Journal of Physics: Conference Series **509** (2014) 012059 doi:10.1088/1742-6596/509/1/012059

Relativistic hydrodynamics on graphics processing units

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Abstract. Hydrodynamics calculations have been successfully used in studies of the bulk properties of the Quark-Gluon Plasma, particularly of elliptic flow and shear viscosity. However, there are areas (for instance event-by-event simulations for flow fluctuations and higher-order flow harmonics studies) where further advancement is hampered by lack of efficient and precise 3+1D program. This problem can be solved by using Graphics Processing Unit (GPU) computing, which offers unprecedented increase of the computing power compared to standard CPU simulations. In this work, we present an implementation of 3+1D ideal hydrodynamics simulations on the Graphics Processing Unit using Nvidia CUDA framework. MUSTA-FORCE (MUlti STAge, First ORder CEntral, with a slope limiter and MUSCL reconstruction) and WENO (Weighted Essentially Non-Oscillating) schemes are employed in the simulations, delivering second (MUSTA-FORCE), fifth and seventh (WENO) order of accuracy. Third order Runge-Kutta scheme was used for integration in the time domain. Our implementation improves the performance by about 2 orders of magnitude compared to a single threaded program. The algorithm tests of 1+1D shock tube and 3+1D simulations with ellipsoidal and Hubble-like expansion are presented.

1. Numerical algorithms

The equations of hydrodynamics can be written as a system of conservative equations

$$\frac{\partial u}{\partial t} + \sum_i \frac{\partial f_i(u)}{\partial x_i} = 0 \tag{1}$$

where the sum runs over all spacial dimensions, and $u = u(x)$ represents a conserved variable, which include the energy density, momenta densities and conserved charge densities. The equation is discretized on a cartesian grid into an ordinary differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \sum_i \frac{1}{|e_i|}\left(f_i\left(u\left(x - \frac{e_i}{2}\right)\right) - f_i\left(u\left(x + \frac{e_i}{2}\right)\right)\right) \equiv L(u) \tag{2}$$

where e_i are the primitive vectors, i.e. vectors between two adjacent cells in the positive direction of dimension i. It is then integrated using a standard, third order Runge-Kutta algorithm. The final discretized operator L depends on a number of neighbouring cells on the lattice in all spacial directions.

1.1. MUSTA-FORCE scheme

In order to obtain a general and accurate algorithm for estimating L we use a hybrid MUSTA (MUlti-STAge) approach [1, 2]. It utilizes a centered flux in a predictor-corrector loop, solving the intercell Riemann problem numerically, without using a priori information about waves that propagate in this system.

The algorithm is extended with the MUSCL scheme, which uses linear interpolation inside the cells for second order accuracy. To reduce oscillations that arise in the vicinity of strong gradients, we used a variety of slope limiting methods.

1.2. WENO scheme

Due to high numerical cost and complexity of the MUSTA algorithm the finite difference WENO scheme was also implemented [3, 4, 5, 6, 7]. It uses polynomial interpolations from multiple stencils, a convex combination of which, with suitably non-linear coefficients, lead to a scheme that stays high order in smooth regions of the solution and discards stencils containing shocks, thus avoiding spurious oscillations.

In our program the fifth (WENO5) and seventh (WENO7) order accurate schemes were implemented.

2. Technical aspects

The code is written using the NVIDIA CUDA programming framework, which produces binaries for NVIDIA graphics processing units. We compared our implementation of the MUSTA-FORCE algorithm with an equivalent implementation for the central processing unit. Using contemporary hardware the GPU code executed approximately 200 times faster.

3. Results

We found that MUSTA-FORCE algorithm is more computationally expensive and gives less accurate description of shock waves compared to WENO algorithms. Thus only results obtained with WENO schemes are presented.

Figures 1 and 2 show a comparison of the WENO5 scheme simulation with an analytical solution to the shock tube problem. The results from WENO7 reproduce shock front better at the cost of slight oscillations in the flat regions.

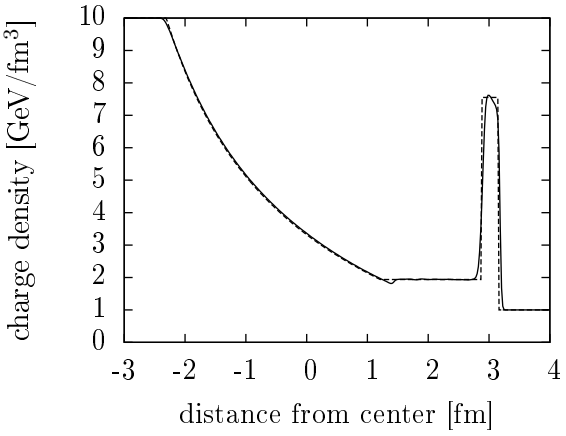

Figure 1. Sod shock tube, charge density.

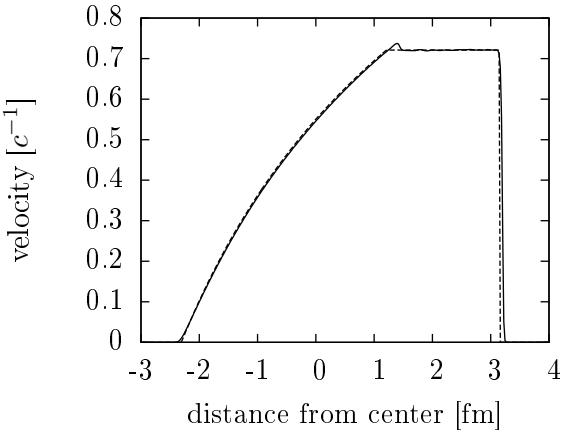

Figure 2. Sod shock tube, velocity.

The following two simulations of ellipsoidal flow solution [8] and Hubble-like solution were done in full 3+1D. On figures 3–6 one dimensional sections are shown. Both WENO5 and WENO7

shown similiar results in these tests—higher order simulations were slightly more accurate in both cases.

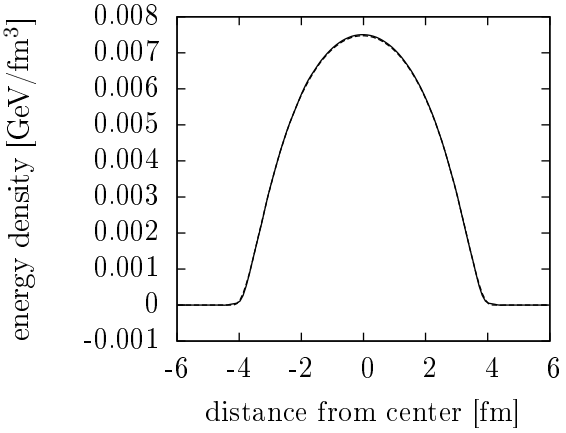

Figure 3. Ellipsoidal flow, energy density (WENO7).

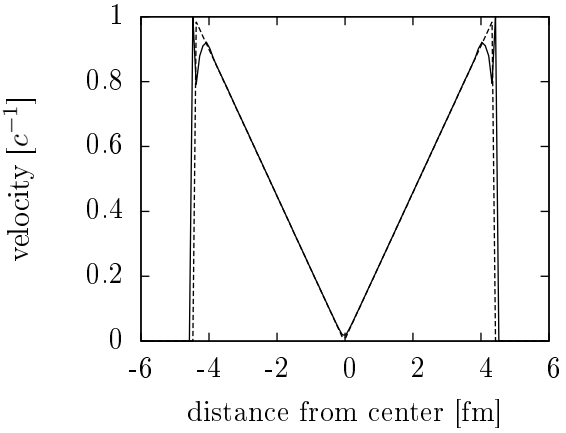

Figure 4. Ellipsoidal flow, velocity (WENO7).

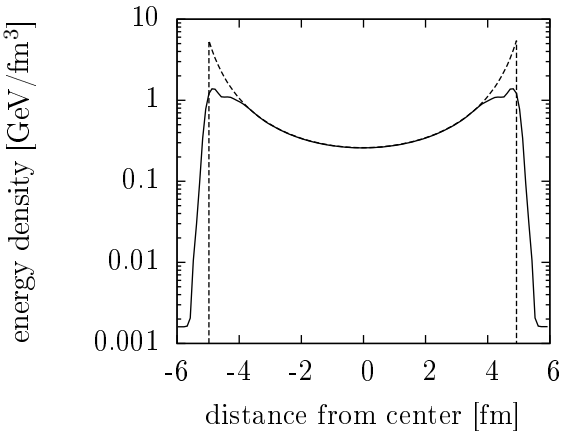

Figure 5. Hubble-like flow, energy density (WENO5).

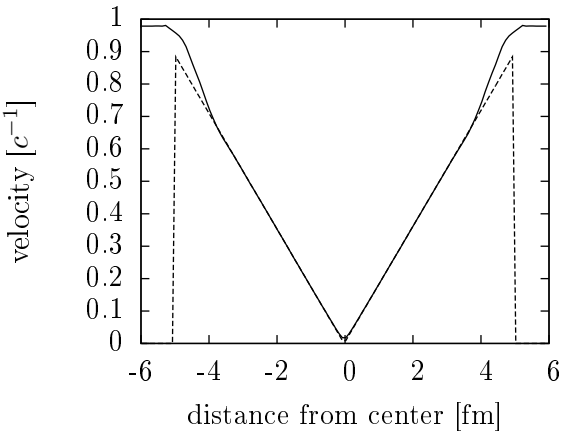

Figure 6. Hubble-like flow, velocity (WENO5).

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