Schrödinger equation, either or not the charge neutrality condition is imposed as a priori one.

Intrigued by this work, we hope to make it clear about not only the role played by this condition, but also the presence of electron mass (i.e., the electron inertia), in the excitation of conventional single-frequency solitary envelops and muliti-frequency oscilliton structures. We pay attention to parallel-propagating cases in cylindrical and Cartesian coordinates, respectively, under electrostatic and electromagnetic conditions. More specific descriptions of the modeling are presented in Ma & Hirose [30, 31].

2. Simple nonlinear waves in a nutshell

Since the first soliton (also called a solitary wave) was noticed by John Scott Russell in 1834, and the first prediction was made about the existence of the non-wave structures in plasmas (called the Bernstein-Green-Kruskal, BGK, mode) in 1957, simple-wave (i.e., single-frequency) solitary structures have been observed and modeled in solving different problems relevant to the construction, maintenance, propagation, and effects of the three kinds of solitary structures (sinusoidal, sawtooth, and spiky/bipolar; see a detailed introduction in[32]).

From the FFT spectra of parallel-propagating ion-acoustic (IA) solitary waves in the absence of electron inertia under charge-neutrality conditions[32, 33], it is easy to find that the wave is featured by a dominant peak-frequency, accompanied by its harmonic peaks, due to the fact that the solitary structures do not have pure sinusoidal waveforms. The farther the shape of solitons away from a sine wave, the more the harmonic peaks to be developed (see, for example, two Fig.3 plots in [30, 31]). It deserves to mention here that the dominant peak in the power spectra may not be located at the fundamental frequency. This warns us that, in data analysis to identify in-situ nonlinear waves, a frequency with the largest amplitude may not represent that the wave is excited at that frequency.

3. Auto-modulation process

For a direct comparison to the simple-wave case under charge-neutrality conditions[33], we choose a cylindrical geometry, where the magnetic field $\mathbf{B} = B\hat{\mathbf{e}}_z$ is along the axial z-direction. A self-similar, two-fluid plasma model under charge non-neutrality conditions is employed for a parameterized study to show the modulation of single-frequency solitary structures to multi-frequency oscillitons. By normalizing ion and electron densities (n_i, n_e) with the equilibrium density n_0, coordinates r and z with the ion gyro-radius ρ_i=c_s/Ω_i (where c_s= $\sqrt{k_B T_e/m_i}$ is the ion acoustic speed, $\Omega_i = eB/m_i$ is the ion gyrofrequency, k_B is the Boltzmann constant, T_e is the Boltzmann electron temperature, e is the electron charge, and m_i is the ion mass), velocity components (u_r, u_ϕ, u_z) with c_s, time t with $1/\Omega_i$, the perturbed potential φ with $k_B T_e/e$, and writing all dimensionless variables by using N_i=n_i/n_0, N_e=n_e/n_0, R=r/ρ_i, Z=z/ρ_i, V_r=u_r/c_s, V_ϕ=u_ϕ/c_s, V_z=u_z/c_s, τ=$\Omega_i t$, Φ=$e\varphi/(k_B T_e)$ (note that φ satisfies $\mathbf{E} = -\nabla\varphi$ where $\mathbf{E}$ denotes the electrostatic electric field) in a self-similar transformation $X = (\alpha_1/M)R + (\alpha_2/M)Z - \tau$ (where M is the Mach number, α_1=sinθ, and α_2=cosθ in which θ is the inclination angle between the propagation direction and the magnetic field), we obtain a dimensionless set of differential equations as follows:

$$\left.\begin{array}{l} \left(1-\frac{U_z}{M}\right)\frac{\mathrm{dln}N_i}{\mathrm{d}X} - \frac{1}{M}\frac{\mathrm{d}U_z}{\mathrm{d}X} = \frac{U_r}{R}\ ,\ \left(1-\frac{U_z}{M}\right)\frac{\mathrm{d}U_r}{\mathrm{d}X} = -\left(\frac{U_\phi}{R}+1\right)U_\phi \\ \left(1-\frac{U_z}{M}\right)\frac{\mathrm{d}U_\phi}{\mathrm{d}X} = \left(\frac{U_\phi}{R}+1\right)U_r\ ,\ \left(1-\frac{U_z}{M}\right)\frac{\mathrm{d}U_z}{\mathrm{d}X} = \frac{1}{M}\frac{\mathrm{d}\left(\frac{N_i}{\xi_T}+\Phi\right)}{\mathrm{d}X} \\ \frac{\mathrm{d}^2\Phi}{\mathrm{d}X^2} = M^2\xi_\omega^2(N_e-N_i)\ ,\ N_e = e^\Phi \end{array}\right\} \quad (1)$$

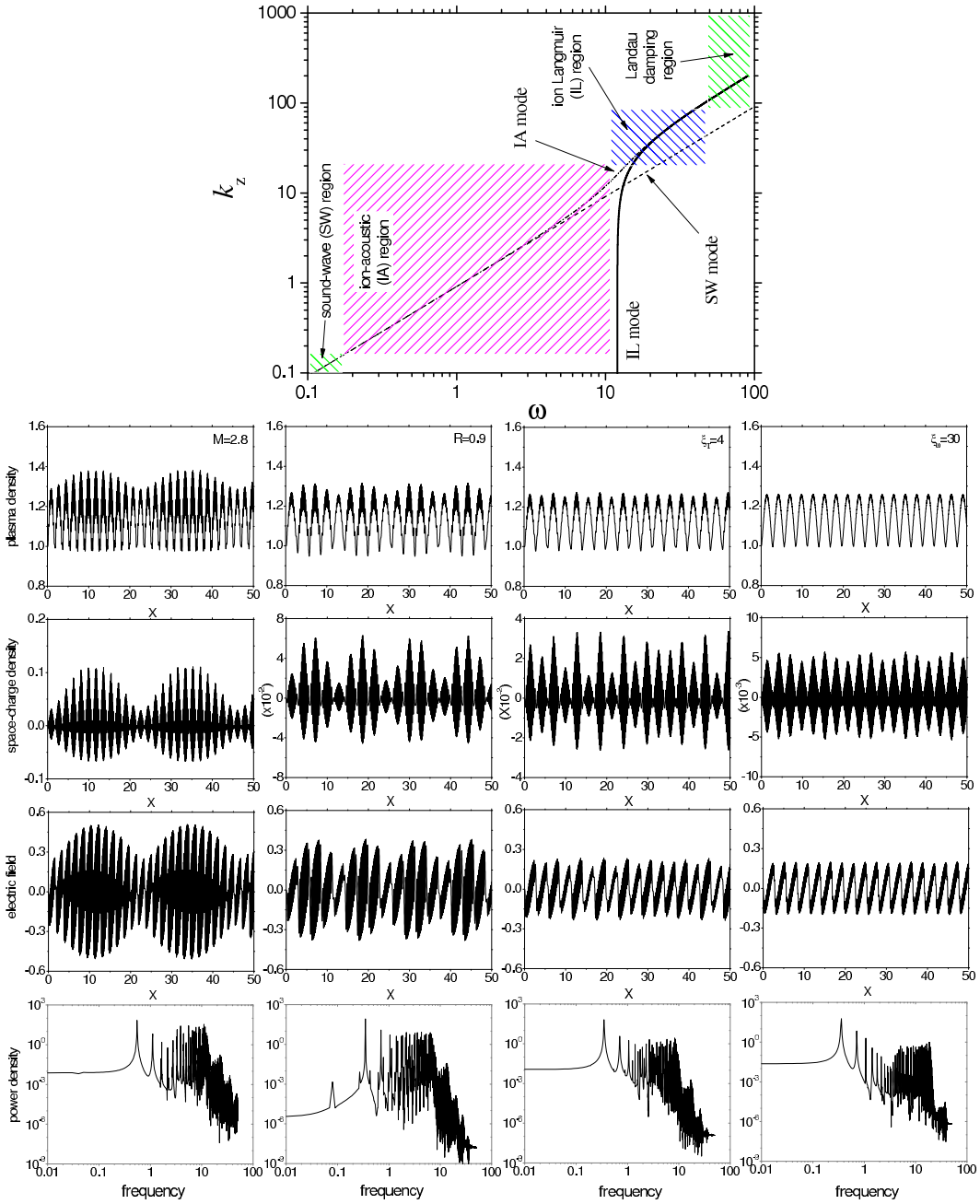


Figure 1. Upper: Background linear oscillation modes in the $\omega-k$ plane with ξ_T=5 and ξ_ω=12. Lower nine panels: Modulated oscillitons by input parameters M (left column), R (middle left column), ξ_T (middle right column), and ξ_ω (right column), respectively, in view of plasma density (top row), space-charge density (second row), electric field (third row), and FFT power density (bottom row), respectively.

in which ξ_T=c_s^2/v_{Ti}^2=$T_e/(2T_{i0})$, and ξ_ω=ω_{pi}/Ω_i= ρ_i/λ_{De} where ω_{pi} is the ion plasma frequency, and λ_{De}=c_s/ω_{pi}= $\sqrt{\epsilon_0 k_B T_e/(n_0 e^2)}$ is the electron Debye length. Naturally, $c_s k_{De}$=ω_{pi} where k_{De}=λ_{De}^{-1} is the electron Debye wave number. Notice that $\theta = 0°$ due to the parallel-propagating case. By linearizing Eq.(1), the background linear oscillation modes in the $\omega-k$ plane are shown

in the upper panel of Fig.1.

The lower nine panels of the figure exhibit multi-frequency oscillitons via the auto-modulation process under charge non-neutrality conditions. The LF envelop is in the sound-wave (SW) mode, while the HF ingredients include the IA mode and the ion-Langmiur (IL) mode. The amplitudes of the SW wave are violently modulated by the IA oscillations, whileas the upward sides of the IA amplitudes are modulated by the IL oscillations of smaller amplitudes, and the downward sides are modulated by hybrid IA/IL oscillations.

4. Mutual modulation process

In cases where the electron inertia comes into play, there exist electron-ion mutual modulations (see details in [30]). For a convenient comparison to the simple-wave case in the absence of the electron inertia [32], we choose a Cartesian geometry. We use following dimensionless parameters: the electron and ion densities n_e and n_i are normalized by n_0; coordinates $\mathbf{r}=\{x,y,z\}$ by electron Debye length $\lambda_{De}=v_{Te}/\omega_{pe}$; velocity $\mathbf{u}_\alpha=\{u_{\alpha x},u_{\alpha y},u_{\alpha z}\}$ by the acoustic speed $c_s=\sqrt{k_BT_{e0}/m_i}$; pressure p_α by $p_{\alpha 0}=n_0k_BT_{\alpha 0}$; time t by ion plasma period $\tau_{pi}=\omega_{pi}^{-1}$ (notice that $\omega_{pi}\lambda_{De}=c_s$); magnetic field $\mathbf{B}$ by a pseudo-magnetic field $B_0=m_e\omega_{pi}/e$; electric field $\mathbf{E}$ by a pseudo-electric field $E_0=c_sB_0$; and, three coefficients are introduced: $\xi_m=m_i/m_e$, $\xi_T=T_{e0}/T_{i0}$, and $\xi_v=v_{Te}/c$. Using $X=x-Mt$, the dimensionless set of self-similar differential equations of the two-fluid model under electromagnetic conditions in a Cartesian frame is as follows:

$$\left.\begin{array}{l}
\frac{\mathrm{d}u_{ex}}{\mathrm{d}X}=-(E_x+u_{ey}B_z-u_{ez}B_y)/\left(u_{ex}-\frac{\xi_m}{u_{ex}}\right)\\
\frac{\mathrm{d}u_{ey}}{\mathrm{d}X}=-(S_y-u_{ex}B_z+u_{ez}B_{x0})/u_{ex}\ ,\ \frac{\mathrm{d}u_{ez}}{\mathrm{d}X}=-(S_z+u_{ex}B_y-u_{ey}B_{x0})/u_{ex}\\
\frac{\mathrm{d}u_{ix}}{\mathrm{d}X}=\xi_m^{-1}(E_x+u_{iy}B_z-u_{iz}B_y)/\left(u_{ix}-\frac{3S_n^2}{\xi_Tu_{ix}^3}\right)\\
\frac{\mathrm{d}u_{iy}}{\mathrm{d}X}=\xi_m^{-1}(S_y-u_{ix}B_z+u_{iz}B_{x0})/u_{ix}\ ,\ \frac{\mathrm{d}u_{iz}}{\mathrm{d}X}=\xi_m^{-1}(S_z+u_{ix}B_y-u_{iy}B_{x0})/u_{ix}\\
\frac{\mathrm{d}E_x}{\mathrm{d}X}=-\xi_mn_{sc}\\
\frac{\mathrm{d}B_y}{\mathrm{d}X}=\xi_v^2S_n\left(\frac{u_{iz}}{u_{ix}}-\frac{u_{ez}}{u_{ex}}\right)/\left(1-\frac{\xi_v^2M^2}{\xi_m}\right)\ ,\ \frac{\mathrm{d}B_z}{\mathrm{d}X}=-\xi_v^2S_n\left(\frac{u_{iy}}{u_{ix}}-\frac{u_{ey}}{u_{ex}}\right)/\left(1-\frac{\xi_v^2M^2}{\xi_m}\right)
\end{array}\right\}\quad(2)$$

in which $S_n=u_{xx0}-M$, $S_y=E_{y0}-MB_{z0}$, $S_z=E_{z0}+MB_{y0}$, $n_e=S_n/u_{ex}$, $n_i=S_n/u_{ix}$, $n_{sc}=n_e-n_i$, $E_y=S_y+MB_z$, $E_z=S_z-MB_y$, and $B_x=B_{x0}$. Following boundary conditions are used: $n_e|_{X=0}=n_0$, $n_i|_{X=0}=n_0$, $u_{xx0}=u_{ex0}|_{X=0}=u_{ix0}|_{X=0}=U_0$-$M$, $E_y|_{X=0}=E_{y0}$, $E_z|_{X=0}=E_{z0}$, $B_x|_{X=0}=B_{x0}$, $B_y|_{X=0}=B_{y0}$, and $B_z|_{X=0}=B_{z0}$. Note that the $u_{\alpha x}$-origin is shifted from "0" to "M", and the density equations and Gauss's law require $S_{ne}=S_{ni}$, written as S_n.

Fig.2 exposes lower-hybrid (LH) oscillitons evolved from single-frequency IA/IC solitary waves under different Mach number, M (top left), and temperature ratio, Te/Ti (top right). To have a qualitative comparison with data, the figure also gives a measurement of "oscilliton" structures in the lower left panel, which was believed to be associated with the LH mode at the plasma sheet boundary (see [34] for details). Under $U_0=(0.2,0,0)$, $E_0=(0.5,0.5,1)$, and $B_0=(0.2,0.5,3)$ with M=1, m_i/m_p=16, T_e/T_i=10, and v_{Te}/c=0.05, the lower right panel depicts a simulation result. Obviously, the electron inertia triggers LH oscillitons characterized by a normal IC-period solitary envelop embedded by LH oscillations which contain higher-frequency but smaller-amplitude IA constituents. This means that the mutual modulation effect between ions and electrons exerts an impact on nonlinear packets via several input parameters like the Mach number and the temperature ratio. The influence of other input parameters (such as ion mass, electron speed) was discussed in [30]. Impressively, Whenever an oscilliton structure is formed, there occurs a density cavity. Its depth can reach up to 20% of the background density, and on both sides of the cavity, there occur density humps. The feature is in accordance with either observations or simulations (see a detailed discussion in [30]).

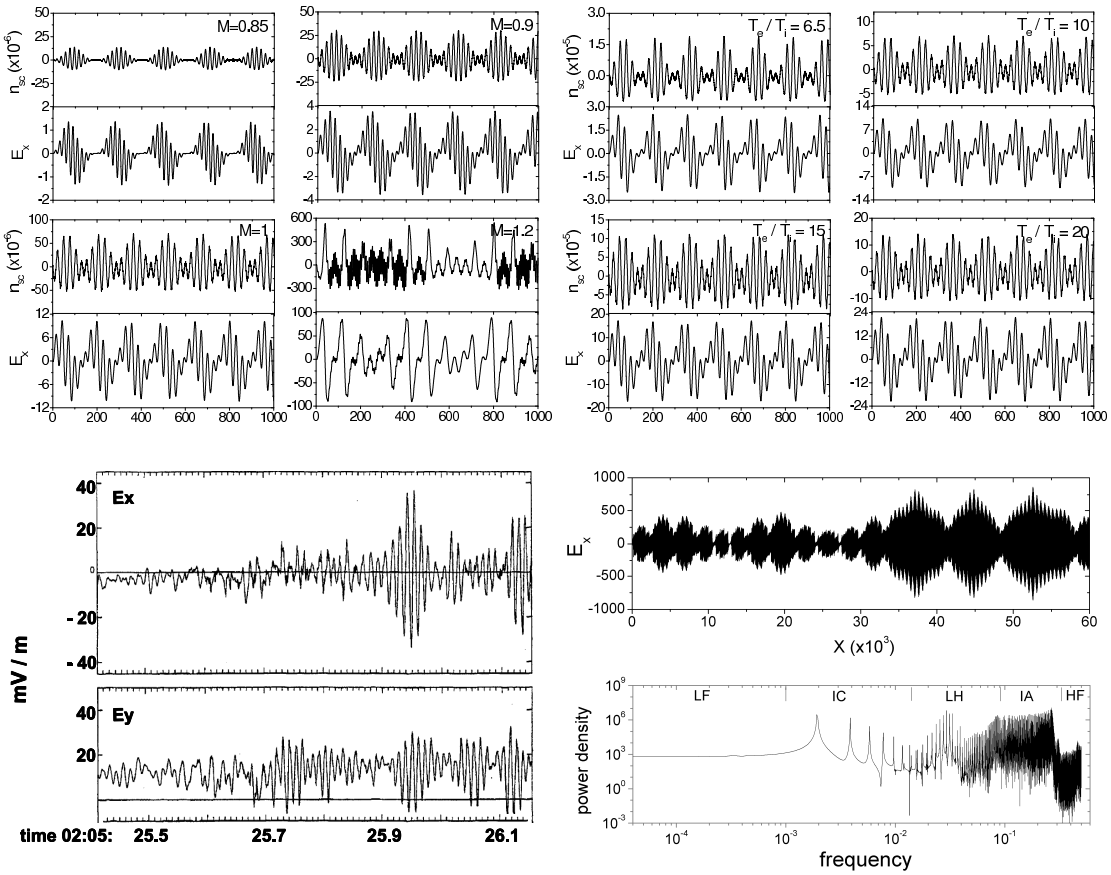


Figure 2. Upper left: Impact of the Mach number M on LH-oscilliton waves. Only space-charge density n_{sc} and electric wavefield E_x-component are shown. Notice that the panels of M=1 is the typical case discussed in the last Section. Upper right: Same as the upper left but the impact of $\xi_T = T_e/T_i$ on LH-oscilliton waves. Lower left: Observed "oscilliton" structures associated with the LH mode by the Polar satellite at the plasma sheet boundary (adapted from Fig.3 of [34]). Lower right: A simulation to observed LH oscillitons in the lower left panel under $U_0 = (0.2, 0, 0)$, $E_0 = (0.5, 0.5, 1)$, and $B_0 = (0.2, 0.5, 3)$ with M=1, m_i/m_p=16, T_e/T_i=10, and v_{Te}/c=0.05. Reproduction of a few figures in [30].

5. Conclusion

Nonlinear oscillitons are evolved from simple solitary waves under certain conditions. If the charge non-neutrality condition is satisfied, an auto-modulation process brings single-frequency solitons into multi-frequency envelops of oscillitons. By contrast, if the electron inertia cannot be neglected, a mutual-modulation process brings solitons into oscillitons.

In the first case, the low-frequency envelop of the oscillitons is in the sound-wave mode, while the high-frequency portion includes the IA mode and the ion-Langmiur mode. The sound-wave amplitudes are violently modulated by the ion-acoustic mode. The upward edges of the ion-acoustic amplitudes are modulated by the ion-Langmiur oscillations of smaller amplitudes, while the downward edges are modulated by hybrid ion-acoustic/ion-Langmiur oscillations.

In the second case, the simple IA/IC waves are developed to lower-hybrid oscillitons where the low-frequency ion-cyclotron envelop is embedded by small-amplitude, high-frequency lower-hybrid oscillations. The packet is superimposed upon by higher-frequency but smaller-amplitude ion-acoustic ingredients. Whenever an oscilliton is triggered, there occurs a density cavity the

depth of which can reach up to 20% of the background density, along with density humps on both sides of the cavity.

In either cases, the oscillitons are shown to be sensitive to input parameters (e.g., the Mach number, the electron-ion mass/temperature ratios, and the electron thermal speed).

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