

# Diffusion of non-Gaussianity in heavy ion collisions

Masakiyo Kitazawa, Masayuki Asakawa, Hirosato Ono

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

E-mail: [kitazawa@phys.sci.osaka-u.ac.jp](mailto:kitazawa@phys.sci.osaka-u.ac.jp)

**Abstract.** We investigate the time evolution of higher order cumulants of bulk fluctuations of conserved charges in the hadronic stage in relativistic heavy ion collisions. The dynamical evolution of non-Gaussian fluctuations is modeled by the diffusion master equation. Using this model we predict that the fourth-order cumulant of net-electric charge is suppressed compared with the recently observed second-order one at ALICE for a reasonable parameter range. Significance of the measurements of various cumulants as functions of rapidity window to probe dynamical history of the hot medium created by heavy ion collisions is emphasized.

## 1. Introduction

Bulk fluctuations of conserved charges, especially higher order cumulants characterizing non-Gaussianity, are believed to be promising experimental observables to reveal thermodynamic properties of the hot medium created by relativistic heavy ion collisions [1, 2]. These observables are experimentally measured by event-by-event analysis, and active analyses of cumulants have been performed at RHIC [3] and LHC [4]; In particular, fourth-order cumulants are measured with a good statistics at RHIC. Numerical analyses of higher order cumulants of conserved charges in equilibrium have been also carried out in lattice QCD Monte Carlo simulations.

An important property of the conserved-charge fluctuations in heavy ion collisions which has been clarified by the recent experimental results is that the experimentally-observed fluctuations are not those of the equilibrated medium at some stage in the hadronic medium. In fact, suppression of the net-electric charge fluctuation observed at ALICE and especially its strong rapidity window dependence [4] clearly show that the fluctuation is not equilibrated at LHC energy. Moreover, a comparison of the fluctuations observed at STAR with the cumulants measured on the lattice [5] also indicates that the fluctuations observed at RHIC are not consistent with the equilibrated values at chemical freezeout. These results show that an appropriate description of the non-equilibrium nature of fluctuations owing to dynamical evolution of the hot medium is inevitably needed to investigate their thermodynamic properties using the fluctuation observables, especially those of conserved charges.

In the present study, we investigate the time evolution of higher order cumulants of conserved charges in the hadronic stage using the diffusion master equation [6]. After hadronization, the fluctuations approach the equilibrated distribution in the hadronic medium, which is approximately given by the Skellam distribution [7]. Our approach can describe this feature of the non-Gaussian fluctuations, while the time evolution of the average and Gaussian fluctuations are consistent with the ones in the stochastic diffusion equation [8]. By analyzing the time evolution of higher order cumulants in this model, we show that the dependence of the cumulants of conserved charges on the size of rapidity window to count the particle number in experiments

show characteristic behaviors reflecting the dynamical history of the time evolution of the hot medium. In particular, we predict that the fourth-order cumulants of conserved charges will be suppressed compared with the second ones at LHC energy for a wide range of parameters.

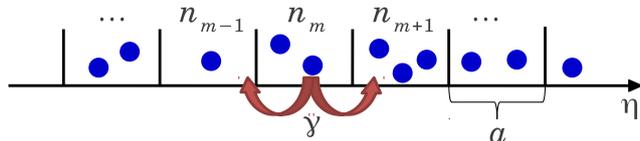
## 2. Stochastic formalism to describe non-Gaussianity in diffusive systems

In relativistic heavy ion collisions with sufficiently large  $\sqrt{s_{NN}}$ , the hot medium created at mid-rapidity has an approximate boost invariance. In a sufficiently large space-time scale where hydrodynamic equations at first order are applicable, the average of the net number of a conserved charge per unit coordinate-space rapidity,  $n(\eta, \tau)$ , follows the diffusion equation

$$\partial_\tau n(\eta, \tau) = D \partial_\eta^2 n(\eta, \tau), \quad (1)$$

with the coordinate-space rapidity  $\eta$ , proper time  $\tau$ , and the diffusion constant  $D$ . Assuming that the kinetic freezeout takes place at a certain proper time  $\tau_{fo}$ , the experimentally-observed conserved-charge number at mid-rapidity at RHIC and LHC is given by  $Q(\tau) = \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta n(\eta, \tau)$  at  $\tau = \tau_{fo}$  with the size of the rapidity window to count the particle number  $\Delta\eta$ .

In order to describe fluctuations around the solution of Eq. (1), one may employ a stochastic model, in which the time evolution of the deterministic part satisfies Eq. (1). A choice of such stochastic models is the theory of hydrodynamic fluctuations [9], in which the hydrodynamic equations are promoted to Langevin equations with stochastic terms representing fast random forces. It is known that these stochastic equations well describe Gaussian fluctuations in fluids. However, extension of this formalism to treat non-Gaussian fluctuations is nontrivial. In fact, one can show that the stochastic force in the theory of hydrodynamic fluctuations for Markov process is of Gaussian [6]. Using this property it is shown that all higher order cumulants of  $Q(\tau)$  vanish in equilibrium unless  $D(\tau)$  is explicitly dependent on  $n$ . This property is not welcome to describe non-Gaussianity in heavy ion collisions, because higher order cumulants are expected to increase toward nonzero equilibrated values in the hadronic medium [1, 3, 4].



**Figure 1.** System described by the diffusion master equation Eq. (2).

In the present study, instead of directly extending the theory of hydrodynamic fluctuations, we investigate the time evolution of higher order cumulants starting from a microscopic model. In this study, as such a model we consider a simple one-dimensional system composed of Brownian particles. Instead of tracking the motion of each Brownian particle separately, however, we represent the system as follows (See, Fig. 1) [6]. First, the coordinate  $\eta$  is divided into discrete cells with an equal length  $a$ . Second, we consider a single species of particle for the moment, and denote the number of particles in each cell, labeled by an integer  $m$ , as  $n_m$ , and the probability that each cell contains  $n_m$  particles as  $P(\mathbf{n}, \tau)$  with  $\mathbf{n} = (\dots, n_{m-1}, n_m, n_{m+1}, \dots)$ . Finally, we assume that each particle moves to adjacent cells with a probability  $\gamma$  per unit proper time. The probability  $P(\mathbf{n}, \tau)$  then follows the differential equation

$$\partial_\tau P(\mathbf{n}, \tau) = \gamma \sum_m [(n_m + 1) \{P(\mathbf{n} + \mathbf{e}_m - \mathbf{e}_{m+1}, \tau) + P(\mathbf{n} + \mathbf{e}_m - \mathbf{e}_{m-1}, \tau)\} - 2n_m P(\mathbf{n}, \tau)], \quad (2)$$

which is referred to as *diffusion master equation*, where  $\mathbf{e}_m$  is the vector that all components are zero except for the  $m$ th one, which takes unity. One can show that the average density and Gaussian fluctuation of  $n(\eta, \tau)$  in Eq. (2) in the continuum limit,  $a \rightarrow 0$ , agree with those in the stochastic diffusion equation [8] with  $D = \gamma a^2$  [6].

### 3. Solution of diffusion master equation

Now, we solve the time evolution of cumulants for the stochastic process Eq. (2). In order to simplify the problem, in the following we limit our attention to the time evolution in an infinitely long system without boundaries. Since we are interested in the solution in the continuum limit,  $a \rightarrow 0$ , we represent the particle numbers  $n_m$  by a function  $n(\eta, \tau)$ . After some algebra [6], one finds that the cumulants of  $Q(\tau)$  with the fixed initial condition  $n(\eta, 0) = M(\eta)$  are given by

$$\langle (Q(\tau))^n \rangle_c = \int_{-\infty}^{\infty} d\eta M(\eta) H_X^{(n)}(\eta), \quad (3)$$

with

$$H_X^{(1)}(z) = I_X(z/\Delta\eta), \quad H_X^{(2)}(z) = I_X(z/\Delta\eta) - I_X(z/\Delta\eta)^2, \quad (4)$$

$$H_X^{(3)}(z) = I_X(z/\Delta\eta) - 3I_X(z/\Delta\eta)^2 + 2I_X(z/\Delta\eta)^3, \quad (5)$$

$$H_X^{(4)}(z) = I_X(z/\Delta\eta) - 7I_X(z/\Delta\eta)^2 + 12I_X(z/\Delta\eta)^3 - 6I_X(z/\Delta\eta)^4, \quad (6)$$

and  $I_X(z) = \int_{-1/2}^{1/2} dx \int dq / (2\pi) e^{-X^2 q^2} e^{iq(x+z)}$ , where  $\Delta\eta$  and  $\tau$  dependences are encoded in the dimensionless parameter  $X = \sqrt{D\tau}/\Delta\eta$ . We note that  $\sqrt{2D\tau}$  is the mean diffusion length of the Brownian particles at  $\tau$ .

In order to examine the time evolution of conserved charges in heavy ion collisions, one must extend the above result to general initial conditions containing fluctuations. We also extend the result to the system with two particle species with densities  $n_1(\eta, \tau)$  and  $n_2(\eta, \tau)$ , and consider cumulants of the difference,  $Q_{(\text{net})}(\tau) = \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta (n_1(\eta, \tau) - n_2(\eta, \tau))$ , in order to compare the results with the cumulants of net charge numbers. In the following, we limit our attention to the solution for the initial conditions which satisfies spatial uniformity and locality with vanishing net-charge number. In this case, second- and fourth-order cumulants of  $Q_{(\text{net})}(\tau)$  are given by

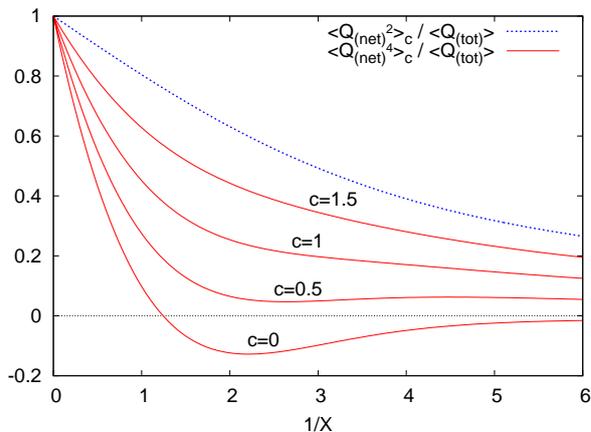
$$\langle Q_{(\text{net})}^2 \rangle_c = \Delta\eta [M_{(\text{tot})}]_c (1 - F_X^{(2)}), \quad (7)$$

$$\langle Q_{(\text{net})}^4 \rangle_c = \Delta\eta \{ 3[M_{(\text{tot})}^2]_c (F_X^{(2)} - 2F_X^{(3)} + F_X^{(4)}) + [M_{(\text{tot})}]_c (1 - 7F_X^{(2)} + 12F_X^{(3)} - 6F_X^{(4)}) \}, \quad (8)$$

with  $F_X^{(n)} = \int_{-\infty}^{\infty} dz [I_X(z)]^n$ , and  $M_{(\text{net}),(\text{tot})}(\eta) = M_1(\eta) \mp M_2(\eta)$ , respectively. In Eq. (8),  $[M_{(\text{tot})}^2]_c$  is the fluctuation of the total number of the particles per unit rapidity at the initial condition. This quantity is not constrained by the conservation laws and strongly depends on the hadronization mechanism [6]. We thus treat this quantity as a parameter that characterizes the hadronization mechanism. From Eqs. (7) and (8), one can check that the distribution of  $Q_{(\text{net})}$  approaches a Skellam one with  $\lim_{\tau \rightarrow \infty} \langle Q_{(\text{net})}^{2n} \rangle_c = \Delta\eta [M_{(\text{tot})}]_c$ . The time evolution of the Gaussian fluctuation, Eq. (7), is equivalent with the one in the stochastic diffusion equation [8].

### 4. Time evolution of cumulants and $\Delta\eta$ dependence

Now, let us consider the cumulants of conserved charges in relativistic heavy ion collisions. To make the argument simple, we assume that a boost invariant system with local equilibration is realized just above the critical temperature of the deconfinement transition. We further assume that the fluctuations of conserved charges vanish at this time, reflecting the small fluctuations in the deconfined phase [1] and the local charge conservations. Due to the diffusion in the hadronic phase, the fluctuations keep on approaching the equilibrated values in the hadronic medium until kinetic freezeout at  $\tau = \tau_{\text{fo}}$ . Provided that this diffusion process is well described by the diffusion master equation,  $\Delta\eta$  dependence of the cumulants of conserved charges at kinetic freezeout are given by Eqs. (7) and (8) with  $\tau = \tau_{\text{fo}}$ .



**Figure 2.** Second and fourth order cumulant of  $Q_{(\text{net})}$  as a function of  $1/X$  with the fixed initial condition.

In Fig. 2, we show the  $1/X$  dependences of Eqs. (7) and (8). Since  $1/X$  is proportional to  $\Delta\eta$  with fixed  $\tau$ , this result can directly be compared with the  $\Delta\eta$  dependence of the cumulants in experiments. The result for the fourth-order is shown with several values of the parameter  $c = [M_{(\text{tot})}^2]_c / [M_{(\text{tot})}]_c$ , which is the quantity sensitive to hadronization mechanism [6].

In the figure, one finds that  $\langle Q_{(\text{net})}^4 \rangle_c$  is suppressed compared with  $\langle Q_{(\text{net})}^2 \rangle_c$  in the parameter range  $c < 1.5$ , while the behavior of  $\langle Q_{(\text{net})}^4 \rangle_c$  depends sensitively on the value of  $c$ . This result indicates that  $\langle (N_Q^{(\text{net})})^4 \rangle_c$  at ALICE, which has not been measured yet, will be suppressed compared with the  $\langle (N_Q^{(\text{net})})^2 \rangle_c$  which has been already measured [4], while the statement is altered for large  $c$ . The same conclusion is also anticipated for the relation between the baryon number cumulants [10],  $\langle (N_B^{(\text{net})})^2 \rangle_c$  and  $\langle (N_B^{(\text{net})})^4 \rangle_c$ . Our results also indicate that experimental measurements of not only the magnitudes of various cumulants at a fixed  $\Delta\eta$  but also their  $\Delta\eta$  dependence enable us to explore various aspects of the time evolution of the hot medium and the hadronization mechanism in the experiments. In particular, these analyses would enable us to estimate the magnitude of the parameter  $c$ , which is sensitive to the hadronization mechanism.

## References

- [1] Koch V 2008 Hadronic Fluctuations and Correlations *Preprint* arXiv:0810.2520 [nucl-th]
- [2] Asakawa M, Ejiri S, and Kitazawa M 2009 Third moments of conserved charges as probes of QCD phase structure *Phys. Rev. Lett.* **103** 262301
- [3] Adamczyk L, *et al.* [STAR Collaboration] 2013 Energy Dependence of Moments of Net-proton Multiplicity Distributions at RHIC *Preprint* arXiv:1309.5681 [nucl-ex]
- [4] Abelev B, *et al.* [ALICE Collaboration] 2013 Net-Charge Fluctuations in Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV *Phys. Rev. Lett.* **110** 152301
- [5] Mukherjee S and Wagner W 2013 Deconfinement of strangeness and freeze-out from charge fluctuations *Preprint* arXiv:1307.6255 [nucl-th]
- [6] Kitazawa M, Asakawa M, Ono H 2013 Nonequilibrium time evolution of higher order cumulants of conserved charges and event-by-event analysis *Preprint* arXiv:1307.2978 [nucl-th]
- [7] F. Karsch and K. Redlich 2011 Probing freeze-out conditions in heavy ion collisions with moments of charge fluctuations *Phys. Lett.* **B695** 136
- [8] Shuryak E V and Stephanov M A 2001 When can long range charge fluctuations serve as a QGP signal? *Phys. Rev. C* **63** 064903
- [9] Kapusta J I, Müller B and Stephanov M 2012 Relativistic Theory of Hydrodynamic Fluctuations with Applications to Heavy Ion Collisions *Phys. Rev. C* **85** 054906
- [10] Kitazawa M and Asakawa M 2012 Revealing baryon number fluctuations from proton number fluctuations in relativistic heavy ion collisions *Phys. Rev. C* **85** 021901R; *ibid.* 2012 Relation between baryon number fluctuations and experimentally observed proton number fluctuations in relativistic heavy ion collisions *Phys. Rev. C* **86** 024904