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Towards the prediction of multiple necking during dynamic extension of round bar : linear stability approach versus finite element calculations

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Abstract. The fragmentation of structures subject to dynamic conditions is a matter of interest for civil industries as well as for Defence institutions. Dynamic expansions of structures, such as cylinders or rings, have been performed to obtain crucial information on fragment distributions. Many authors have proposed to capture by FEA the experimental distribution of fragment size by introducing in the FE model a perturbation. Stability and bifurcation analyses have also been proposed to describe the evolution of the perturbation growth rate. In the proposed contribution, the multiple necking of a round bar in dynamic tensile loading is analysed by the FE method. A perturbation on the initial flow stress is introduced in the numerical model to trigger instabilities. The onset time and the dominant mode of necking have been characterized precisely and showed power law evolutions, with the loading velocities and moderately with the amplitudes and the cell sizes of the perturbations. In the second part of the paper, the development of linear stability analysis and the use of salient criteria in terms of the growth rate of perturbations enabled comparisons with the numerical results. A good correlation in terms of onset time of instabilities and of number of necks is shown.

1. Introduction

Dynamic fragmentation of expanding structures has been widely studied experimentally since Mott [1], by using explosive loadings [2, 3] or electromagnetic devices [4–6]. Three steps of the fragmentation process can been observed: the stable expansion, the development of instabilities (shear bands or necks), some of them lead to fracture, the others being arrested. Numerical simulations with salient perturbations have been used to reproduce those fragmentation experiments, see [7–9]. All those studies were mostly interested in the description of the distributions of fragments. Stability and bifurcation analyses were also developed in order to describe the evolution of perturbation instantaneous growth rate, so that dominant instability modes could be determined. Understanding the onset of instabilities is essential for the good description of the later fragmentation. The goal of the present work is to compare analytical theory and simulations in terms of onset time and number of necks. A numerical study of multiple necking during dynamic extension of copper round bar using perturbations on the yield stress has been performed. A criterion and a numerical methodology are defined to capture the onset time and the number of necks. Their evolution with the loading velocity and the

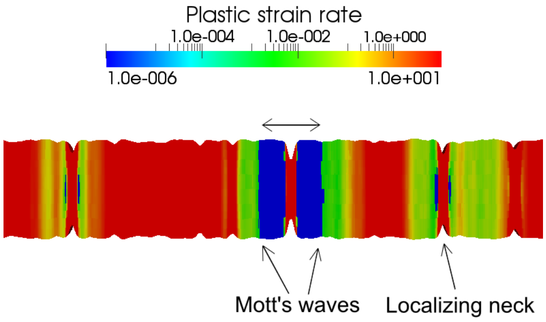


Figure 1. Localized necking and Mott's waves in the bar subject to traction.

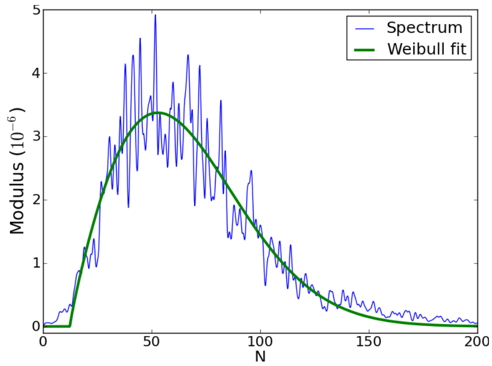


Figure 2. Fourier Transform of the cross section profile and associated Weibull law (v_0 = 2100 m/s, $A = 5\%$ and $S = S1$).

perturbation characteristics have been evaluated. In a second step, a linear stability analysis is developed to describe the stability of the round bar extension. Finally, an analytical/numerical comparison is achieved.

2. Numerical modeling

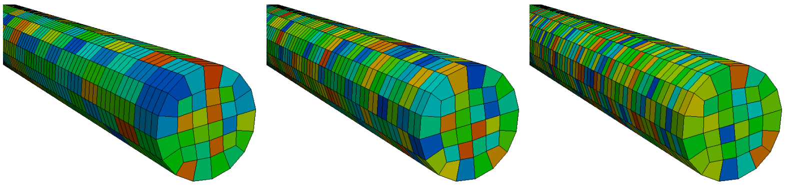
A 128.805 mm long and 1 mm^2 circular section copper bar is considered in the OURANOS 3D Lagrangian code developed by the *Commissariat à l'Énergie Atomique* (CEA). This code demonstrates that without any perturbation, the bar can be over-elongated with no appearance of instabilities. So a perturbation, used to trigger necking, is then applied on the yield stress $\sigma_y(\epsilon_p, \dot{\epsilon}_p, T)$, taken as in [10], where ϵ_p, $\dot{\epsilon}_p$ and T are respectively the plastic strain, the plastic strain rate and the temperature. This perturbation is defined by a cell size S and an amplitude of perturbation A (see figure 3). The bar is finally subjected to constant loading velocities v_0 varying from 150 to 2100 m/s.

2.1. Definition of the instability characteristics

Two features have been defined to describe the evolution of the instabilities. The first one is the onset time of localized necking t_{neck} and is associated with the time when an element first experiences an elastic release due to a localizing neck. Figure 1 shows the unloading in the vicinity of localized necks. The second one defines the number of necks N_{neck} given by the dominant frequency at time t_{neck}. It has been evaluated by fitting the Fourier transform of the section profile along the bar with a Weibull law, and by associating the abscissa of its maximum with N_{neck} as represented on figure 2 for example. Besides the fact that the Weibull law is commonly used in fragmentation models (see [1, 11], it shows advantages in the description of internecking mass distributions (see the appendix).

2.2. Parametric studies

Thanks to these two instability characteristics, parametric studies have been achieved. First of all, as in [12] where mesh convergence was demonstrated for "pre-fragments" mass distributions, mesh convergence has been observed in terms of t_{neck} and N_{neck}. The second study considers different random seeds for each configuration and shows that the values of t_{neck} and N_{neck} are then slightly modified. However based on the central limit theorem, the use of five different seeds enables the evaluation of mean values with satisfactory standard deviations. Both previous studies were done with constant perturbation parameters A and S. Finally, the effect of A and S on the development of instabilities is analysed. Four amplitudes A (1%, 5%, 10% and 20%)



(a) S1 (b) S1/2 (c) S1/4

Figure 3. Studied cell sizes: S1 is the initial cell size, S1/2 is the half length cell size and S1/4 is the quarter length cell size.

Table 1. t_{neck} and N_{neck} fitted coefficients.

t_{neck}				N_{neck}			
K	a	b	c	K'	a'	b'	c'
460±9	-0.181±0.004	-0.155±0.003	-0.079±0.006	3.39±0.15	0.388±0.006	-0.045±0.0042	-0.042±0.009

and three sizes S (see figure 3) are tested for different loading velocities. The mean values of t_{neck} and N_{neck} obtained for those configurations show powerlaw evolutions in the form: $t_{neck} = K\ v_0^a\ A^b\ S^c$ and $N_{neck} = K'\ v_0^{a'}\ A^{b'}\ S^{c'}$, where K and K' are constant parameters depending on the geometry and the material of the specimen. These laws demonstrate in particular that N_{neck} usually weakly depends on the perturbation (see table 1). On the contrary, t_{neck} is depending upon the perturbation features.

3. Comparisons with analytical results

The linear stability analysis developed in [3] for cylindrical bars has been employed to validate the trends observed numerically for t_{neck} and N_{neck}. Two types of criteria applied to this analysis have been tested. The first is the relative growth rate $\eta/\dot{\epsilon}$ (as in [13]), η being the perturbation growth rate and $\dot{\epsilon}$ the homogeneous strain rate. The dominant perturbation mode is selected when this parameter reaches a given critical value. An integral indicator $\int \eta dt$ is proposed secondly to ensure some history effect in the selection of the dominant perturbation mode. The comparisons between those criteria and some numerical results are presented on figure 4. A better agreement is obtained with the integral criterion type.

4. Conclusion and future direction

A new methodology is proposed to derive the onset time and the number of necks in a round bar subject to dynamic extension. In particular, the use of a Weibull law to evaluate the number of necks gives interesting results. From numerical studies, it is found a powerlaw dependency with respect to the loading velocity and to the parameters of the perturbation introduced in the model. These trends are in close agreement with analytical results obtained via a linear stability analysis and an integral criterion for a large range of loading velocities. Finally, an electromagnetic device, similar to the one presented in [6], is currently under development to evaluate experimentally the onset time and number of necks in ring expansion experiments. In the future, not only the dominant mode of perturbation, but the whole distribution of modes is expected to be analysed.

Acknowledgments

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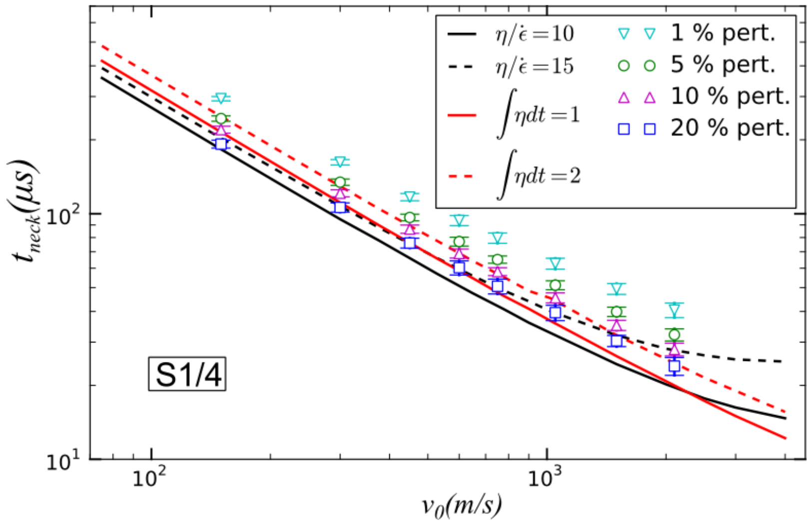
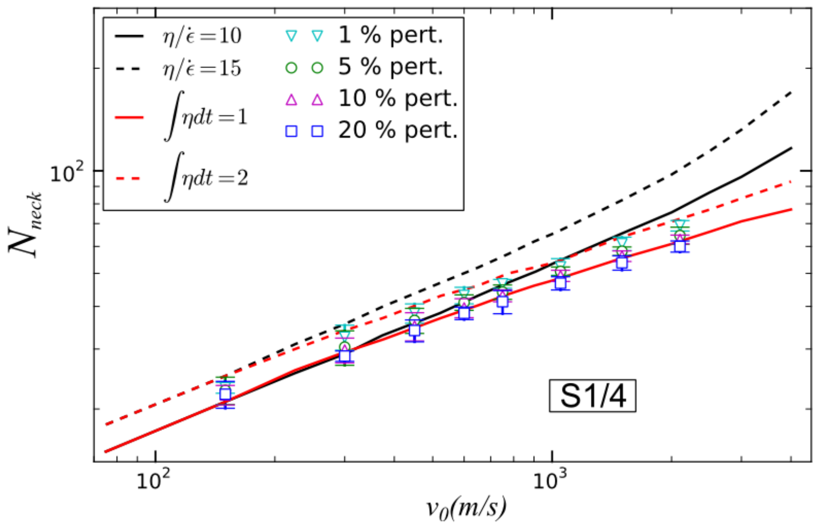


(a) Onset time of necking



(b) Number of necks

Figure 4. Analytical results using two values for each criterion compared to the numerical results with the cell size S1/4.

Appendix

The Weibull laws describing the number of necks for each loading velocity (see section 2.1) are assumed to be some harmonic probability density functions of perturbation modes. New functions for internecking length distributions are then inferred by change of variable, thanks to which statistical histograms representative of ten experiments can be generated, as observed in figure 5. The cumulative length ratio is also plotted in plain line in this figure. These histograms show very similar trends compared to the experimental observations in [6]: sharper distributions with loading velocities, a non zero minimum internecking length, and some lonely long sections for low velocity cases.

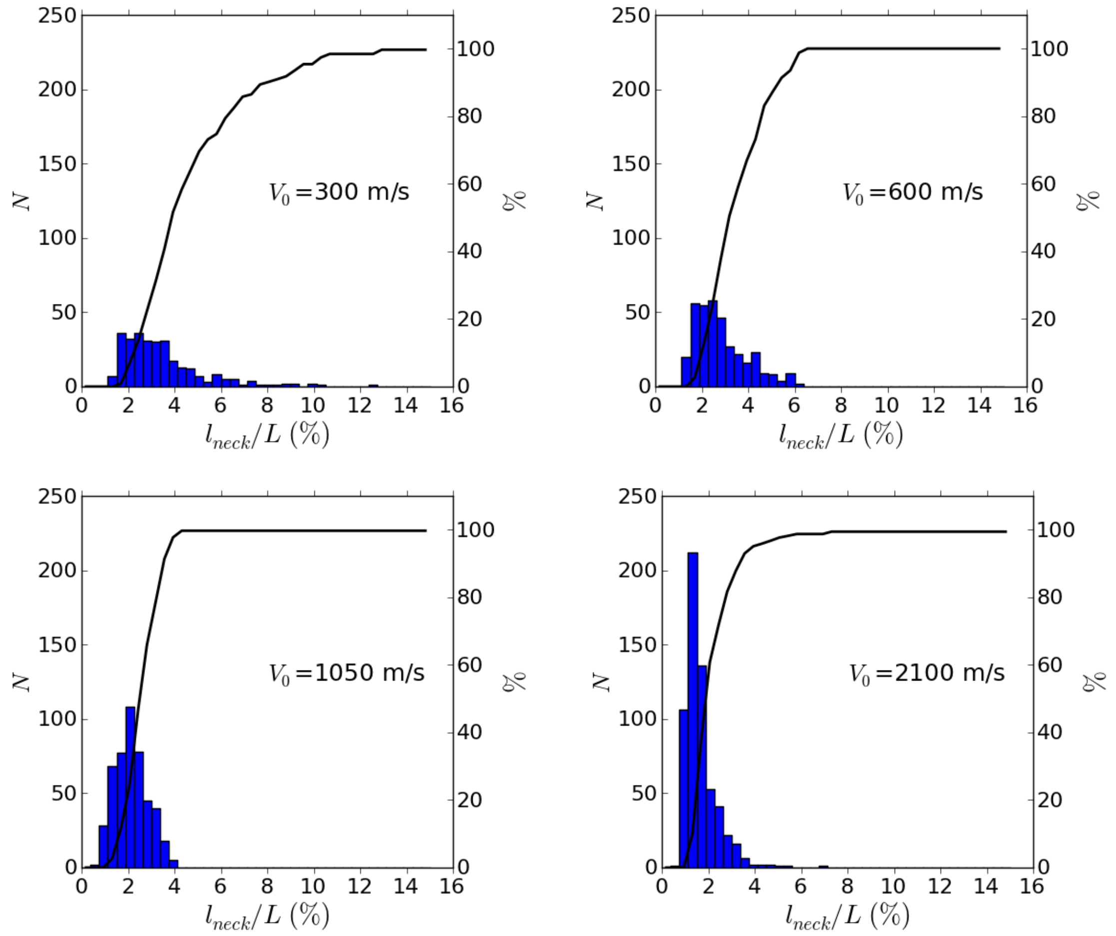


Figure 5. Distribution of internecking lengths for random seeds relative to ten experiments with $v_0 = 300$, 600, 1050 and 2100 m/s and $A = 5\%$ on a L =128.805 mm long bar.

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