

Determination of the effective mechanical properties of inclusionary materials using bulk elastic waves

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Abstract. For many years, the study of material with locally-resonant inclusions has known a great interest. Due to their attractive macroscopic acoustic properties, which do not exist in natural materials, many applications governed by the contrast between the constituent properties can be achieved: enhanced absorption, flat perfect lens, wave trapping... Such structures containing inclusions are usually modelled as effective homogeneous materials with complex-valued and frequency-dependent effective wave number and mass density. In this context, we derived a method for the simultaneous determination of both parameters using an immersed sample. First, its reliability is demonstrated using noisy synthesised temporal-waveform. Finally, the characterisation method is applied to experimental waveforms measured on a standard glass panel.

1 Introduction

Materials with inclusions are studied for many years in acoustics [1-5]. This kind of heterogeneous materials exhibits interesting acoustical macroscopic properties, such as strong absorption, band gap or negative index, for which the dynamical mass density and stiffness can take negative values.

An inclusionary medium can be considered as a homogeneous medium with the same acoustic behaviour than the heterogeneous one. For a viscoelastic homogeneous medium, it is known that its stiffness is complex valued and frequency dependent, in order to model the visco-thermal losses. Now, if we consider an inclusionary medium, its effective stiffness is also complex valued and frequency dependent in order to model the acoustic damping due to multiple scattering, in addition with eventual viscosity losses. Moreover, its mass density is also complex valued and frequency dependent, in order to take into account the inertial effects in between the different phases occurring when the wave propagates in the medium.

Many ultrasonic methods are proposed in the literature for characterising the phase celerity and attenuation of waves propagating in a homogeneous material. Here, in order to characterise materials with inclusions, we need a method that allows the identification of both the wave number and the dynamical mass density at the same time. Unfortunately, only few existing techniques retrieve the dynamical mass density. Adamowski [6] developed a characterisation method for the mass density of liquids, using reflected signals at different interfaces. Meanwhile, He [7] recovered the phase celerity and attenuation of a material using reflected and transmitted waveforms by a plate. These two techniques cannot be used to characterise materials with inclusions, because the mass density is assumed to be real valued and constant with the angular frequency ω . In addition, since we do not want to be subjected to any *a priori* model describing the variation of the material properties with the frequency, the characterisation procedure is performed “frequency by frequency”.



An identification procedure using reflection and transmission of an immersed plate is developed here. This method for the retrieval of the wave number and complex-valued dynamical mass density is based on that proposed by Fokin [8]. It is suitable for the experimental setup presented in section 2. Here, the technique is applied on temporal waveforms synthesised from the acoustic response of a viscoelastic homogeneous immersed plate. Its reliability on a simple case will ensure a part of reliability on other cases. More, the sensitivity of the technique regarding the measurement uncertainties is investigated.

2 Identification method

2.1 Experimental setup

The identification method presented here is based on the measurement of the reflection and transmission coefficients of a plate. The experimental setup contains a water tank within which a plate shape material is immersed. Two transducers are placed on both sides of the sample, and thus are able to catch the signals reflected and transmitted by the material (Fig. 1). The transducers send a temporal signal with a finite temporal length and a large frequency-band, centred at the central frequency of the transducers. Six signals can be measured by the setup, as it is shown on figure 1.

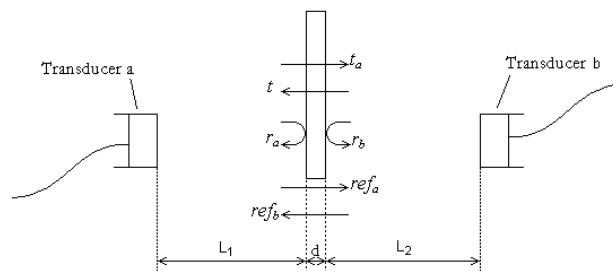


Figure 1: scheme of the experimental setup, and all the measurable signals.

Depending on the plate material, the overall shape of the reflected and transmitted signals can be slightly different (Fig. 2). Indeed, two general cases can be observed. First (Fig. 2a), the echoes coming from the interfaces can be separated if the thickness is large enough, or if the absorption is small enough. In fact, more the plate thick is, more the shift between the echoes large is, so it will be easier to separate the different echoes. It is interesting to note that, whatever the attenuation level is, the successive echoes have decreasing amplitude, until the total absorption of the signal. Second (Fig. 2b and 2c), the echoes cannot be separated, if the attenuation is large enough (Fig. 2b), or if the thickness is small enough (Fig. 2c). For thin plates, all the echoes are contained in the main signal. Their number is *a priori* unknown, since it is large for weak absorption or equal to zero for weak absorption. In the case of strong absorption, if no acoustic signal is transmitted by the plate, the identification method is not applicable, since the knowledge of both the reflection and transmission spectra at the same time is required. In such case, the measurement of the only reflection spectrum does not allow the two-parameter recovery.

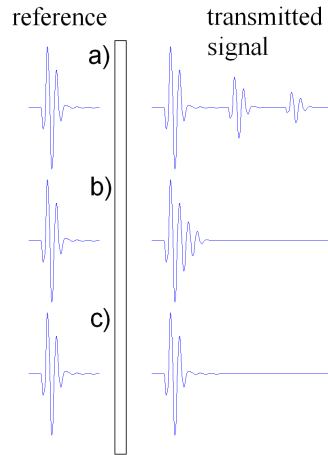


Figure 2: different configurations depending on the material properties / thickness values: a) the echoes are separable (thick plate); b) all echoes are contained in the main signal (thin plate); c) only the direct signal is present, because of the strong absorption in the plate (no back and forth reflections within the plate)

In this paper, the subscript 0 refers to the external medium (water), the subscript 1 refers to the plate material, and c is the longitudinal wave celerity. The properties we will identify are the wave number $k(\omega) = \omega/c(\omega) + i\alpha(\omega)$, with α standing for the attenuation, and the complex-valued dynamical mass density $\rho(\omega) = \rho'(\omega) + i\rho''(\omega)$, with $(\rho', \rho'') \in \mathbb{R}^2$.

Let t_a, t_b , be the signals transmitted through the plate respectively from transducer a to transducer b, and from transducer b to transducer a. The signals r_a and r_b are, in the same way, the signals reflected by the plate. Finally, ref_a and ref_b are the reference signals, only propagating through the water without the sample.

2.2 Characterisation method

2.2.1 Identification using total reflection and total transmission

Let $T_a, T_b, R_a, R_b, Ref_a$ et Ref_b be the spectra coming from the temporal-waveforms recorded by the experimental setup. These spectra can be expressed using the reflection and transmission coefficients of the plate, respectively $R(\omega)$ and $T(\omega)$, as well as the transfer functions of the transducers ($X_a(\omega), X_b(\omega)$ in emission, and $Y_a(\omega), Y_b(\omega)$ in reception):

$$\begin{aligned}
 T_a(\omega) &= X_a(\omega) e^{ik_0 L_1} T(\omega) e^{ik_0 L_2} Y_b(\omega), \\
 T_b(\omega) &= X_b(\omega) e^{ik_0 L_2} T(\omega) e^{ik_0 L_1} Y_a(\omega), \\
 R_a(\omega) &= X_a(\omega) e^{ik_0 L_1} R(\omega) e^{ik_0 L_1} Y_a(\omega), \\
 R_b(\omega) &= X_b(\omega) e^{ik_0 L_2} R(\omega) e^{ik_0 L_2} Y_b(\omega), \\
 Ref_a(\omega) &= X_a(\omega) e^{ik_0 (L_1 + d + L_2)} Y_b(\omega), \\
 Ref_b(\omega) &= X_b(\omega) e^{ik_0 (L_2 + d + L_1)} Y_a(\omega),
 \end{aligned} \tag{1}$$

with $k_0 = \omega/c_0$ and where the term $e^{-i\omega t}$ has been omitted. In the system (1), we suppose that the R and T functions are the same in both propagation paths. This assumption allows us to extract the coefficients R and T :

$$\begin{aligned}\frac{T_a}{Ref_a} &= \frac{T_b}{Ref_b} = T e^{-ik_0 d}, \\ \frac{R_a}{Ref_a} \frac{R_b}{Ref_b} &= R^2 e^{-2ik_0 d}.\end{aligned}\tag{2}$$

These combinations of spectra eliminate the unknown transfer functions of the transducers, as well as the distances between the plate and the transducers. More, this measurement procedure of signals avoids the knowledge of a reference signal usually needed for the measurement in reflection.

The analytical expressions for R and T are functions of the plate properties [8]:

$$\begin{aligned}R &= \frac{\left(\frac{1}{Z} - Z\right) \tan(k_1 d) i}{2 - \left(\frac{1}{Z} + Z\right) \tan(k_1 d) i}, \\ T &= \frac{2}{\cos(k_1 d) \left[2 - \left(\frac{1}{Z} + Z\right) \tan(k_1 d) i\right]},\end{aligned}\tag{3}$$

$$\text{with } Z = \frac{Z_1}{Z_0} = \frac{\rho_1 k_0}{\rho_0 k_1}.$$

Using the experimental measurement of these coefficients, an identification procedure of the material parameters is developed here. It consists, for a given frequency, to invert analytically the system (3), and then to extract analytical expressions for the wave number k_1 and the mass density ρ_1 :

$$\cos(k_1 d) = \frac{1 - R^2 + T^2}{2T} = A,$$

and then

$$k_1 = \frac{\pm \cos^{-1}(A) + 2\pi n}{d}, (n \in \mathbb{Z})\tag{4}$$

$$\left(\frac{1}{Z} + Z\right) = \left(2 - \frac{2}{T \cos(k_1 d)}\right) i \tan(k_1 d) = B,$$

and then

$$Z = \frac{B \pm \sqrt{B^2 - 4}}{2}.\tag{5}$$

The wave number k_1 is defined with sign uncertainty as well as branch selection incertitude (with n). The sign of the wave number is set for its imaginary part to be positive. Indeed, a negative imaginary part corresponds to an amplification of the wave during its propagation through the material (non passive material). We observe that the acoustic impedance Z is also defined with a sign uncertainty (Eq. 5).

2.2.2 Identification using partial reflection and partial transmission

When the different echoes in a signal are separable, the sought parameters can be identified using the direct reflection R_1 , and the direct transmission T_1 , without any multiple reflections within the plate. These coefficients can be expressed, like R and T , using the properties of the material:

$$\begin{aligned} R_1 &= \frac{1-Z}{1+Z}, \\ T_1 &= \frac{4Z}{(1-Z)^2} e^{ik_1 d}. \end{aligned} \quad (6)$$

As previously, we can analytically revert this system:

$$e^{ik_1 d} = \frac{T_1}{1-R_1^2} = C,$$

and then

$$k_1 = \frac{-i \ln(C) + 2\pi n}{d}, (n \in \mathbb{Z}) \quad (7)$$

$$\frac{Z}{(1+Z)^2} = \frac{T_1}{4} e^{-ik_1 d} = D,$$

and then

$$Z = \frac{1-2D \pm \sqrt{1-4D}}{2D}. \quad (8)$$

3 Stability of the characterisation

In order to quantify the characterisation stability, we simulate the experimental setup with synthetic data. The material is chosen to be linear viscoelastic ($c_1(\omega) \in \mathbb{C}, \rho \in \mathbb{R}^+$), and to respect causal relations between phase celerity and attenuation [9, 10] (Fig. 3). This causal mechanical system is based on relaxation functions, which consider appropriate probability densities of relaxation times. These judicious choices lead one to explicit relations between the mechanical characteristics of homogeneous linear viscoelastic media and observable acoustic properties (wave number).

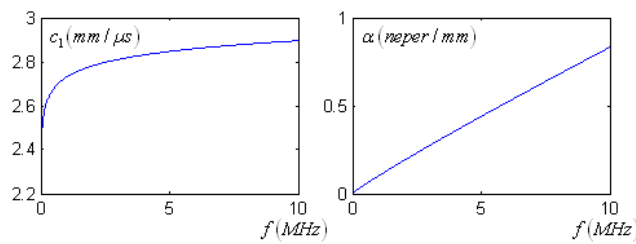


Figure 3: phase celerity and attenuation of the material respecting causality. ($\rho_1 = 1.56 \text{ g/cm}^3$).

The reflected and transmitted signals are simulated using the coefficients R and T . The reference signal is an experimental signal, measured with two transducers with a central frequency of 150kHz (Fig. 4).

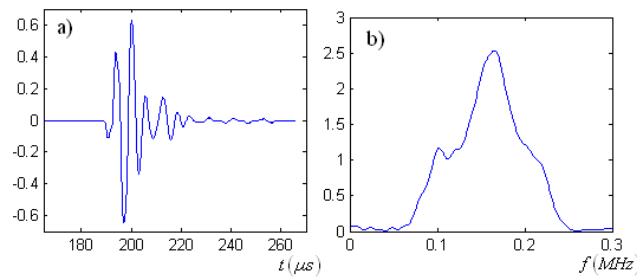


Figure 4: a) reference signal (au); b) modulus of the spectra of the reference signal (au)

The characterisation procedure applied to synthetic signals gives exactly the original data used for the simulation. The goal is then to observe the reliability of the identification for noisy signals. The white noise is expressed in dB with a signal to noise ratio (SNR), defined as the energy ratio between the original signal and the noise. We choose here a 24dB SNR. The signals represented in figures 5 and 6 have been simulated for two thicknesses (10mm and 1mm). The results of the identification are shown in figures 7 and 8.

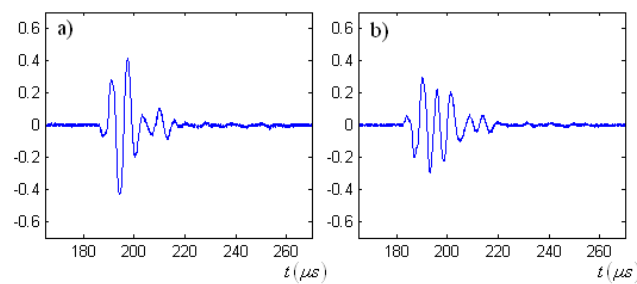


Figure 5: temporal waveform simulated in transmission a) and in reflection b) for a 10mm-thick plate, with a 24dB SNR.

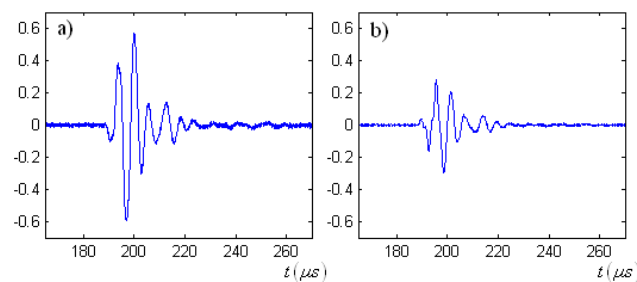


Figure 6: temporal waveform simulated in transmission a) and in reflection b) for a 1mm-thick plate, with a 24dB SNR.

We can observe that, with the same noise level, the identification of the wave number is less accurate for a thin plate. Concerning the mass density, the identification has the same sensitivity as that of the wave number for the 10mm-thick plate, and its sensitivity is of the same level as that of the 1mm-thick plate. This result is directly linked to the ratio of the wave length regarding the plate thickness. Indeed, if the wave length is much larger than the plate thickness, the propagation inside the plate is very short, so the acoustic response of the thin plate is less dependent upon the wave number than for the thick plate. At the central frequency of the reference signal (160kHz), the wave length is equal to 16mm, which is much larger than the thickness of the thinnest plate (1mm).

It seems that a thickness comparable to the wave length of the signal is required to have an accurate retrieval of the dynamic parameters.

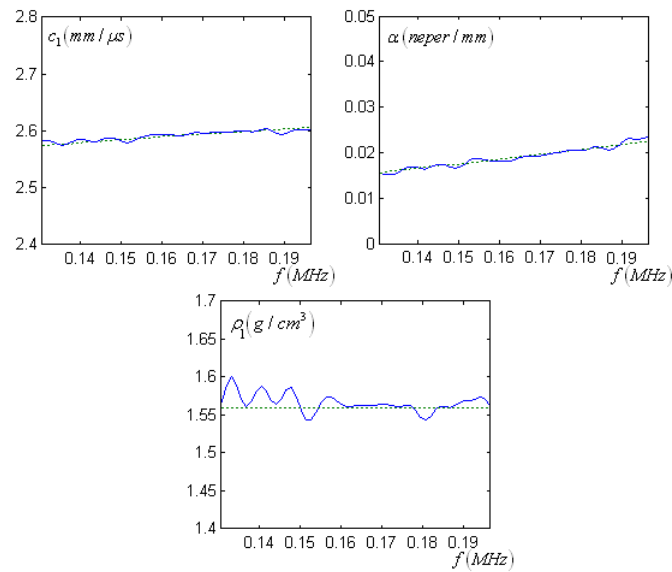


Figure 7: phase celerity, attenuation and mass density (retrieved properties in blue, original data in green), recovered using signals simulated for a 10mm-thick plate with a 24dB SNR.

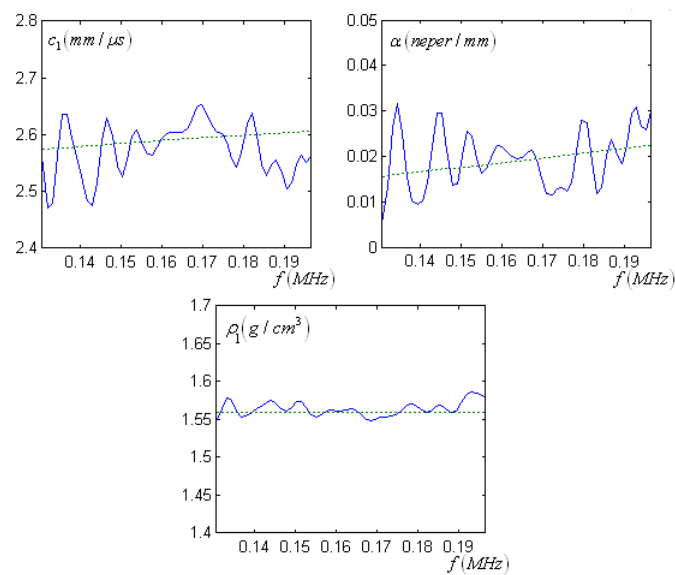


Figure 8: phase celerity, attenuation and mass density (retrieved properties in blue, original data in green), recovered using signals simulated for a 1mm-thick plate with a 24dB SNR.

4 Experimental results

The reflection and transmission coefficients have been measured with a 7.97mm-thick glass plate (with a pair of 5MHz central-frequency transducers).

Figure 9 shows the results of the characterisation method applied on the experimental waveforms. Due to the high central frequency of the transducers, the direct response of the plate (without multiple reflections inside the plate) can be extracted from the signals. While the retrieved phase celerity corresponds to values found in literature, the attenuation is higher than expected. This overestimating can be explained by the fact that the experimental setup is not fully modelled. Indeed, an erroneous alignment of the transducers reduces the amplitude of the received signal, and so increases the recovered attenuation. More, the diffraction of the acoustic field, due to the finite size of the transducers also reduces the received signal amplitude.

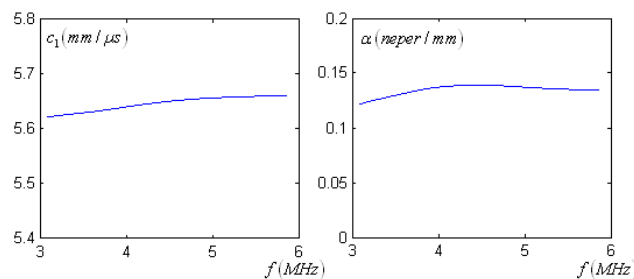


Figure 9: phase celerity and attenuation recovered from the measured reflection and transmission of a 7.97mm thick glass plate.

The retrieved mass density is represented in figure 10. As a function of the sign we choose for the acoustic impedance (Eq. 5), two values can be proposed for the mass density. The real part of the first value of the identified mass density (in blue in figure 10) corresponds to the real density of glass (2.51g/cm^3), but its imaginary part is higher than expected (since the attenuation is also overestimated). The second solution for the mass density is very low, for its real part as well as for its imaginary part and does not correspond with the expected one.

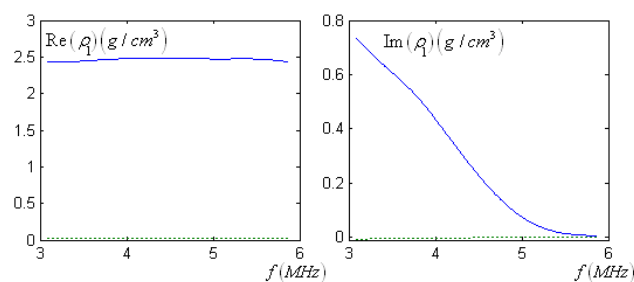


Figure 10: two values for the mass density (real and imaginary parts) recovered from the measured reflection and transmission of a 7.97mm-thick glass plate (first value in solid blue line, and second value in dotted green line).

5 Conclusions

An ultrasonic characterisation technique for *longitudinal* wave properties of inclusionary materials has been developed. The method retrieves both the effective wave number and the effective mass density from the

reflection and transmission of an immersed plate. Since no model for the evolution of the parameters is taken into account, explicit formulas for these parameters have been derived depending on the frequency.

We then studied numerically the sensitivity study of the method to the noise in waveforms. The characterisation procedure is found to be more sensitive to the noise when the plate is thin (especially for the wave number)

Finally, the experimental characterisation of a glass plate has shown that the current procedure naturally overestimates the attenuation.

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References

- [1] Kuster G T and Toksöz M N 1974, Velocity and attenuation of seismic waves in two-phase media: Part I. Theoretical formulations, *Geophysics* **39** (5), p. 587-606
- [2] Kinra V K, Petraitis M S and Datta S K 1979, Ultrasonic wave propagation in a random particulate composite, *Int. J. Sol. Struct.* **16**, p. 301-312
- [3] Waterman P C and Truell R 1961, Multiple scattering of waves, *J. Math. Phys.* **2** (4), p. 512
- [4] Aristégui C and Angel Y C 2007, Effective mass density and stiffness derived from P-wave multiple scattering, *Wave Motion* **44**, p. 153-164
- [5] Audoly C 1994, Acoustic analysis of panels made with viscoelastic materials containing resonant cavities, *Acta acustica* **2**, p. 393-402
- [6] Adamowski J C, Buiocchi F, Simon C and Silva E C N 1995, Ultrasonic measurement of density of liquids, *J. Acoust. Soc. Am.* **97** (1), p. 354-361
- [7] He P 2000, Measurement of acoustic dispersion using both transmitted and reflected pulses, *J. Acoust. Soc. Am.* **107** (2), p. 801-807
- [8] Fokin V, Ambati M, Sun C and Zhang X 2007, Method for retrieving effective properties of locally resonant acoustic metamaterials, *Physical review B* **76**, 144302
- [9] Wintle H J 1999, Kramers-Kronig analysis of polymer acoustic data, *J. App. Phys.* **85** (1), p. 44-48
- [10] Wintle H J 2000, Power law absorption in polymers and other systems, *J. Acoust. Soc. Am.* **107** (3), p. 1770-1773