

Resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ within the extended Linear Sigma Model

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Abstract. In the framework of the extended Linear Sigma Model (eLSM) we investigate the masses and decays of the three scalar-isoscalar resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. The degrees of freedom of the eLSM are (pseudo)scalars and axial(vectors) as well as the scalar glueball. Although still preliminary, present results, based on the physical masses of the above mentioned resonances, show that $f_0(1710)$ is the predominantly glueball state. However, acceptable decays for this resonance can be obtained only for a (very) large gluon condensate.

1. Introduction

In this work we study the scalar-isoscalar resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ [1] in the framework of the extended Linear Sigma Model (eLSM) [2, 3, 4, 5]. The eLSM is built accordingly to two fundamental properties of QCD: chiral symmetry and dilatation invariance, the former being spontaneously broken by a Mexican-hat potential, the latter being explicitly broken in such a way to mimic the trace anomaly of QCD [6, 7]. As a consequence of these two requirements, the eLSM Lagrangian contains only a finite number of terms. Moreover, (axial)vector d.o.f. are included from the very beginning in the effective model. The inclusion of the latter in a chiral invariant way makes the model more complete and has a strong influence on the phenomenology.

In Ref. [2] the eLSM has been first studied, both in the baryonic and mesonic sectors, in presence of two-flavours, $N_f = 2$. The inclusion of the dilaton/glueball field for $N_f = 2$ has been carried out in Ref. [3], where it has been shown that the glueball is predominantly contained in either $f_0(1500)$ or $f_0(1710)$ (the former case being favoured). Still, the study was not complete because scalar mesons containing the s -quark were not included. A detailed study of the eLSM for three flavours, $N_f = 3$, has been performed in Ref. [4], where a very good description of various meson masses and decays was achieved. Yet, the dilaton, although formally included to justify the form of the Lagrangian, was inert (i.e., with zero width, in agreement with large- N_c limit). The scalar-isoscalar system was studied with the nonstrange quarkonium field $\sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ and with the hidden-strange quarkonium field $\sigma_S \equiv \bar{s}s$ only. Ref. [4] shows through a fit to experimental data that these quark-antiquark states are heavy (above 1 GeV); similarly, the scalar isovector and isodoublet states are identified with the heavy resonances $a_0(1450)$ and $K_0^*(1430)$, respectively. Consequently, the scalar mesons below 1 GeV shall be interpreted differently: tetraquarks or molecular/dynamically generated states are the most prominent options, e.g. Refs. [8, 9] and refs. therein.

In the scalar-isoscalar sector a three-body mixing problem must be solved: the three bare



fields σ_N , σ_S , and, in addition, the dilaton $G \equiv gg$ mix and generate the physical resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. The aim is to determine the mixing matrix in such a way that the masses and the decays of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ are in agreement with the experimental results listed in Ref. [1]. Such a mixing problem was investigated in a variety of phenomenological models, see Ref. [10, 11, 12] and refs. therein. However, a full calculation involving a *full* chiral approach has not yet been achieved for $N_f = 3$. In Ref. [5] the first step toward this direction has been performed, but the attention was focused on the masses only (no decays were calculated). Based on the parameter determination obtained in Ref. [4], a correct description of the masses $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ implies that the predominant glueball content is located in the resonance $f_0(1710)$ (and not in $f_0(1500)$).

In this work we continue the study of this system by calculating, for the first time, the decay widths of the scalar-isoscalar states. It is shown that the mixing matrix of Ref. [5] implies too large decay widths for $f_0(1500)$ and $f_0(1710)$, and must be discarded. Therefore, we search for other solutions. The system is extremely dependent on subtle issues such as constructive and destructive interference effects; for this reason, a fit (which is the most straightforward thing to do) could not yet deliver acceptable results. Anyhow, an interesting partial solution could be obtained by a simple ‘guess and try’ procedure. In this solution, the gluon condensate is very large and the glueball is to a very good extent described by the resonance $f_0(1710)$ only. The phenomenology of $f_0(1710)$ and $f_0(1370)$ can be qualitatively described, but the kaon-kaon decay of $f_0(1500)$ is still too large. Future studies will show if this novel solution is phenomenologically acceptable or not.

This proceeding is organized as follows: In Sec. 2 we present the effective Lagrangian of the eLSM, in Sec. 3 we discuss the results and in Sec. 4 we provide a summary and an outlook for work in progress.

2. The eLSM Lagrangian

For the purpose of studying the mixing behavior in vacuum of the scalar-isoscalar mesons below 2 GeV we use eLSM Lagrangian developed in Refs. [2, 3, 4]:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{dil} + \text{Tr}[(D^\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \left(\frac{G}{G_0} \right)^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 - \lambda_2 \text{Tr}[(\Phi^\dagger \Phi)^2] \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \text{Tr}[H(\Phi^\dagger + \Phi)] + \text{Tr} \left[\left(\frac{m_1^2}{2} \left(\frac{G}{G_0} \right)^2 + \Delta \right) (L^{\mu 2} + R^{\mu 2}) \right] \\ & - \frac{1}{4} \text{Tr} [L^{\mu \nu 2} + R^{\mu \nu 2}] + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[L_\mu L^\mu + R_\mu R^\mu] + h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] \\ & + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu] + \dots, \end{aligned} \quad (1)$$

where $D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu)$. The nonstrange $\sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ and the strange $\sigma_S \equiv \bar{s}s$ bare quark-antiquark mesons are contained in the (pseudo)scalar multiplet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}. \quad (2)$$

The matrices L_μ and R_μ describe (axial)vector fields, see Ref. [4] for details. The scalar glueball $G \equiv gg$ is described by the dilaton Lagrangian \mathcal{L}_{dil} , in which a logarithmic term with the energy

scale Λ mimics the trace anomaly of the pure Yang-Mills sector of QCD [6, 7]:

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right). \quad (3)$$

The three scalar fields σ_N , σ_S , and G condense, leading to the following shifts: $\sigma_N \rightarrow \sigma_N + \phi_N$, $\sigma_S \rightarrow \sigma_S + \phi_S$, and $G \rightarrow G + G_0$. As a consequence, bilinear mixing terms $\sim \sigma_N \sigma_S$, $G \sigma_N$ and $G \sigma_S$ arise. The corresponding potential reads:

$$V(\sigma_N, G, \sigma_S) = \frac{1}{2}(\sigma_N, G, \sigma_S) \begin{pmatrix} m_{\sigma_N}^2 & z_{G\sigma_N} & z_{\sigma_S\sigma_N} \\ z_{G\sigma_N} & M_G^2 & z_{G\sigma_S} \\ z_{\sigma_S\sigma_N} & z_{G\sigma_S} & m_{\sigma_S}^2 \end{pmatrix} \begin{pmatrix} \sigma_N \\ G \\ \sigma_S \end{pmatrix} \quad (4)$$

where $z_{G\sigma_N} = -2m_0^2\phi_N/G_0$, $z_{G\sigma_S} = -2m_0^2\phi_S/G_0$ and $z_{\sigma_N\sigma_S} = 2\lambda_1\phi_N\phi_S$. The physical states are obtained upon diagonalization:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = B \begin{pmatrix} \sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \\ G \equiv gg \\ \sigma_S \equiv \bar{s}s \end{pmatrix}. \quad (5)$$

The aim is to determine the mixing matrix B .

3. Results

We use the parameters determined in Ref. [4]. These parameters allow for a correct description of a variety of vacuum properties of mesons up to 1.5 GeV. However, four parameters could not be uniquely determined in Ref. [4]: these are m_G , Λ , λ_1 and h_1 . In Ref. [5] three of them (m_G , Λ and λ_1) were determined in order to obtain the measured experimental masses of the resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$: $M_{f_0(1370)} = (1200-1500)$ MeV, $M_{f_0(1500)} = (1505 \pm 6)$ MeV and $M_{f_0(1710)} = (1720 \pm 6)$ MeV [1]. A solution in which $f_0(1500)$ is predominantly gluonic could not be found. On the contrary, the masses could be well described for $m_G = 1.580$ GeV, $\Lambda = 0.93$ GeV, and $\lambda_1 = 2.03$. The resulting mixing matrix reads:

$$B = \begin{pmatrix} 0.92 & -0.39 & 0.05 \\ -0.22 & -0.40 & 0.89 \\ -0.33 & -0.83 & -0.45 \end{pmatrix}. \quad (6)$$

We have now tested this scenario by evaluating the decay widths (the corresponding mathematical expressions will be presented in a forthcoming publication [13]). For $h_1 = 0$ (large- N_c limit), one finds the decay widths into kaons and pions: $\Gamma_{f_0(1710) \rightarrow \pi\pi} = 0.83$ GeV, $\Gamma_{f_0(1710) \rightarrow KK} = 0.42$ GeV, $\Gamma_{f_0(1500) \rightarrow \pi\pi} = 0.22$ GeV, $\Gamma_{f_0(1500) \rightarrow KK} = 1.14$ GeV, $\Gamma_{f_0(1370) \rightarrow \pi\pi} = 1.78$ GeV, $\Gamma_{f_0(1370) \rightarrow KK} = 0.89$ GeV. These results are clearly *too large* and cannot be cured by varying the only remaining free parameter h_1 (which should anyhow be small). Thus, the decay widths do not support this scenario as physical. Note, such a large decay width of the predominantly glueball state is in agreement with the study of Ref. [14].

The search for an acceptable solution is extremely difficult due to interference effects in the decay amplitudes. A direct fit to the known decay widths was so far not successful. This is why we have searched a solutions by trying to guess the right area of the parameter space. First, we use as an input the bare glueball mass $m_G = 1.7$ GeV in agreement with lattice QCD [15]. Then, due to the fact that $f_0(1710)$ was too broad in the previous solution, we have increased the value of the dimensionful parameter Λ : for $\Lambda \simeq 2$ GeV the resonance $f_0(1710)$ is sufficiently narrow. By further tuning $\lambda_1 \simeq -10$ and $h_1 \simeq -5$, we obtain the mixing matrix

$$B = \begin{pmatrix} 0.90 & -0.05 & 0.41 \\ -0.42 & -0.03 & 0.90 \\ -0.04 & -0.99 & -0.05 \end{pmatrix}. \quad (7)$$

The resonance $f_0(1710)$ is (almost) a pure glueball. The masses that are determined by these parameters are acceptable: $M_{f_0(1370)} = 1.06$ GeV, $M_{f_0(1500)} = 1.48$ MeV and $M_{f_0(1710)} = 1.70$ GeV. The resulting decay widths are: $\Gamma_{f_0(1710) \rightarrow \pi\pi} = 0.082$ GeV, $\Gamma_{f_0(1710) \rightarrow KK} = 0.064$ GeV, $\Gamma_{f_0(1500) \rightarrow \pi\pi} = 0.14$ GeV, $\Gamma_{f_0(1500) \rightarrow KK} = 0.13$ GeV, $\Gamma_{f_0(1370) \rightarrow \pi\pi} = 0.12$ GeV, $\Gamma_{f_0(1370) \rightarrow KK} = 0.07$ GeV.

Thus, while the decays of $f_0(1370)$ and $f_0(1710)$ are at least in qualitative agreement with the experiment, this is not the case for $f_0(1500)$: the decays are still too large. Note also that the very large value of Λ implies a large gluon condensate: lattice results [16] suggest that $\Lambda \lesssim 0.6$ GeV, see the discussion in Ref. [3]. Thus, at this level this solution is not yet conclusive, but can point to an interesting direction where to look for it: large bare glueball mass in agreement with lattice (1.7 GeV) and a large value of the gluon condensate.

4. Conclusions and outlook

In this work we have studied the masses and the decays of the resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ within the eLSM. Presently, the favoured glueball seems to be $f_0(1710)$, in agreement with Refs. [12, 17], but further studies are needed. Namely, decay widths which are -at least qualitatively- in agreement with data could only be found for a large (eventually too large!) value of the gluon condensate.

Another possibility is an improvement of the underlying effective model of Ref. [4], by studying the influence of a quadratic mass term in the (pseudo)scalar sector. This is a minimal change of Ref. [4], which however can have interesting phenomenological implications due to the fact that the strange current quark mass is not negligible. For value of the gluon condensate in agreement with lattice, a not too broad glueball can only be found if destructive interferences between the different amplitudes occur. This is why an improved numerical analysis, which allows to study in detail the whole parameter space, would be also helpful.

More in general, one can extend the study of glueballs with other quantum numbers. In Ref. [18] the pseudoscalar glueball was investigated within the eLSM. A similar program can be carried out for a tensor glueball with a mass of about 2.2 GeV, see e.g. Ref. [19] and refs. therein, as well as heavier glueballs, such as the vector and pseudovector glueball states, with lattice predicted masses of about 3.8 and 3 GeV, respectively.

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