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Which came first, spacetime or clocks?

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Abstract. Emergent quantum mechanics seeks a deeper level theory, anticipating that such a theory will provide a clearer picture of the relation between the quantum and classical worlds. In this work we show that the quantum-classical divide is a manifestation of the transition from Newton's absolute time to relativity's path-dependent time. The prior theory in this case is that *particles are intrinsic clocks*. The emergence of separate classical and quantum behaviour is seen by considering different continuum limits in a single digital clock model. A continuum limit that constructs a continuous worldline provides a simple basis for Minkowski spacetime. An alternative limit in which the clock itself contains boost information leads to the Dirac equation.

Introduction

The four major theories of the twentieth century, special relativity (SR), quantum theory (QT), general relativity and quantum field theory are all considered more fundamental than their predecessor, Newtonian mechanics (NM). For our purposes, general relativity and quantum field theory shall be considered extensions of SR and QT respectively, and the transition from Newtonian mechanics to QM and SR will be our main concern.

Quantum mechanics, while empirically accurate, is currently a prescription rather than a mathematical expression of physical principles. Since there are a myriad of interpretations of QM, it has been difficult to determine through which interpretation we are to find a logically prior theory. If quantum theory is emergent, the question remains 'emergent from what?'.

By comparison, special relativity seems straightforward. From Einstein's two physically motivated postulates we arrive at Minkowski spacetime without the encumbrance of multiple interpretations. However, having arrived at spacetime by virtue of the light speed postulate, it is clear that we have *missed* quantum mechanics in the process. A sensible question is then; what is *wrong* with the conventional route to Minkowski space that it misses quantum mechanics?

One prime suspect for quantum incompatibility is the worldline. In Newtonian mechanics particles have worldlines and this is carried over to SR. However, worldlines are not a feature of quantum mechanics outside of the Bohm picture. If we are to question the existence of worldlines when in fact objects appear to *have* worldlines on macroscopic scales, it seems that the continuum limit involved in time-keeping is in question, and it is to this that we turn.

In section 1 we review a simple model of a digital clock that illustrates the transition to Newtonian absolute time. We see the logical steps necessary to go from a digital clock to an analog clock to absolute time. The continuum limit here uncovers unimodular complex numbers as model clocks.

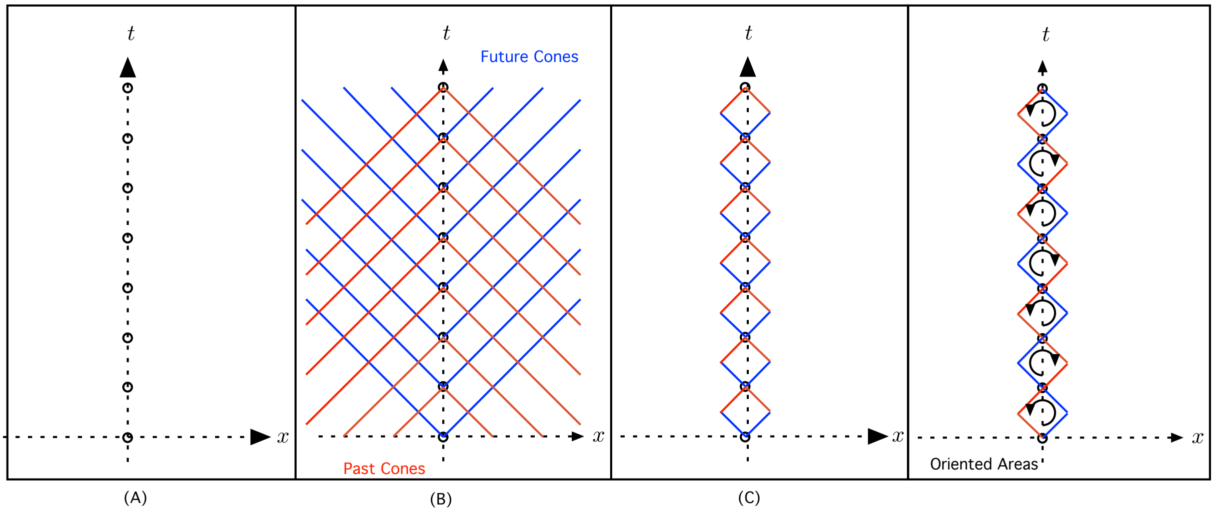

Figure 1. (A) Periodic events in a two dimensional space-time. (B) Past and future light-cones for the events. Units are chosen so that $c = 1$. (C) The causal areas between events. (D) The area boundary is chosen to provide a single traversal. This results in an alternating orientation of causal areas.

In section 2 we show how the clock must be modified for consistency with the Lorentz transformation. The continuum limit in this case uncovers Clifford algebra in place of complex numbers and absolute time is replaced by Minkowski spacetime.

In section 3 we take a closer look at the clock model that generated Minkowski spacetime and show that if we require a clock to *contain* the boost information of the Lorentz transformation, the transfer matrix of Minkowski spacetime becomes a generating function that satisfies the Dirac equation. Section 4 discusses the implication of this result.

1. Newtonian Timekeeping

Following[1] consider a periodic process in a stationary frame that provides events at fixed intervals. To make contact with special relativity in the next section we shall assume a speed of light $c = 1$ much greater than any other speed in the model. The Galilean transformation shall be assumed.

Fig. 1(A) represents a periodic sequence of events, stationary at the origin in an inertial frame. (B) shows the past and future light cones of these events and (C) the 'causal' areas that lie between the sequential events. Here the term causal is used because, regardless of any actual clock 'mechanism', the areas represent the maximum time-like domain of influence that successive ticks have in common. In (D) we colour the area boundaries to show that we can imagine the boundaries to be drawn as a single continuous curve from t=0 to the latest event and thence back to the origin. Drawing the boundary this way gives the causal areas an orientation and places the events at intersections of smooth segments of the two curves. If we think of the chain of areas in terms of the boundary curves there are two kinds of events, the outside corners and the path-crossing points where the curves intersect. We call the latter on-worldline events and the former off-worldline events. Our clock, from a path perspective, is equivalent to a couple of featureless photons confined between two walls that generate a 'tick' at crossing points.

Our periodic process is just a digital clock that ticks at the events. To construct an analog clock from this we require a continuum limit in which we allow for ever higher frequency ticks. We can do this using the same clock model, as illustrated in Fig. 2. To take the continuum limit for this sequence of clocks we represent the clock algebraically via the encoding illustrated in

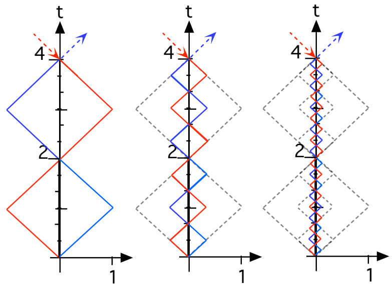

Figure 2. A full clock cycle for the original clock with similar clocks running at three and nine times the original frequency. As the frequency is increased, the density of events on the t-axis increases and the causal area between on-worldline events decreases. In the continuum limit the causal area shrinks to zero and the event sequence becomes a worldline.

Fig. 3. In this figure, a two-component 'state vector' records projections onto the left and right light cones and because the area boundaries are themselves null segments, successive states are orthogonal. The states consist of a period 4 sequence

$$s_k \in S_0 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\} \tag{1}$$

and the actual sequence is a clock *signal* that can be inverted to establish the time up to integer accuracy. If we think of the column vectors as the active display of a clock, algebraically the clock mechanism corresponds to left multiplication by the transfer matrix

$$T_0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

where we note that $T_0^2 = -I_2$, a square root of minus the identity and a fourth root of the identity. In terms of the Pauli matrices T_0 is just $-i\sigma_y$. If the clock is in state s_k at time k then the subsequent state, after the next event is

$$s_{k+1} = T_0 s_k. \tag{3}$$

Since then

$$s_k = T_0^k s_0 \tag{4}$$

the power of the transfer matrix corresponds to the discrete displacement in time.

To make this clock analog, we have to construct a process that agrees with the clock at the integer events where the clock ticks, but provides extra events between ticks. The simplest way to do this is to construct a clock, similar to the original, but running at a higher frequency. Fig. 2 shows an example of the original clock with similar clocks running at three and nine times the original frequency.

In terms of the transfer matrix T_0, a clock running at n times the frequency will have a transfer matrix T_n such that $T_n^n = T_0$ so that we can take T_n as the n-th root of T_0 via an eigenvalue expansion:

$$T_n = \begin{pmatrix} \cos\left(\frac{\pi}{2n}\right) & -\sin\left(\frac{\pi}{2n}\right) \\ \sin\left(\frac{\pi}{2n}\right) & \cos\left(\frac{\pi}{2n}\right) \end{pmatrix} \tag{5}$$

We can use this to take the limit as $n \to \infty$, in which case we get:

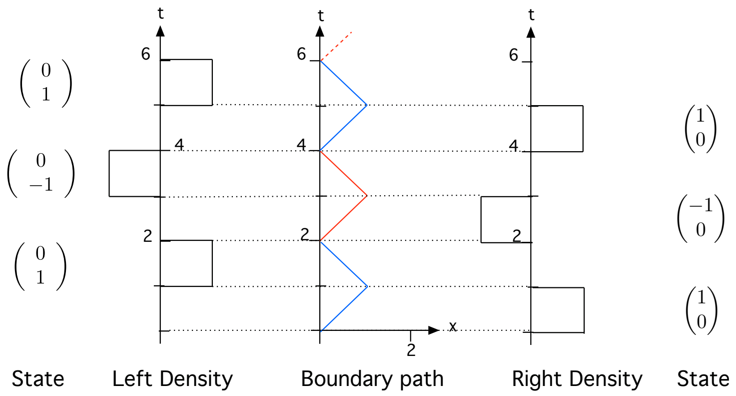

Figure 3. The right boundary of the chain of causal areas can be in any one of four states. The boundary densities and column vectors are illustrated. On-worldline events occur at the even integers. The continuum limit (6) suggest that the left and right densities taken together can be replaced by a 'clock face' that interpolates between the four states imitating a unimodular complex number in polar form.

$$
\begin{aligned}
T(t) = \lim_{n\to\infty} T_n^{nt} &= \begin{pmatrix} \cos\left(\frac{\pi t}{2}\right) & -\sin\left(\frac{\pi t}{2}\right) \\ \sin\left(\frac{\pi t}{2}\right) & \cos\left(\frac{\pi t}{2}\right) \end{pmatrix} \\
&= \mathrm{I}_2 \cos\left(\frac{\pi \mathrm{t}}{2}\right) + \mathrm{T}_0 \sin\left(\frac{\pi \mathrm{t}}{2}\right) \qquad (6)
\end{aligned}
$$

where I_2 is the 2×2 identity matrix. Notice that $T(n) = T_0^n$ for integer n so that our 'analog clock' agrees with the digital clock at the integers where the original digital clock has events.

From equation (6) we can see that the analog transfer matrix $T(t)$ is just a rotation matrix and that the continuum limit replaces the set of four state vectors S_0 with the set of all column unit vectors $S = \left\{(u,v)^T \mid u^2+v^2 = 1, u, v \in \Re\right\}$. From the form of (6) we see that the transfer matrix looks like a complex number in polar form where T_0 is just an implementation of the unit imaginary. To see the connection to complex numbers more directly we can consider a change of variables $(x,y)^T = \frac{1}{\sqrt{2}}(I_2 + T_0)(u,v)^T$. This diagonalizes the original transfer matrix T_0 which becomes $i\sigma_z$ and equation(6) becomes

$$
\begin{aligned}
T(t) &\to \begin{pmatrix} \exp[i\left(\frac{\pi t}{2}\right)] & 0 \\ 0 & \exp[-i\left(\frac{\pi t}{2}\right)] \end{pmatrix} \\
&= \exp[\left(\frac{i\pi\sigma_z t}{2}\right)] \qquad (7)
\end{aligned}
$$

From equation (7) we see that the diagonal form of the transfer matrix has two unimodular complex numbers on its diagonal, both synchronized but rotating in opposite directions as t increases. As a result, both components of the resulting state vector are unimodular complex numbers rotating in opposite directions. Either component could be inverted over half periods to extract t as a function of the component itself. Notice in Fig. 4, the continuum limit smoothly interpolates a four state process creating a spiral that is conveniently described by a unimodular complex number.

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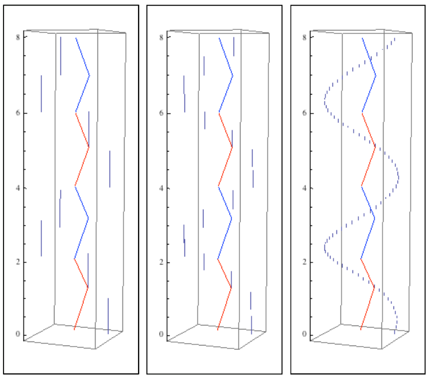

Figure 4. The initial digital clock has only four states, however the continuum limit builds an extra dimension that appears algebraically through the arrival of complex numbers.

In a Newtonian world where the Galilean transformation holds, the above 'clock' measures time intervals without being affected by relative motion. The continuum limit would give the same result, regardless of the inertial frame. Since the clock *signal* is the same unimodular complex number for all such clocks, regardless of inertial frame, the clock argument t can be taken as absolute and assigned its own dimension in a two dimensional space-time. In a Newtonian world we can ascribe a separate reality to t as a universal time while assuming that real clocks emit *signals* such as $e^{i\omega t}$ *as particular implementations of time measurement.*

The above model has just illustrated the fact that the transition from a discrete clock to a continuum one is conveniently provided through the use of complex numbers. The unimodular complex number is the mathematical counterpart of an analog wristwatch where the counting of digital events is replaced by a continuous increase in phase. By assuming the Gallilean transformation, we have ensured that an ensemble of inertial clocks moving at different speeds keep the same time. The universality of our clock model, the fact that all discrete sequences of events can be interpolated this way, provides a justification of the *priority of time over clocks.*

In the next section, we modify the Galilean invariance of the above **G**-clock to be consistent with the light-speed postulate. In the context of our model, we shall see that this illustrates the transition from complex numbers to Clifford algebras.

2. Minkowski space

In the original digital clock the transition to successive states is accomplished algebraically through left multiplication by T_0. Even and odd states are not distinguished by T_0 as the causal areas are symmetric rectangles, Fig. 5. However, relativistically, a clock moving at constant speed v in the x direction has to stay in states 1 and 3 longer than states 2 and 4, Fig. 6. We need a new transfer matrix T_{m+n} that has to function as an $(m+n)$-th root of T_0^2 while weighting the occupation of state 1 more heavily than state 2 ($m > n$). This means increasing the magnitude of the 1-1 element and decreasing the magnitude of the 2-2 element. To see how this may be done expand $T(t)$ to first order in $t = 1/n$:

$$T(1/n) \simeq \begin{pmatrix} 1 & -\frac{\pi}{2n} \\ \frac{\pi}{2n} & 1 \end{pmatrix}$$

where we assume $n \gg 1$. The ones on the diagonal equally weight 1-1 and 2-2 transitions. If we consider the replacement $T(1/n) \to T_v(1/n) = T(1/n) + w\frac{\pi}{2n}\sigma_z$ this will effect an increased residence time in the odd states ($w > 0$) and a decreased residence time in the even states as suggested by Fig. 6. We choose w so that the 'moving clock runs slow' in accordance with the

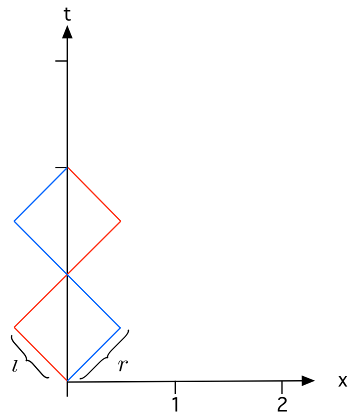

Figure 5. A clock cycle in a co-moving frame. The null boundaries cross at successive on-worldline events. Successive causal regions have the same area but opposite orientations.

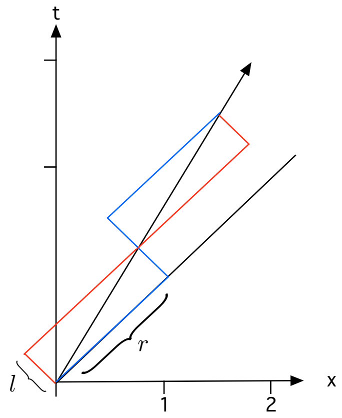

Figure 6. A clock cycle in a moving frame. The null boundaries remain null but the left and right projections change length. The signs and magnitudes of the causal areas are invariant.

Lorentz transformation and the area preservation of the light clock[2]. To be in accordance with Fig. 6 the first on-worldline event should take place at $t = 2\gamma$ where $\gamma = 1/\sqrt{1-v^2}$ is the time dilation factor. So we must have:

$$T_v(2\gamma) = \lim_{n\to\infty} T_v(1/n)^{2n\gamma} \quad = \quad -I_2. \tag{8}$$

For equation (8) to hold, the eigenvalues of T_v raised to the power $2\gamma n$ must be -1 in the limit as $n \to \infty$. The eigenvalues of T_M to lowest order in $1/n$ are $\lambda = 1 \pm \frac{i\pi\sqrt{1-w^2}}{2n}$ giving

$$\lim_{n\to\infty} \lambda^{2\gamma n} = e^{\mp i\pi\sqrt{1-w^2}\gamma}. \tag{9}$$

For this to equal -1 we have $w = \pm v$. Choosing the positive value of v for w we can calculate the transfer matrix for a clock moving at speed v with respect to the lab frame whose transfer matrix was given by (6). The matrix is

$$T_v(t) \quad = \quad \mathrm{I}_2 \cos\left(\frac{\pi \mathrm{t}}{2\gamma}\right) + \gamma(\mathrm{v}\sigma_{\mathrm{z}} - \mathrm{i}\sigma_{\mathrm{y}}) \sin\left(\frac{\pi \mathrm{t}}{2\gamma}\right). \tag{10}$$

Equation (10) is the appropriate transfer matrix for the moving clock and, as in the Newtonian case, the continuum limit increases the density of on-worldline events based on the assumption that clocks have worldlines and can emit continuous signals. The mathematical convenience and physical justification for Minkowski spacetime can be seen by viewing the transfer matrix as a set of instructions for constructing an analog time-signal along a worldline. Notice that the transfer matrix has a structure that can be partitioned into 'kinematic' and 'dynamic' information. Setting the base frequency of the above **M**-clock at ω we have

$$T_v(t) = \mathrm{I}_2 \cos\left(\frac{\omega \mathrm{t}}{\gamma}\right) + \underbrace{\gamma(v\sigma_z - i\sigma_y)}_{\text{kinematics}} \overbrace{\sin\left(\frac{\omega t}{\gamma}\right)}^{\text{dynamics}}. \tag{11}$$

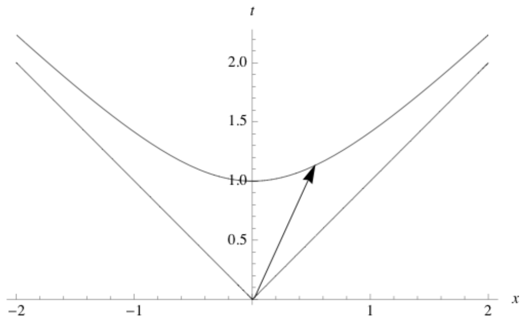

Figure 7. The odd part of the transfer matrix is a vector. The unit vector starts at the origin and ends on the first on-worldline event. The Euclidean length of the vector is direction dependent, however the causal area it represents is a Lorentz invariant.

The dynamic information is clock-specific and contains the frequency and phase information. Comparing this to the transfer matrix for the **G**-clock equation (6) we see that the matrix implementation of the unit imaginary T_0 has been replaced by the (bi)vector $\mathbf{J_v} = \gamma(v\sigma_z - i\sigma_y)$ that has similar properties. Notice that $\mathbf{J_v}^2 = -\mathrm{I}_2$ and that time evolution generated by T_v exhibits the time dilation appropriate for the Lorentz transformation. Unlike the unit imaginary, the vector $\mathbf{J_v}$ has a directional aspect, providing as it does, the direction of the proper time axis of the moving clock, Fig. 7. The intrinsic normalization of $\mathbf{J_v}$ is through spacetime *area* as opposed to length. It is this feature that leads to the mixed signature of spacetime. As we shall see in the next section, the 'wave-particle duality' of quantum mechanics is an alternative manifestation of the spacetime area-invariance of the Lorentz transformation.

In terms of information, equation (10) is a direct algebraic encoding of the first half of Wheeler's famous maxim[3]:

"Spacetime tells matter how to move; matter tells spacetime how to curve."

Equation (10) is literally the instruction for the clock to generate a spiral signal as it moves along its worldline at constant velocity. The coefficient of the vector part of (10) is plotted in Fig. 8 as a colour map showing the phase of the clock along the worldline. In the continuum limit, the transfer matrix has filled in the worldline of the clock with an increasing density of events that is mapped onto the phase of the trig function. The vector $\mathbf{J_v}$ ensures the phase is properly distributed according to the Lorentz transformation for each velocity. Fig. 9] shows the ensemble of clock phases for all velocities.

Since for all clocks, the kinematic structure of $T_v(t)$ is independent of the clock's base frequency, this suggests that we ascribe this feature of clocks to an ambient *spacetime* using frame vectors σ_x and $-i\sigma_y$ for the x and t coordinates respectively. With this convention, boosts are handled algebraically like rotations in a Euclidean space. This is a mathematical convenience, but one that has precedence in Newton's clock. In that case we concluded that because all discrete periodic clocks could be smoothly interpolated along their worldline to give a unimodular complex number with time linearly related to phase, time itself became 'fundamental' and clock signals became the physical approximations of 'reality'.

In this relativistic case, although we can no longer divorce time from space via the Galilean transformation, the algebraic analog is clear. We simply ascribe the kinematic information of clocks to 'spacetime'. In doing so, we are keeping the worldline feature of macroscopic clocks intact and inventing spacetime as a global attribute of Nature that controls boosts. Here spacetime becomes the reality that all physical clocks approximate. Individual clock *signals* are then the derivative concept, not the spacetime that represents all clocks.

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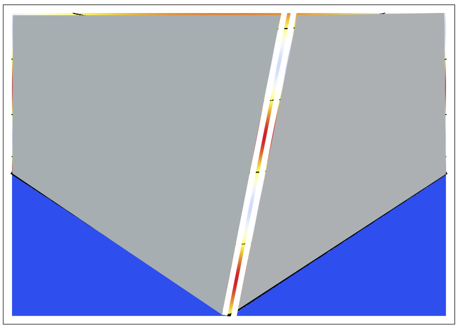

Figure 8. The coefficient of the vector term in equation (10) plotted as a colour map along the direction of $\mathbf{J_v}$ inside the light cone.

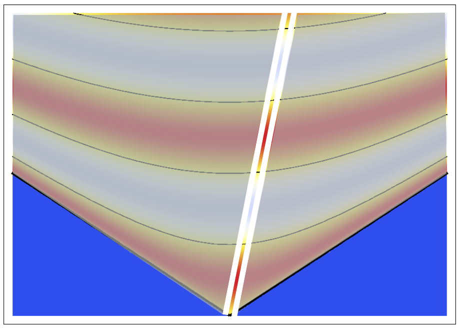

Figure 9. The coefficient of the vector in equation (10) plotted against a background of velocities $\{-1 < v < 1\}$ inside the light cone.

The argument that Newton's absolute time and Minkowski's spacetime are logically prior concepts to clock signals is rooted in the idea that real objects, including clocks, have worldlines. While macroscopically this is true, microscopic verification is simply not possible below some physical scale. Furthermore, the uncertainty principle calls the concept of smooth worldlines into question, suggesting fractal rather than smooth paths[4] as a basis for quantum propagation. This being the case, the above argument for a logically prior spacetime is weakened by its assumption of a continuum limit that 'fills in' the worldline to arbitrarily fine scales. In the next section we relinquish worldlines in favour of a continuum limit that packages the boost information with the clock itself.

3. The Dirac clock

In the previous two sections we associated discrete periodic clocks with chains of oriented areas, Fig. 1. We argued that all such periodic clocks give rise to either Newton's clock or Minkowski's clock depending on how boosts affect timekeeping. The universality of the transcription to absolute time or Minkowski spacetime encouraged a perspective in which time or spacetime comes first and clock signals are derivative concepts. The continuum limit is however in question since the worldlines of quantum particles are not verifiable. As a result, placing the continuum limit along the worldline, as we have done in the previous sections, is suspect. By placing the continuum limit along worldlines we risk building the quantum Zeno effect into the clock model through this limit, thus excluding quantum propagation!

A less restrictive continuum limit can be obtained by 'repackaging' the information that we commonly ascribe to spacetime. Consider an isolated periodic clock in empty space in which we are a 'special'[1]observer. We observe a single discrete event of the clock and we use that to choose the origin of our spacetime coordinate system. With a single observation we do not know the speed or direction of the clock with respect to our chosen frame so the orientation of our time axis will not in general coincide with that of the clock, neither will the 'instruction set' for the clock be written in terms of our arbitrary reference frame. In fact, if we do not believe that Nature uses an informative spacetime to 'tell the clock how to move', the clock itself must have prepackaged information that we as observers can *access* from whatever spacetime coordinate system we construct. The situation in comparison to previous clocks is visually depicted in figures [10] and [11]. In Fig. 10 the continuum limit is taken along the worldline with spacetime providing instructions that keep the timekeeping appropriate to the clock's inertial frame. In

[1] Special in the sense that we can observe the process without disturbing it in any way.

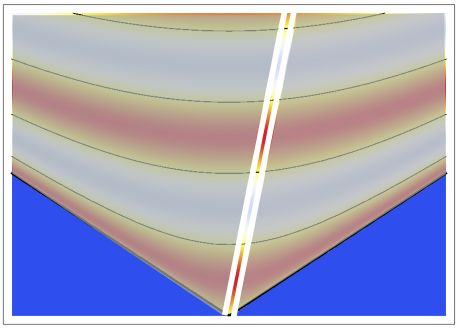

Figure 10. The coefficient of the vector term in equation (10) plotted as a colour map along the worldline of the clock. The continuum limit for Minkowski space is taken along the worldline of the clock. The transfer matrix 'tells the clock how to keep time' with the clock moving at fixed v.

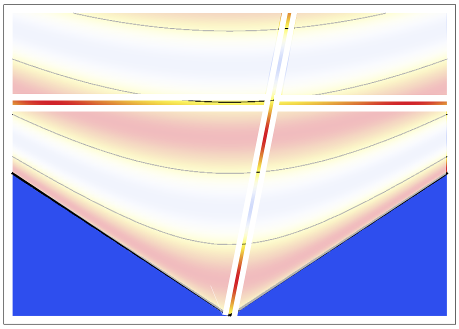

Figure 11. The continuum limit for the generating function for boosts is taken at fixed t. It encodes the information the clock must access if it can be read from an arbitrary inertial frame. The resulting generating function satisfies the Dirac equation.

Fig. 11 we assume the clock itself contains the information needed for the whole ensemble of relative velocities from which our chosen spacetime is drawn. Shifting this information from spacetime to the clock itself corresponds to associating the clock with the *generating function* for the transfer matrices that we associate with Minkowski's clock. In this case the continuum limit is no longer along the worldline, it is at whatever value of t we choose to observe the clock in our own frame. This changes the role of the continuum limit from implementing a continuous evolution along the worldline to one that encodes the temporal history along the spatial direction.

To implement the transfer matrix as a generating function we use complex exponentials to count ticks of the clock[5, 6]. Algebraically we replace I_2 by $\exp[-ip\sigma_z\epsilon]$ in the $t = \epsilon << 1$ expansion of the transfer matrix, take the continuum limit, and then find the state of the clock at a specific *spacetime* point by inverting what has become a Fourier transform. Thus we consider replacing the short-time transfer matrix by

$$\begin{pmatrix} 1 & -m\epsilon \\ m\epsilon & 1 \end{pmatrix} \rightarrow \begin{pmatrix} e^{ip\epsilon} & -m\epsilon \\ m\epsilon & e^{-ip\epsilon} \end{pmatrix}.$$

The continuum limit of this generating function gives

$$\begin{aligned} T(p,t) &= \lim_{\epsilon\rightarrow 0} \begin{pmatrix} \exp[ip\epsilon] & -m\epsilon \\ m\epsilon & \exp[-ip\epsilon] \end{pmatrix}^{t/\epsilon} \\ &= \cos(Et)I_2 - \frac{1}{E}(ip\sigma_z + im\sigma_y)\sin(Et) \end{aligned} \quad (12)$$

where $E = \sqrt{p^2+m^2}$ and for generality we have replaced $\pi/2$ by m and $1/n$ by ϵ. Notice here the generating function form of the transfer matrix still has a multivector structure that is a combination of a scalar and a unit vector since

$$(\frac{-1}{E}(ip\sigma_z + im\sigma_y))^2 = \frac{1}{E^2}(-p^2-m^2)I_2 = -I_2. \quad (13)$$

The generating function has singled out one-dimensional subspaces of a 2-dimensional domain via a unit vector. To return to 'configuration' space notice that the generating function is also a Fourier transform and can be inverted as such. Namely

$$\begin{aligned} T(x,t) &= \lim_{\epsilon\to 0}\frac{1}{2\pi\epsilon}\int_{-\pi/\epsilon}^{\pi/\epsilon} e^{ix(p\epsilon)}\begin{pmatrix} \exp[ip\epsilon] & -m\epsilon \\ m\epsilon & \exp[-ip\epsilon]\end{pmatrix}^{t/\epsilon} d(p\epsilon) \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ipx}\left(I_2\,\cos(Et)+\frac{1}{E}(ip\sigma_z+im\sigma_y)\sin(Et)\right)dp\,, \qquad |x|<t, \quad (14) \end{aligned}$$

the inverse Fourier transform of $T(p,t)$. Here the division of the integral by ϵ in the continuum limit gives $T(x,t)$ as a density. Notice the relation between Eqn. (14) and Fig. 11. In Eqn. (14), t and m are fixed and the integral over p samples all possible boosts. This corresponds to all possible values of v in Minkowski's clock in Fig. 11. The continuum limit in the generating function assigns a phase to each point in Minkowski's spacetime based on the fundamental frequency m of the clock. We call this model Dirac's clock because (14) satisfies the free-particle Dirac equation[1, 7, 8]. The only difference between the **M**-clock and this clock is the role of the continuum. The limit for the **D**-clock does not form worldlines and the causal areas between events is not reduced to zero in the limit. This means that the paths associated with the transfer matrix products, squashed into worldlines in the **M**-clock, expand into the future cone of the origin for the **D**-clock Fig. 12.

4. Discussion

The Dirac equation is commonly regarded as a unique harbinger of relativistic quantum mechanics, yet the generating function (14) satisfies this enigmatic equation without any appearance of a quantum context. How can this be?

The answer lies at the heart of the quest for an emergent quantum mechanics. In their conventional formulations, QM and SR are based on very different paradigms. The former is about process and propagation, while the latter is about events and objects. The conventional merger of the two at the Dirac equation gives the appearance of a theory more fundamental than either constituent. However, obtaining a fundamental equation by merging two approximations is an inspired kludge, not a derivation.

The above clock models show us how spacetime and quantum propagation emerge from periodic processes in a relativistic world and why, once emerged, they appear to represent conflicting paradigms. Minkowski spacetime arises through a continuum limit that creates a worldline for a clock, the causal area between events going to zero in the continuum limit (Fig. 2). Timekeeping information is ultimately transmitted along rays of constant velocity in a two dimensional spacetime that acquires responsibility for the Lorentz transformation. That is, a fair statement of the significance of the **M**-clock is:

> *Any* periodic sequence of events, stationary in an inertial frame, may be smoothly interpolated to form a worldline described by a transfer matrix of the form (11).

This may be construed to mean that it is a property of an ambient spacetime that all digital clocks behave in this way, and that both spacetime and the worldline are physically real in functionality, if not observability. This interpretation, clearly useful for macroscopic objects, is questionable in its assumption of scale invariance.

As an analogy, in the case of diffusion, the assumption of scale invariance is the condition that the mean free time of a diffusing particle be zero in the continuum limit. In the derivation of the diffusion equation from kinetic theory the vanishing of the mean free time is analogous to the disappearance of the causal areas between events in the **M**-clock. While it is easy to recognize the 'non-physical' aspects of this in the diffusion equation[2], it takes a model clock to

[2] For example, shrinking the mean free time to zero creates Wiener paths of infinite length with fractal dimensions of 2. It also gives the diffusion equation an infinite signal velocity. These features are non-physical, but have approximate validity over scales much larger than the actual mean free time.

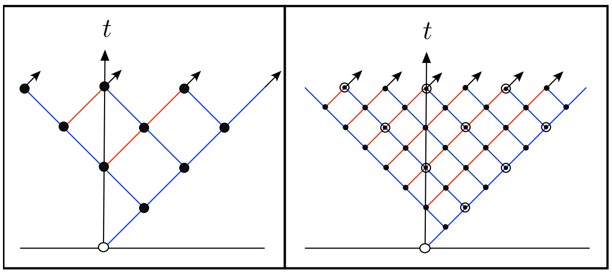

Figure 12. Refining the lattice for the generating function (Dirac's clock Eqn. (14)). Plotted are four steps in time for the $(1, 1)$ element of T_0 and its first refinement at half the original step size. Comparing this to the refinement of the step size for Minkowski's clock (Fig. 2), notice the proliferation of paths rather than the constriction of a causal chain.

demonstrate that the spacetime of Minkowski contains assumptions of scaling that are ultimately unphysical.

Dirac's clock addresses the implicit scale invariance of the 'Spacetime telling matter how to move' assumption in Minkowski's clock. It simply shifts the boost information from a containing spacetime to the clock itself, preserving the dynamical information that Minkowski's clock omits. This is done mathematically by replacing the transfer matrix by a generating function(Fig. 12). It reverses the roles of time and clock-signal that the **M**-clock inherits from its Newtonian predecessor. Whereas clock-signals are functions of time for Minkowski, spacetime is a function of clock-signals for Dirac. This latter perspective has the advantage that mass is inextricably linked to spacetime, as is the Fourier uncertainty principle. Conventional treatments have to reintroduce these to Minkowski's spacetime (via dynamics and quantization respectively) since they are both removed in the **M**-clock's continuum limit.

The parallel emergence of Minkowski spacetime and the Dirac equation shown here(Fig. 13) exposes a weakness in conventional interpretations of both constructs. The clock model shows that spacetime is an abstraction of properties that are shared by all clocks *from the perspective of a continuum limit in which the clock period is always much smaller than* **any** *other timescale under consideration.* Spacetime is clearly relevant for macroscopic objects but the scale invariance eliminates the possibility of intrinsic timekeeping for microscopic objects.

Similarly the conventional picture of the Dirac equation arising as a transplant of the quantum mechanics of Schrödinger into the relativistic domain creates the illusion that the origin of quantum propagation is not itself relativistic. The **D**-clock shows that this is highly unlikely. The path-dependent phase of the **D**-clock is a direct inheritance of the time dilation present in the **M**-clock *but absent in the* **G**-*clock.* In this view, Schrödinger's equation inherits its path-dependent phase from relativity. Its apparent independence from relativity is an artifact similar to the apparent independence of kinetic energy from 'c' in non-relativistic mechanics.

Returning to the question in the title: 'Which came first, spacetime or clocks?' the three clock models illustrate that the answer illuminates the boundary between classical and quantum physics. If the spacetime of Minkowski is a reality that is independent of individual clocks and holds to arbitrarily small scales, then all clocks appear as classical objects with worldlines. Quantum propagation is consequently 'squeezed out' and must ultimately have a separate origin. On the other hand, if spacetime is a scale-dependent ensemble concept and there exist clocks that contain all the boost information they need to be consistent with an ambient spacetime, Dirac propagation is an intrinsic property of such clocks. In this case, as is increasingly suggested by both theory and experiment,[9, 10, 7, 11, 12], "particles are clocks".

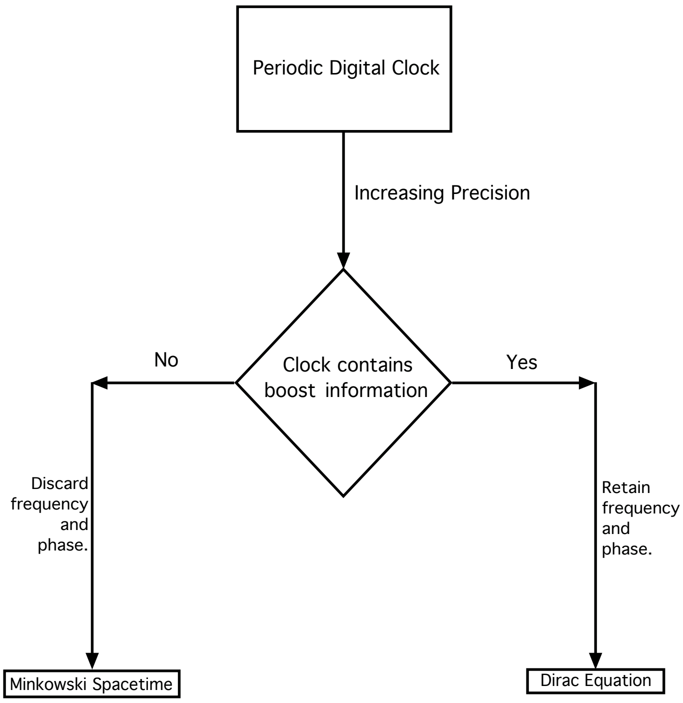

Figure 13. The relation between boost information and clocks in the continuum limit determines the choice of quantum or classical behaviour. If boost information is relegated to spacetime via worldlines in the continuum limit, clocks measure an ambient spacetime and the physics is classical. If clocks include the information for boosts, the resulting generating function obeys the Dirac equation. This suggests that the physics is no longer classical.

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References
[1] Ord G 2012 *J. Phys.: Conf. Ser.* **361** 012007
[2] Ord G and Mann R 2011 *Int. J. Theor. Phys.* **51** 652–666 `DOI 10.1007/s10773-011-0945-1`
[3] Ford K and Wheeler J A 2010 *Geons, Black Holes, and Quantum Foam: A Life in Physics* (W. W. Norton and Company)
[4] Ord G N 1983 *J. Phys. A* **16** 1869–1884
[5] Thompson C J 1979 *Mathematical Statistical Mechanics* (Princeton University Press)
[6] Ord G N 1992 *J. Stat. Phys.* **66** 647–659
[7] Ord G N 2009 *Ann. Phys.* **324** 1211–1218 `DOI 10.1016/j.aop.2009.03.007`
[8] Ord G N and Mann R B 2003 *Phys. Rev. A* **67** 022105
[9] Penrose R 2013 *Ideas Roadshow* ed Burton H (Open Agenda Publishing) URL `http://www.ideasroadshow.com/issues/roger-penrose-2013-07-12`
[10] Lan S Y, Kuan P C, Estey B, English D, Brown J M, Hohensee M A and Müller H 2013 *Science* **339** 554–557 (*Preprint* `http://www.sciencemag.org/content/339/6119/554.full.pdf`)
[11] Poirier B 2013 *talk at this conference*
[12] Jizba P 2013 *this volume*