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Emergence of quantization: the spin of the electron

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Abstract. In previous papers, the quantum behavior of matter has been shown to emerge as a result of its permanent interaction with the random zero-point radiation field. Fundamental results, such as the Schrödinger and the Heisenberg formalism, have been derived within this framework. Further, the theory has been shown to provide the basic QED formulas for the radiative corrections, as well as an explanation for entanglement in bipartite systems.

This paper addresses the problem of spin from the same perspective. The zero-point field is shown to produce a helicoidal motion of the electron, through the torque exerted by the electric field modes of a given circular polarization, which results in an intrinsic angular momentum, of value $\hbar/2$. Associated with it, a magnetic moment with a g-factor of 2 is obtained. This allows us to identify the spin of the electron as a further emergent property, generated by the action of the random zero-point field.

1. Introduction

In previous work, the quantum behavior of matter has been shown to emerge as a result of its continuous interaction with the random zero-point radiation field (ZPF). This has been, in essence, the program of stochastic electrodynamics (SED). Fundamental quantum results, such as the Schrödinger and the Heisenberg formalism, have been recently obtained within this framework [1]. Contact has been made with (nonrelativistic) quantum electrodynamics, by deriving the main formulas for the radiative corrections, notably the Lamb shift and the atomic lifetimes [1], [2]. Further, the study of bipartite systems has shown the ZPF to lie at the origin of entanglement and the symmetrization postulate [3].

Another most fundamental problem in quantum theory relates to the nature of the spin of the electron: is it something inherent, as is usually considered, or is it a result of the dynamics, as has been postulated time and again? A theory intended to explain the genesis of quantization should be expected to provide an answer to this query, instead of *a priori* taking the spin as one more innate property of the particle, like the mass or the electric charge.

During the initial period of SED [4] the electron was in fact taken as a spinless particle, the only exception being the analysis by Braffort and Taroni [5] of some effects due to spin. As of 1981, a number of models for spin as an acquired property have been proposed [6]-[12] (there were of course earlier models, such as that of Huang [13]). Though based on classical models, the various SED calculations exhibit the ZPF as the source of a kind of (nonrelativistic) zitterbewegung that gives rise to an intrinsic angular momentum of the electron, with a mean square value of order $\hbar^2$ and projections of order $\hbar$.

This paper re-addresses the problem of spin from the perspective offered by present SED. Before entering into the subject matter, a brief introduction to recent work on SED is provided,

focusing on those results that will be used for the present derivations. It then will be shown that in addition to giving rise to position, momentum and energy fluctuations, the ZPF induces a helicoidal motion, as a result of the torque exerted by the random Lorentz (electric) force on the particle. The field modes of a given circular polarization are shown to give rise to an (intrinsic) spin angular momentum of the electron, of value $\hbar/2$. Additionally, the corresponding magnetic moment with a g-factor of 2 is derived. The paper ends with some brief additional remarks on the physical meaning of the results obtained.

2. Emergence of quantum mechanics

2.1. The original stochastic problem

This section contains the basic steps that take us from the original dynamical equation for the particle embedded in the random ZPF, to the Schrödinger formulation of quantum mechanics [1]. The system under study is a particle of mass m and charge e —typically an electron—, subject to an external (binding) potential V in addition to the stationary ZPF. The motion of the particle is described by

$$m\ddot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + m\tau\dddot{\boldsymbol{x}} + e\boldsymbol{E}(t), \tag{1}$$

where $m\tau\dddot{\boldsymbol{x}}$, with $\tau = 2e^2/3mc^3$, represents the reaction force due to Larmor radiation, and $\boldsymbol{f}(\boldsymbol{x})$ is the force due to the potential V. A nonrelativistic description is assumed to be sufficient, so that the magnetic term of the Lorentz force is neglected. The random field $\boldsymbol{E}(t)$ is taken in the long-wavelength approximation, anticipating that those field modes that become eventually dominant have wavelengths much larger than the characteristic dimensions of the motion. The random field has zero mean value, i.e., $\overline{\boldsymbol{E}(t)}^{(i)} = 0$, where $\overline{(\cdot)}^{(i)}$ denotes the average over all realizations (i) of the field, and a spectral energy density corresponding to a mean energy $\hbar\omega/2$ per frequency mode, i. e.,

$$\rho_0(\omega) = \frac{\omega^2}{\pi^2 c^3}\frac{1}{2}\hbar\omega = \frac{\hbar\omega^3}{2\pi^2 c^3}. \tag{2}$$

This expression corresponds to the autocorrelation function

$$\overline{E_i(t)E_j(t')}^{(i)} = \delta_{ij}\varphi(t-t'), \tag{3}$$

where

$$\varphi(t-t') = \frac{4\pi}{3}\int_0^\infty \rho_0(\omega)\cos\omega(t-t')d\omega. \tag{4}$$

If there is an extra component due to an external field, as could be a thermal equilibrium radiation or any other excitation of the field modes, the corresponding contribution must be added to $\rho_0(\omega)$ in equation (2).

2.2. Statistical description from phase space to configuration space

The detailed motion of a *single* particle depends on the specific realization (i) of the field, which is unknown. Therefore a statistical description is made, by constructing the equation of evolution for the particle phase-space probability density $Q(\boldsymbol{x}, \boldsymbol{p}, t)$, taking the ZPF as given and starting from the dynamical equations obtained from (1),

$$m\dot{\boldsymbol{x}} = \boldsymbol{p}, \quad \dot{\boldsymbol{p}} = \boldsymbol{f}(\boldsymbol{x}) + m\tau\dddot{\boldsymbol{x}} + e\boldsymbol{E}(t). \tag{5}$$

Through a standard projection procedure [14], the generalized Fokker-Planck equation (GFPE)

$$\frac{\partial Q}{\partial t} + \frac{1}{m}\frac{\partial}{\partial x_i}p_i Q + \frac{\partial}{\partial p_i}f_i Q + m\tau\frac{\partial}{\partial p_i}\dddot{x}_i Q = e^2\frac{\partial}{\partial p_i}\hat{\mathcal{D}}_i Q \tag{6}$$

is obtained, with the diffusion operator $\hat{\boldsymbol{\mathcal{D}}}(t)$ defined by means of the expression

$$\hat{\mathcal{D}}_i(t)Q = \hat{P}E_i\hat{G}\frac{\partial}{\partial p_j}E_j\sum_{k=0}^{\infty}\left[e\hat{G}\frac{\partial}{\partial p_l}(1-\hat{P})E_l\right]^{2k}Q, \tag{7}$$

and the projection operator $\hat{P}$ and the evolution operator $\hat{G}$ given by

$$\hat{P}A = \overline{A}^{(i)}, \quad \hat{G}A(\boldsymbol{x},\boldsymbol{p},t) = \int_{-\infty}^{t} e^{-\hat{L}(t-t')}A(\boldsymbol{x},\boldsymbol{p},t')dt', \tag{8}$$

where $\hat{L}$ is

$$\hat{L} = \frac{1}{m}\frac{\partial}{\partial x_i}p_i + \frac{\partial}{\partial p_i}\left(f_i + m\tau\dddot{x}_i\right). \tag{9}$$

The operator $e^{-\hat{L}(t-t')}$ in equation (8) makes $\boldsymbol{x}(t')$, $\boldsymbol{p}(t')$ $(t' < t)$ evolve towards $\boldsymbol{x}(t)$, $\boldsymbol{p}(t)$ as final conditions, following a deterministic path.

Right after particle and field start to interact, the system is far from equilibrium. In this initial regime the main effect of the ZPF on the particle is due to the high-frequency modes, which produce violent accelerations and randomize the motion. Eventually, however (after a transient period which is estimated to be of the order of $\hbar/mc^2 \simeq 10^{-20}$ s for an electron), the interplay between the electric field force and radiation reaction is expected to drive the system close to equilibrium. In this new (time-reversible) regime the Markovian approximation applies, and the GFPE (6) reduces to a true Fokker-Planck equation,

$$\frac{\partial Q}{\partial t} + \frac{1}{m}\frac{\partial}{\partial x_i}p_iQ + \frac{\partial}{\partial p_i}\left(f_i + m\tau\dddot{x}_i\right)Q = \frac{\partial}{\partial p_i}D_{ij}^{pp}\frac{\partial Q}{\partial p_j} + \frac{\partial}{\partial p_i}D_{ij}^{px}\frac{\partial Q}{\partial x_j}, \tag{10}$$

with the diffusion coefficients given to lowest order in e^2 by

$$D_{ij}^{pp} = e^2\int_{-\infty}^{t}dt'\varphi(t-t')\frac{\partial p_j}{\partial p_i'}, \quad D_{ij}^{px} = e^2\int_{-\infty}^{t}dt'\varphi(t-t')\frac{\partial x_j}{\partial p_i'}. \tag{11}$$

A statistical description *in configuration* space is made in terms of a hierarchy of equations, obtained by multiplying (10) successively by p_i^k $(k = 1, 2, \ldots$, where p_i stands for the different components of the momentum $\boldsymbol{p}$) and integrating over momentum space (assuming the integrand to vanish in the limits). In the time-reversible regime, the radiative terms become small and tend to balance each other in the mean, so any additional effect they may have on the dynamics becomes negligible. When the radiationless approximation is made (i. e., to zero order in e^2), the first two equations of the hierarchy can be recast in the form of a Schrödinger-like equation for a complex function $\psi(\boldsymbol{x},t)$,

$$-\frac{2\eta^2}{m}\boldsymbol{\nabla}^2\psi + V\psi = 2i\eta\frac{\partial\psi}{\partial t}, \tag{12}$$

and the corresponding complex conjugates, with $\psi^*\psi = \int_{-\infty}^{\infty}d^3pQ(\boldsymbol{x},\boldsymbol{p},t) = \rho(\boldsymbol{x},t)$, and η a free parameter. The value of this parameter is determined by the energy-balance condition that must hold in the equilibrium regime (see below).

2.3. Some important relations for average values

Equation (6) (or its Markovian version, (10)) contains a wealth of statistical information about the dynamics of the system, part of which is lost in the transition to the reduced, radiationless description in configuration space. To recover some of this information we take the (phase-space) average of a generic function $\mathcal{G}(\boldsymbol{x},\boldsymbol{p})$ that has no explicit time dependence. Equation (6) multiplied from the left by $\mathcal{G}$ and integrated over the entire phase space gives thus

$$\frac{d}{dt}\langle\mathcal{G}\rangle = \left\langle \dot{x}_i \frac{\partial\mathcal{G}}{\partial x_i}\right\rangle + \left\langle f_i \frac{\partial\mathcal{G}}{\partial p_i}\right\rangle + m\tau\left\langle \ddot{x}_i \frac{\partial\mathcal{G}}{\partial p_i}\right\rangle - e^2\left\langle \frac{\partial\mathcal{G}}{\partial p_i}\hat{\mathcal{D}}_i\right\rangle. \tag{13}$$

In particular, for any $\mathcal{G}=\xi(\boldsymbol{x},\boldsymbol{p})$ representing a classical ('radiationless') integral of the motion, we have

$$\frac{d}{dt}\langle\xi\rangle = m\tau\left\langle \ddot{x}_i \frac{\partial\xi}{\partial p_i}\right\rangle - e^2\left\langle \frac{\partial\xi}{\partial p_i}\hat{\mathcal{D}}_i\right\rangle. \tag{14}$$

In the Markovian limit mentioned above, this equation takes the form

$$\frac{d}{dt}\langle\xi\rangle = m\tau\left\langle \ddot{x}_i \frac{\partial\xi}{\partial p_i}\right\rangle + \left\langle D_{ij}^{pp}\frac{\partial^2\xi}{\partial p_i\partial p_j}\right\rangle + \left\langle D_{ij}^{px}\frac{\partial^2\xi}{\partial p_i\partial x_j}\right\rangle, \tag{15}$$

with the diffusion coefficients given by (11). For instance, for the particle Hamiltonian (defined without the radiative terms, i. e., to zero order in e^2),

$$H = \frac{1}{2m}\boldsymbol{p}^2 + V(\boldsymbol{x}), \tag{16}$$

equation (15) gives

$$\frac{d}{dt}\langle H\rangle = \tau\langle\ddot{\boldsymbol{x}}\cdot\boldsymbol{p}\rangle + \frac{1}{m}\langle\mathrm{tr}\boldsymbol{D}^{pp}\rangle. \tag{17}$$

This equation gives the contributions of the dissipative and diffusive terms to the energy balance. Stationarity is reached when the terms cancel each other, which should occur when both field and particle are in their ground state, i.e.,

$$\tau\langle\ddot{\boldsymbol{x}}\cdot\boldsymbol{p}\rangle_0 = -\frac{1}{m}\langle\mathrm{tr}\boldsymbol{D}^{pp}\rangle_0. \tag{18}$$

Notice that for the calculation of these terms (to the lowest order of approximation) one must use the solutions of the Schrödingerlike equation (12), since the system is already in the time-reversible regime. This means, in particular, that in equations (11) the following replacements must be made:

$$\frac{\partial p_j}{\partial p_i'} \to \frac{1}{2i\eta}[\hat{x}_i', \hat{p}_j], \quad \frac{\partial x_j}{\partial p_i'} \to \frac{1}{2i\eta}[\hat{x}_i', \hat{x}_j]. \tag{19}$$

Explicit calculation gives that for the energy-balance condition (18) to be satisfied, the parameter η *must* have precisely the value $\eta=\hbar/2$ [1]. This is, then, the point of entry of Planck's constant into the Schrödinger equation, obtained from (12),

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}. \tag{20}$$

The time-reversible and radiationless regime, in which the mechanical system is correctly described by this equation, is therefore called the *quantum regime*. Notice that, from this perspective, $\hbar$ provides a direct measure of the intensity of the fluctuations impressed by the ZPF upon the (quantum) particle.

Let us now go back to equation (15), and apply it to the angular momentum $\boldsymbol{L}$, which is a 'classical' constant of the motion for any central-force problem. By taking $\xi = L_{ij} = x_i p_j - x_j p_i$ $(i \neq j)$, we get

$$\frac{d}{dt}\langle \boldsymbol{L} \rangle = m\tau \langle \boldsymbol{x} \times \dddot{\boldsymbol{x}} \rangle - \langle \boldsymbol{D}^A \rangle , \tag{21}$$

where $\boldsymbol{D}^A$ is the antisymmetric tensor with components $D^{px}_{ij} - D^{px}_{ji}$. This equation tells us that for the angular momentum to be conserved, the radiative terms must cancel each other,

$$m\tau \langle \boldsymbol{x} \times \dddot{\boldsymbol{x}} \rangle = \langle \boldsymbol{D}^A \rangle . \tag{22}$$

Finally, for the square of the angular momentum $\boldsymbol{L}^2$, equation (14) gives

$$\frac{d}{dt}\langle \boldsymbol{L}^2 \rangle = m\tau \left\langle \dddot{x}_i \frac{\partial \boldsymbol{L}^2}{\partial p_i} \right\rangle + \left\langle D^{pp}_{ij} \frac{\partial^2 \boldsymbol{L}^2}{\partial p_i \partial p_j} \right\rangle + \left\langle D^{px}_{ij} \frac{\partial^2 \boldsymbol{L}^2}{\partial p_i \partial x_j} \right\rangle , \tag{23}$$

so that for $\boldsymbol{L}^2$ to be conserved, we must have

$$m\tau \left\langle \dddot{x}_i \frac{\partial \boldsymbol{L}^2}{\partial p_i} \right\rangle = - \left\langle D^{pp}_{ij} \frac{\partial^2 \boldsymbol{L}^2}{\partial p_i \partial p_j} \right\rangle - \left\langle D^{px}_{ij} \frac{\partial^2 \boldsymbol{L}^2}{\partial p_i \partial x_j} \right\rangle . \tag{24}$$

These latter results, first proposed in [9], will be used in section 3.1.

2.4.Linear response of the particle in the stationary regime

A close analysis of the stationary states attained by the particle in the time-reversible regime described above, has proved particularly revealing of some general properties of quantum systems. For such analysis a more straightforward approach, complementary to the previous one, has been developed [1], starting again from the same equation of motion (1), and assuming that any stationary solution of it (characterized by the index α) can be written as an expansion of the form

$$x_\alpha(t) = \sum_\beta \tilde{x}_{\alpha\beta} a_{\alpha\beta} e^{i\omega_{\alpha\beta} t}, \tag{25}$$

and similarly for any dynamical variable,

$$A_\alpha(t) = \sum_\beta \tilde{A}_{\alpha\beta} a_{\alpha\beta} e^{i\omega_{\alpha\beta} t}, \tag{26}$$

where $a_{\alpha\beta}$ stands for the random amplitude of the ZPF mode of frequency $\omega_{\alpha\beta}$,

$$E_\alpha(t) = \sum_\beta \tilde{E}_{\alpha\beta} a_{\alpha\beta} e^{i\omega_{\alpha\beta} t}. \tag{27}$$

In these equations the coefficients $\tilde{x}_{\alpha\beta}$, $\tilde{A}_{\alpha\beta}$ are in principle functions of the amplitudes $a_{\alpha\beta}$, and hence stochastic variables. By introducing these expansions into equation (1), the system is found to respond *resonantly* to certain field modes $(\alpha\beta)$; the rest of the ZPF (to be neglected in the expansions) represents just a background noise. The values of the corresponding $\omega_{\alpha\beta}$ (called relevant frequencies), as well as of the coefficients $\tilde{x}_{\alpha\beta}$, depend on the specific problem. By imposing the condition of ergodicity on the stationary solutions, the $\{\tilde{x}_{\alpha\beta}\}$ turn out to be independent of the $\{a_{\alpha\beta}\}$, which means, according to (25), that the response of the system to the respective field modes is *linear* in the field variables. Further, those modes controlling the stationary states are found to satisfy certain properties, which can be summarized in terms of the *chain rule* for the random field coefficients, $a_{\alpha\beta'} a_{\beta'\beta''} a_{\beta''\beta'''} \cdots a_{\beta^{(n-1)}\beta} = a_{\alpha\beta}$, whence

$a_{\alpha\beta} = e^{i\varphi_{\alpha\beta}}$ with $\varphi_{\alpha\beta} = \phi_\alpha - \phi_\beta$ a random phase. Similarly for the corresponding relevant frequencies, the relation $\omega_{\alpha\beta'} + \omega_{\beta'\beta''} + ... + \omega_{\beta^{(n-1)}\beta} = \omega_{\alpha\beta}$ applies, which means that $\omega_{\alpha\beta}$ is of the form

$$\omega_{\alpha\beta} = \Omega_\alpha - \Omega_\beta. \tag{28}$$

As a result of these properties, the coefficients in the above expansions satisfy a matrix algebra; thus, for instance, if $\hat{x}$ is the matrix with elements $\tilde{x}_{\alpha\beta}$, we have

$$(x^n)_\alpha = \sum_\beta (\widetilde{x^n})_{\alpha\beta} a_{\alpha\beta} e^{i\omega_{\alpha\beta}t}, \tag{29}$$

with $(\widetilde{x^n})_{\alpha\beta}$ given by the element $\alpha\beta$ of the corresponding matrix product, $(\widetilde{x^n})_{\alpha\beta} = (\hat{x}^n)_{\alpha\beta}$.

It is clear that the time-dependence of $x_\alpha(t)$ can be transferred to every single factor $\tilde{x}_{\alpha\beta}$, so that the evolving matrix $\hat{x}(t)$ has as elements the coefficients $\tilde{x}_{\alpha\beta}e^{i\omega_{\alpha\beta}t}$. This allows to write the equation satisfied by the stationary solutions in the form

$$m\frac{d^2\hat{x}(t)}{dt^2} = \hat{f}(t) + m\tau\frac{d^3\hat{x}(t)}{dt^3} + e\hat{E}(t). \tag{30}$$

In the radiationless approximation, the last two terms are neglected and one is left with

$$m\frac{d^2\hat{x}(t)}{dt^2} = \hat{f}(t). \tag{31}$$

With these results one may construct the law of evolution for the matrix $\hat{A}(t)$ associated with the (generic) dynamical variable A; the outcome is

$$i\frac{d\hat{A}(t)}{dt} = \left[\hat{A}(t), \hat{\Omega}\right], \tag{32}$$

with the matrix elements of $\hat{\Omega}$ given by $\Omega_{\alpha\beta} = \Omega_\alpha\delta_{\alpha\beta}$. Notice that, as with the previous derivation presented in section 2.2, the radiationless approximation has deleted any explicit trace of the ZPF. No stochastic variable is contained in equations (30)-(32); instead, they are expressed in terms of operators. In section 2.3, the missing value of the paramerter η in equation (12) was obtained by imposing the energy-balance condition; this led unequivocally to the Schrödinger equation (20). In the present case, it is again the scale of the solution what is missing in equation (32). The loss is repaired by, first, finding that the canonical commutator $[\hat{x}, \hat{p}]$, with

$$\hat{p} = m\frac{d\hat{x}(t)}{dt} = -im\left[\hat{x}(t), \hat{\Omega}\right], \tag{33}$$

has the *universal* form $[\hat{x}, \hat{p}] = C\mathbb{I}$, with C constant, and, second, finding the value of C. This is readily achieved by applying the above equations to the harmonic oscillator of natural frequency ω_0 in its ground state, when it is in equilibrium with the ZPF mode of the same frequency. One thus obtains $C = i\hbar$, whence

$$[\hat{x}, \hat{p}] = i\hbar\mathbb{I}. \tag{34}$$

The value of the commutator represents therefore a direct measure of the intensity of the fluctuations impressed upon the particle by the ZPF, just as was concluded in section 2.3. Finally, from the above results it can be readily seen that the relevant frequencies are given by

$$\omega_{\alpha\beta} = \Omega_\alpha - \Omega_\beta = \hbar^{-1}\left(\mathcal{E}_\alpha - \mathcal{E}_\beta\right), \tag{35}$$

where $\mathcal{E}_\alpha$ is the eigenvalue of the Hamiltonian in state α, and the final form of equation (32) is therefore

$$i\hbar\frac{d\hat{A}(t)}{dt} = \left[\hat{A}(t), \hat{H}\right], \tag{36}$$

i. e., the Heisenberg equation for the operator $\hat{A}(t)$.

3. Revealing the spin of the electron

To disclose the rotational effect of the ZPF on the particle, let us briefly go back to the original (stochastic) equation of motion (1) and rewrite it as

$$\dot{\boldsymbol{p}} = \boldsymbol{f} + m\tau\dddot{\boldsymbol{x}} + e\boldsymbol{E}(t), \tag{37}$$

where $\boldsymbol{p} = m\dot{\boldsymbol{x}}$. We shall consider that there is no external torque; the (central) force can then be written as $\boldsymbol{f} = g(r)\boldsymbol{x}$, $r = |\boldsymbol{x}|$. By taking the vector product of this equation with $\boldsymbol{x}$ we get

$$\frac{d}{dt}\boldsymbol{L} = \boldsymbol{x} \times \dot{\boldsymbol{p}} = m\tau\boldsymbol{x} \times \dddot{\boldsymbol{x}} + e\boldsymbol{x} \times \boldsymbol{E}, \tag{38}$$

which gives the instantaneous change of the angular-momentum due to the torque exerted by both radiation reaction and the random ZPF. The average of this expression over the field realizations is

$$\frac{d}{dt}\overline{\boldsymbol{L}}^{(i)} = m\tau\overline{(\boldsymbol{x} \times \dddot{\boldsymbol{x}})}^{(i)} + e\overline{(\boldsymbol{x} \times \boldsymbol{E})}^{(i)}. \tag{39}$$

Since in the quantum regime the dynamical variables satisfy the condition of ergodicity, according to our discussion in section 2.4, this equation is equivalent to (21). Equation (39), however, has the advantage of allowing us to directly identify the last term as the effective torque exerted by the Lorentz force of the ZPF on the particle. For the angular momentum to be conserved, we must have (compare with equation (22))

$$m\tau\overline{(\boldsymbol{x} \times \dddot{\boldsymbol{x}})}^{(i)} = -e\overline{(\boldsymbol{x} \times \boldsymbol{E})}^{(i)}. \tag{40}$$

Let us assume that the system is in its ground state, so that there is no orbital angular momentum. Making the usual substitution $\dot{\boldsymbol{p}} \to \boldsymbol{f}$ (which is valid to zero order in $\tau \sim e^2$) and writing $\boldsymbol{f}$ as $g(r)\boldsymbol{x}$, we get explicitly

$$\tau\overline{g(r)\boldsymbol{L}}^{(i)}\Big|_0 = -me\ \overline{(\boldsymbol{x} \times \boldsymbol{E})}^{(i)}\Big|_0. \tag{41}$$

Since only the fluctuating component of $\boldsymbol{x}$ can contribute to the average $\overline{(\boldsymbol{x} \times \boldsymbol{E})}^{(i)}$ (because $\boldsymbol{E}(t)$ is purely random), it is clear that all the angular momentum thus generated is due to the random motion around the mean trajectory followed by the particle; thus, it is independent of the system of coordinates, and has an 'internal' nature, usualy taken as intrinsic.

3.1. The electron's intrinsic angular momentum

The above discussion suggests looking for a procedure that can bring to the surface the 'intrinsic' angular momentum acquired by the electron through its interaction with the ZPF. For this purpose we recall the experimental observation that the interaction of the electron with the radiation field takes place via the circular polarized modes of the field (or modes of a certain helicity). This is known to be the case for the photonic field, which, from the perspective of SED, is the excited state of the radiation field, additional to the zero-point component. Now, it is natural to assume that the modes of the field in its ground state, i. e., of the ZPF, interact in a similar way with the electron. To analyze the effect of such interaction, we should therefore consider the ZPF as composed of modes of both right- and left-handed circular polarization. When the ensemble of field modes is considered in its entirety, as is usually done, any such effect is concealed; yet by focusing on one of the two subensembles of a given polarization, the effective rotation induced on the corresponding particles should be disclosed.

Let us therefore consider a situation in which the particle is in a stationary state, under the action of the background field of a given circular polarization with respect to an axis $\boldsymbol{k}$. To start

with, we shall consider the particle in its ground state, so as to ensure that there is no orbital angular momentum. We therefore take

$$\langle L_{ij}\rangle_0 = \langle 0|\left(\hat{x}_i\hat{p}_j - \hat{x}_j\hat{p}_i\right)|0\rangle \tag{42}$$

and analyze separately the contributions arising from each of the two circular polarizations, characterized by the (circularly polarized) vectors

$$\boldsymbol{\epsilon}_{\boldsymbol{k}\pm} = \tfrac{1}{\sqrt{2}}\left(\boldsymbol{\epsilon}_{\boldsymbol{k}i} \pm i\boldsymbol{\epsilon}_{\boldsymbol{k}j}\right), \tag{43}$$

with $\boldsymbol{\epsilon}_{\boldsymbol{k}i}$, $\boldsymbol{\epsilon}_{\boldsymbol{k}j}$ unit Cartesian vectors orthogonal to the axis $\boldsymbol{k}$.

Since according to the results reported in section 2.4, the response of the particle to the field is linear, to describe its motion under the action of a circular field mode one should write the variable $\boldsymbol{x}$ in cylindrical coordinates, i. e. $\boldsymbol{x} = x^+\boldsymbol{\epsilon}_{\boldsymbol{k}+} + x^-\boldsymbol{\epsilon}_{\boldsymbol{k}-} + x_k\hat{\boldsymbol{k}}$, with

$$x^\pm = \tfrac{1}{\sqrt{2}}\left(x_i \mp ix_j\right), \tag{44a}$$

$$x_i = \tfrac{1}{\sqrt{2}}\left(x^+ + x^-\right), \quad x_j = i\tfrac{1}{\sqrt{2}}\left(x^+ - x^-\right). \tag{44b}$$

Taking into account that $x^\pm_{n0} = \left(x^\mp_{0n}\right)^*$, equation (42) becomes explicitly

$$\langle L_{ij}\rangle_0 = m\sum_n \omega_{n0}\left(x^+_{0n}x^-_{n0} - x^-_{0n}x^+_{n0}\right) = m\sum_n \omega_{n0}\left(\left|x^+_{0n}\right|^2 - \left|x^-_{0n}\right|^2\right). \tag{45}$$

In the ground state, $\langle L_{ij}\rangle_0 = 0$; hence the two sums on the right-hand side contribute with equal magnitude and opposite sign to the k-component of the total angular momentum, as should be the case for a nonpolarized vacuum. Taken separately, these contributions are

$$\langle L_{ij}\rangle^+_0 = m\sum_n \omega_{n0}\left|x^+_{0n}\right|^2, \quad \langle L_{ij}\rangle^-_0 = -m\sum_n \omega_{n0}\left|x^-_{0n}\right|^2. \tag{46}$$

Since, on the other hand, the mean value of the commutator (34) gives the sum rule

$$m\sum_n \omega_{n0}\left|x^\pm_{0n}\right|^2 = \frac{\hbar}{2}, \tag{47}$$

the size of each separate contribution to (45) is just $\hbar/2$. In order to distinguish these contributions from the (orbital) component of the angular momentum we write $\langle S_{ij}\rangle^\pm$ instead of $\langle L_{ij}\rangle^\pm_0$; thus $\langle L_{ij}\rangle_0 = \langle S_{ij}\rangle^+ + \langle S_{ij}\rangle^-$, with

$$\langle S_{ij}\rangle^\pm = \pm\frac{\hbar}{2}. \tag{48}$$

Direct calculation of the square of the angular momentum, using (24), is more cumbersome. As a simple expedient, let us carry out this calculation for the isotropic harmonic oscillator in its ground state. The (radiationless) approximation $\dot{p}_i = f_i$ allows us to write in this case $m\dddot{x}_i = -\omega_0^2 p_i$. Further, since $\boldsymbol{L}^2 = \boldsymbol{x}^2\boldsymbol{p}^2 - (\boldsymbol{x}\cdot\boldsymbol{p})^2$, we have

$$p_i\frac{\partial \boldsymbol{L}^2}{\partial p_i} = 2\boldsymbol{L}^2, \quad \frac{\partial^2 \boldsymbol{L}^2}{\partial p_i \partial p_i} = 4\boldsymbol{x}^2, \quad \frac{\partial^2 \boldsymbol{L}^2}{\partial x_i \partial p_i} = 0.$$

With these results equation (24) transforms into

$$\left\langle \boldsymbol{L}^2\right\rangle_0 = \frac{mc^3}{e^2\omega_0^2}\left\langle D^{pp}\boldsymbol{x}^2\right\rangle_0, \tag{49}$$

with D^{pp} given by equation (11). Now in the case of the harmonic oscillator, using equations (2), (4) and (19) we obtain a constant value for the diffusion coefficient,

$$D^{pp} = \frac{\hbar e^2 \omega_0^3}{c^3}, \tag{50}$$

whence using $\langle \boldsymbol{x}^2 \rangle_0 = 3\langle \hat{x}^2 \rangle_0 = 3\hbar/(2m\omega_0)$, equation (49) becomes finally

$$\left\langle \boldsymbol{L}^2 \right\rangle_0 = m\hbar\omega_0 \left\langle \boldsymbol{x}^2 \right\rangle_0 = \frac{3}{2}\hbar^2. \tag{51}$$

This result was obtained for the first time by Marshall in 1965 [15], and taken as an additional contribution to the orbital angular momentum due to the ZPF. In [9] the same result was obtained, but interpreted as an intrinsic (spin) angular momentum of double the correct value. However, in line with the present approach, we separate again the full ensemble into two subensembles corresponding to the different circular polarizations, thus obtaining

$$\left\langle \boldsymbol{L}^2 \right\rangle_0 = \left\langle \boldsymbol{L}^2 \right\rangle_0^+ + \left\langle \boldsymbol{L}^2 \right\rangle_0^- , \tag{52}$$

with each partial contribution to the mean square angular momentum given by

$$\left\langle \boldsymbol{L}^2 \right\rangle_0^+ = \left\langle \boldsymbol{L}^2 \right\rangle_0^- = \tfrac{3}{4}\hbar^2. \tag{53}$$

Using the notation introduced above (see equation (48)), which identifies this as an intrinsic angular momentum, we write

$$\left\langle \boldsymbol{S}^2 \right\rangle^+ = \left\langle \boldsymbol{S}^2 \right\rangle^- = \tfrac{3}{4}\hbar^2. \tag{54}$$

The fact that this result does not depend on the oscillator's frequency ω_0, suggests that it holds in the general case, and for the free particle in particular. Therefore, we conclude that when the transformation (44) possesses physical meaning, so that the decompositions $\langle L_{ij} \rangle_0 = \langle L_{ij} \rangle_0^+ + \langle L_{ij} \rangle_0^-$ and $\left\langle \boldsymbol{L}^2 \right\rangle_0^+ = \left\langle \boldsymbol{L}^2 \right\rangle_0^+ + \left\langle \boldsymbol{L}^2 \right\rangle_0^-$ make sense, equations (48) and (54) tell us that there exists an angular momentum that does *not* correspond to an orbital motion of the particle and can therefore be considered as *intrinsic*. For an electron, which (as stated above) interacts with the radiation field via its circular polarized modes, the transformation (44) is indeed physically meaningful, and the angular momentum thus induced can therefore be identified with its spin.

3.2. General derivation of the spin angular momentum

In the preceding section we have disclosed the existence of the spin angular momentum for an electron in its ground state. Let us now extend our analysis to the general case, including excited states with orbital angular momentum.

According to the above discussion, we should separate the contributions to the angular momentum arising from the two circular polarizations of the field. Denoting with the index n (or k) the set of quantum numbers that characterize a state of the particle, including the orbital angular momentum and its projection along the z-axis, we have (for simplicity in the writing we use $x_i = x$, $x_j = y$, and $x_k = z$)

$$\begin{aligned} \left\langle \hat{L}_z \right\rangle_n &= \langle n | \hat{L}_z | n \rangle = \sum\nolimits_k \left(x_{nk} p_{ykn} - y_{nk} p_{xkn} \right) = \\ &= im \sum\nolimits_k \omega_{kn} \left(x_{nk} y_{kn} - y_{nk} x_{kn} \right). \end{aligned} \tag{55}$$

Under the same procedure that led to (45), equation (55) transforms into

$$\left\langle \hat{L}_z \right\rangle_n = m \sum\nolimits_k \omega_{kn} \left(\left| x_{nk}^+ \right|^2 - \left| x_{nk}^- \right|^2 \right). \tag{56}$$

This expression can be rewritten as

$$\left\langle \hat{L}_z \right\rangle_n = \langle O_z \rangle_n^+ + \langle O_z \rangle_n^- , \tag{57}$$

with $\langle O_z \rangle_n^\sigma$ given by ($\sigma = \pm$)

$$\langle O_z \rangle_n^\sigma = \sigma m \sum\nolimits_k \omega_{kn} \left| x_{nk}^\sigma \right|^2 . \tag{58}$$

Using again the sum rule $m \sum_k \omega_{kn} \left| x_{nk} \right|^2 = m \sum_k \omega_{kn} \left| y_{nk} \right|^2 = \frac{1}{2}\hbar$, one obtains from (58)

$$\hbar = m \sum\nolimits_k \omega_{kn} \left(\left| x_{nk}^+ \right|^2 + \left| x_{nk}^- \right|^2 \right) = \langle O_z \rangle_n^+ - \langle O_z \rangle_n^- , \tag{59}$$

which combined with (57) gives

$$\langle O_z \rangle_n^\sigma = \tfrac{1}{2} \left\langle \hat{L}_z \right\rangle_n + \sigma \tfrac{1}{2} \hbar. \tag{60}$$

This quantity $\langle O_z \rangle_n^\sigma$ contains, for every polarization state σ, both the corresponding part of the orbital angular momentum, and the spin associated with that state.

To construct the operator associated with the vector $\boldsymbol{S}$ introduced in section 3.1, we observe that the mean value $\langle \hat{L}_z \rangle_n$ does not depend on σ, whereas the term $\sigma\hbar/2$ does not depend on n. This indicates that the operator $\hat{L}_z$ and the operator to be associated with $\sigma\hbar/2$ (which we shall call $\hat{\Sigma}_z$) belong to different Hilbert spaces. In order to express $\langle O_z \rangle_n^\sigma$ as the average of an operator, we must therefore extend the Hilbert space to include the dichotomous variable σ in addition to the quantum index n. The result is the product space $\mathcal{H} = \mathcal{H}_n \otimes \mathcal{H}_2$, with $\mathcal{H}_2$ a bidimensional vector space spanned by an orthonormal basis having as elements the vectors $\{|\sigma\rangle\} = \{|+\rangle, |-\rangle\}$. In terms of $|n\sigma\rangle = |n\rangle \otimes |\sigma\rangle$, equation (60) rewrites as

$$\langle O_z \rangle_n^\sigma = \tfrac{1}{2} \langle n\sigma | \hat{L}_z | n\sigma \rangle + \tfrac{1}{2}\hbar \langle n\sigma | \hat{\Sigma}_z | n\sigma \rangle , \tag{61}$$

with $\hat{\Sigma}_z$ an operator that has $|\sigma\rangle$ as eigenvector,

$$\langle n\sigma | \hat{\Sigma}_z | n\sigma \rangle = \langle \sigma | \hat{\Sigma}_z | \sigma \rangle = \sigma. \tag{62}$$

Expressing $\hat{\Sigma}_z$ in terms of the Pauli matrices gives

$$\hat{\boldsymbol{\Sigma}}_z = a_0 \mathbb{I} + a_z \hat{\sigma}_z + a^+ \hat{\sigma}_+ + a^- \hat{\sigma}_-, \tag{63}$$

where $a^\pm = (a_x \mp i a_y)/\sqrt{2}$, and $\hat{\sigma}_+ = \sqrt{2} |+\rangle \langle -|$, $\hat{\sigma}_- = \sqrt{2} |-\rangle \langle +|$ are ladder operators. Condition (62) imposed on $\hat{\boldsymbol{\Sigma}}_z$ gives $a_0 = 0$, $a_z = 1$. Further, since we are here considering the variables (x^+, x^-, z), the polarization vectors (43) fix $\hat{\boldsymbol{z}}$ as the preferred axis, whence $a^\pm = 0$, $\hat{\boldsymbol{\Sigma}}_z = \hat{\sigma}_z$, and equation (61) becomes

$$\langle O_z \rangle_n^\sigma = \langle n\sigma | \left(\tfrac{1}{2} \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}} \right) \cdot \hat{\boldsymbol{z}} \, | n\sigma \rangle , \tag{64}$$

with $\hat{\boldsymbol{S}}$ the vector operator defined as $\hat{\boldsymbol{S}}_z = \hbar \frac{1}{2} \hat{\Sigma}_z$, i.e.,

$$\hat{\boldsymbol{S}} = \tfrac{1}{2} \hbar \hat{\boldsymbol{\sigma}}. \tag{65}$$

The identification of the operator $\hat{\boldsymbol{S}}$ with the spin of the electron is thus justified. The independence of $\langle\hat{L}_z\rangle_n$ from σ and of $\langle\hat{\Sigma}_z\rangle$ from n, indicates that under the present conditions, the fluctuations associated with the spin are not correlated with those that characterize the kinematics of the particle in the configuration space: $\boldsymbol{L}$ and $\boldsymbol{S}$ constitute independent dynamical variables. Of course the spaces of the spin and the orbital angular momentum may become connected by the presence of magnetic fields.

It should be stressed that even if $\hat{\boldsymbol{L}}$ and $\hat{\boldsymbol{S}}$ are combined under one expression for the angular motions, e. g. equation (64), the spin is not an orbital angular momentum. Indeed, there are fundamental differences between $\hat{\boldsymbol{L}}$ and $\hat{\boldsymbol{S}}$. In particular, the mean value of $\hat{L}_z$, say, can be freely determined by adjusting external parameters, and may acquire a whole spectrum of values. However, only the sign of the projection $\hat{S}_z$ can be subject to external adjustment; its absolute value is determined by the fundamental commutator, which in its turn is fixed by the ZPF. It is because of the *universal* value of the commutator that the spin of the electron is the same for all electrons under all circumstances, which reinforces its 'intrinsic' nature.

The connection of the commutator $[\hat{x},\hat{p}_x] = i\hbar$ with the spin of the electron deserves a further comment. As pointed out in previous sections, Planck's constant $\hbar$ is a direct measure of the size of the fluctuations, both those of the ZPF and those impressed by it on the particle. Specifically, since the commutator implies that the fluctuations of x and p_x have a minimum value adjusted to the rule $\sigma_x^2\sigma_{p_x}^2\big|_{\min} = \hbar^2/4$ (σ_x^2, $\sigma_{p_x}^2$ are variances), one may write the *numerical* relation $\left|\langle\pm|\,\hat{S}_z\,|\pm\rangle\right| = \hbar/2 = \sigma_x\sigma_{p_x}|_{\min}$, which emphasizes the fact that the *value* of the electron spin is determined by the irreducible fluctuations of the phase-space variables x, p_x around the instantaneous position of the particle.

3.3. The spin gyromagnetic factor

It was discovered experimentally that the g-factor associated with the spin magnetic moment of the electron has an approximate value $g_S = 2$, whereas for the orbital magnetic moment the g-factor is $g_L = 1$. This characteristic value of g_S is incorporated into nonrelativistic quantum theory by hand, usually without further elaboration. The issue is normally resolved by resorting to the Dirac equation, which predicts $g_S = 2$. Given that the present theory produces the electron spin, it seems appropriate to investigate the value predicted by it for g_S. For this purpose consider the electron acted on, in addition to the external force $\boldsymbol{f}(\boldsymbol{x})$, by a static uniform magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$. The contribution of the orbital angular momentum $\boldsymbol{L}$ to the Hamiltonian is given by

$$\hat{H} = -\hat{\boldsymbol{\mu}}\cdot\boldsymbol{B} = -\mu_z B, \tag{66}$$

where $\hat{\boldsymbol{\mu}} = -(g_L\mu_0\hat{\boldsymbol{L}})/\hbar$ is the magnetic moment due to $\hat{\boldsymbol{L}}$, $\mu_0 = |e|\,\hbar/(2mc)$ is the Bohr magneton (with $-e = |e|$), and $g_L = 1$. Therefore the mean energy is

$$\mathcal{E} = \frac{\mu_0}{\hbar}B\langle\hat{L}_z\rangle. \tag{67}$$

Consider now a situation in which the spin projection along $\hat{\boldsymbol{z}}$ has a well-defined value, say $\langle\hat{S}_z\rangle = +\hbar/2$. This means that one should consider only the subensemble that corresponds to $\sigma = +$. Resorting to equation (60) to write the corresponding contribution to $\langle L_z\rangle$ as $(\langle\hat{L}_z\rangle+\hbar)/2$, the component of $\mathcal{E}$ of interest is

$$\mathcal{E}^+ = \frac{\mu_0 B}{\hbar}\left(\frac{1}{2}\langle\hat{L}_z\rangle + \frac{\hbar}{2}\right) = \frac{\mu_0 B}{\hbar}\left(\frac{1}{2}\langle\hat{L}_z\rangle + \langle\hat{S}_z\rangle\right)^+. \tag{68}$$

An analogous result holds for the subensemble with $\sigma = -$, for which $\langle\hat{S}_z\rangle = -\hbar/2$,

$$\mathcal{E}^- = \frac{\mu_0 B}{\hbar}\left(\frac{1}{2}\langle\hat{L}_z\rangle - \frac{\hbar}{2}\right) = \frac{\mu_0 B}{\hbar}\left(\frac{1}{2}\langle\hat{L}_z\rangle + \langle\hat{S}_z\rangle\right)^-. \tag{69}$$

The corresponding Hamiltonian describing the total magnetic interaction of the electron follows from the sum of these contributions; it is therefore

$$\hat{H}_{LS} = \frac{\mu_0 B}{\hbar}\left(\hat{L}_z + 2\hat{S}_z\right). \tag{70}$$

This contains the correct g-factor of 2 for the spin of the electron, in the radiationless approximation. It is clear from the derivation that this value is determined by the two degrees of freedom associated with the polarization of the ZPF.

Notice that the result (70) gives a precise meaning to the operator appearing in equation (64). Indeed, from this latter equation we can write $\hat{\boldsymbol{O}} = (\hat{\boldsymbol{L}} + 2\hat{\boldsymbol{S}})/2$, whence

$$\hat{H}_{LS} = \frac{\mu_0}{\hbar}\boldsymbol{B}\cdot\left(\hat{\boldsymbol{L}} + 2\hat{\boldsymbol{S}}\right) = -\hat{\boldsymbol{\mu}}\cdot\boldsymbol{B}, \tag{71}$$

with $\hat{\boldsymbol{\mu}} = -\frac{2\mu_0}{\hbar}\hat{\boldsymbol{O}}$, which directly relates $\hat{\boldsymbol{O}}$ with the *total* magnetic-moment operator of the atomic electron.

4. Final remarks

The present results give strong support to the representation of the electron spin as an acquired angular momentum. They reaffirm the image, suggested in previous SED work, of spin as a helicoidal motion around the local mean path followed by the particle, produced by the high-frequency (circular polarized) modes of the fluctuating vacuum.

If, according to the results presented, a charged particle acquires spin 1/2, the question arises as to whether a similar effect shouldn't be expected to appear in the case of scalar bosons. A possible answer to this question is that the spin 1/2 is acquired by elementary particles, such as the electron, whereas composite particles may acquire it or not, depending on their internal structure. This suggests that bosons are in general composite structures, with an even number of elements (if of fermion type).

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