

# Experimental measurements of the coherent field resulting from the interaction of an ultrasonic shock wave with a multiple scattering medium

Nicolas Viard, Bruno Gianmarinaro, Arnaud Derode, Christophe Barrière

Institut Langevin, 1 rue Jussieu, 75005 Paris, France

E-mail: nicolas.viard@espci.fr

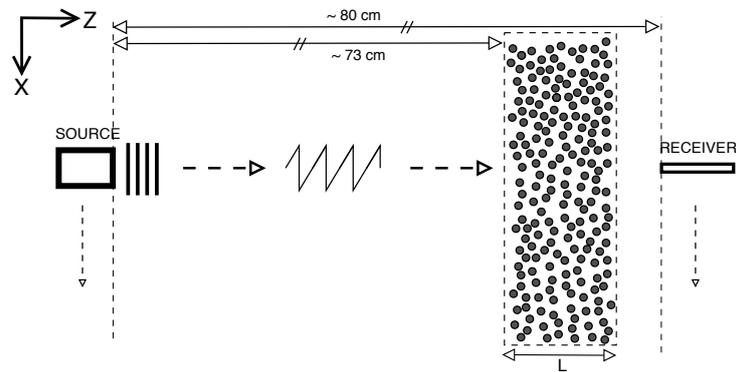
**Abstract.** Whereas multiple scattering and shock wave formation are known to be antagonistic phenomena, this work concentrates on the interaction of an ultrasonic shock wave with a random multiple scattering medium. The shock wave is generated by long distance propagation of a short pulse (4 periods at a 3.5 MHz central frequency) in water before it encounters the scattering medium (a slab-shaped random set of parallel metallic rods). Transmitted waves are recorded over hundreds of positions along the lateral dimension of the slab to estimate the ensemble-averaged transmitted field  $\langle\phi(t)\rangle$ , also known as the coherent wave. Experiments are repeated for different thicknesses  $L$  of the slab and different emission amplitudes. The elastic mean free path  $l_e$  (i.e the typical distance for the decreasing of the coherent intensity  $|\langle\phi(t)\rangle|^2$  due to scattering) is determined as well as the harmonic rate of the averaged transmitted wave. Experimental results are discussed and compared to the linear case.

## 1. Introduction

Acoustic wave propagation is intrinsically nonlinear and classical acoustic nonlinearities act as a cumulative effect [1]. For a plane wave propagating in a homogeneous non dissipative fluid, one usually defines the shock formation distance  $L_s = (k\beta M)^{-1}$  as the distance the wave has to travel before a discontinuity appears in its pressure waveform. It is inversely proportional to the wavenumber  $k = 2\pi/\lambda$ , to the fundamental nonlinear parameter  $\beta$ , and to the Mach number  $M = v_0/c_0$ , where  $v_0$  is the particle velocity and  $c_0$  is the sound speed. Linear approximation is usually made if the travelled distance is much shorter than  $L_s$ . Otherwise, if the travelled distance is approximately equal or superior to  $L_s$ , the propagation regime is nonlinear. Multiple scattering of wave is a general phenomenon. It can concerns electrons, optical waves, acoustic waves as well as seismic waves for example. For the acoustic waves, it comes out that multiple scattering could be beneficial on as well as antagonistic towards the classical acoustic nonlinearities. Indeed, multiple scattering lengthens travelled paths in the medium which is beneficial on the cumulative nonlinearities, while it also spreads the energy in the medium, yielding to lower values of  $v_0$  corresponding to higher values of  $L_s$ .

In this work, we investigate the interaction of an ultrasonic shock wave with a multiple scattering medium. Experimental measurements of the coherent wave field, i.e., the ensemble-averaged field, are compared for different emission amplitudes corresponding to linear and nonlinear regimes. The ultrasonic shock wave is formed in water before it reaches the





**Figure 1.** Experimental set-up. A short ultrasonic pulse around 3.5 MHz is emitted by a planar circular transducer ( $\varnothing$  38 mm). It propagates into water before it reaches the multiple scattering slab ( $10 < L < 40$  mm). Transmitted scattered waves are recorded by an hydrophone ( $\varnothing$  200  $\mu$ m).

heterogeneous medium. The latter consists of parallel steel rods randomly placed in water. Transmitted scattered waves are recorded and averaged to form the coherent wave. In the linear approach, the coherent wave has an effective wavenumber  $k_{eff}$  whose imaginary part accounts for losses due to scattering. As the wave propagates into the medium, the intensity of its coherent part decreases exponentially with a characteristic length  $l_e = 1/(2 \text{Im}(k_{eff}))$  called the elastic mean free path. Harmonic components of the coherent wave are compared for different sample thicknesses. The elastic mean free path  $l_e$  is estimated as a function of frequency. Because of nonlinear propagation, the increase of the emission amplitude extends the frequency band and therefore allows to estimate the elastic mean free path at higher frequencies. At a given frequency, we observe that, estimates at high emission amplitudes do not change compare to estimates at a low amplitude, which implies that multiple scattering in this kind of medium is purely a linear phenomenon.

## 2. Experimental procedure

Experimental setup is depicted in figure 1. A 4-period tone burst at a 3.5 MHz central frequency with a gaussian amplitude modulation is generated by a Tektronix arbitrary waveform generator (AFG 3101) and amplified by an AR amplifier (75A250A, 75W, 10kHz-250Mhz). It is emitted by a planar circular transducer ( $2r = 38$  mm, nominal central frequency  $f_0 = 3.5$  MHz,  $\Delta f/f_0 = 100\%$  at  $-6$ dB) in water, where the corresponding wavelength is 0.43 mm. The wave propagates over 73 cm in water ( $c_0 = 1500$  m/s,  $\beta = 3.5$ ) before it reaches the multiple scattering medium. It is a random set of 0.8 mm diameter steel rods with density 12 rods/cm<sup>2</sup>. A set of slabs with different thicknesses allows us to vary the total thickness of the multiple scattering medium from 10 to 40 mm. As a reminder, the density of steel  $\rho_{steel}$  is 7800 kg/m<sup>3</sup>, the longitudinal wavespeed  $c_L$  is 5.7 mm/ $\mu$ s and the transversal wavespeed  $c_T$  is 3 mm/ $\mu$ s. Using an HGL-200 hydrophone from ONDA, one records the scattered wave that are transmitted through the sample at a constant distance from the source, i.e., 80 cm. Such a distance ensures an optimized configuration in terms of spectral components and diffracted waveform. Indeed, at a shorter distance from the surface of the transducer, due to interferences between the direct and the edge waves from the transducer, the amplitude oscillates between zero and a maximal value along the axis  $z$  of the transducer. At a greater distance, because of diffraction, the ultrasonic beam diverges. It is commonly referred to the Rayleigh distance  $d_R = \pi r^2/\lambda$ , the distance from which the acoustic beam is no longer collimated. Here,  $d_R > 2$  m. At last, it

is important to compare the shock formation distance  $L_s$  to some characteristic length for the intrinsic attenuation, whose effect limits the creation of higher order harmonics. From [2], we obtain the characteristic length for the intrinsic attenuation in pure water,  $L_a = 3.4$  m at 3.5 MHz. The shock formation distance  $L_s$  is calculated for the lowest and the highest emission amplitudes, respectively  $v_0 = 0.002$  m/s and  $v_0 = 0.05$  m/s, at  $f = 3.5$  MHz.  $L_s$  is found equal to 14.6 m and 0.6 m. Therefore, the regime of propagation can be considered as highly nonlinear for the highest emission amplitude.

In order to estimate the coherent wave, the recording of the transmitted wave is repeated over a few hundreds positions along the medium by translating both the transducer and the hydrophone. Rigourosly the coherent wave -i.e., the ensemble averaged transmitted field- is given by the following equation :

$$\langle \phi(t) \rangle = \frac{1}{N} \sum_{i=1}^N \phi_i(t), \quad N \rightarrow \infty \quad (1)$$

where  $i$  denotes the number of positions -or realisations of the disorder. Nevertheless, in our experimental configuration, ensemble-averaged of the transmitted field cannot be achieved because of the finite size of our sample. Therefore, the number of positions  $N$  is determined, both by the spatial step  $p$  and the length of the scan over the lateral dimension  $x$ . Spatial step  $p$  was determined so that two adjacent positions are uncorrelated, whereas the length of the scan was chosen in order not to be perturbed by the edges of the sample. With a spatial step  $p = 0.5$  mm and a lateral scan over 20 cm, we obtain  $N = 401$ . When one scan is finished, the electrical power transmitted to the transducer is increased by changing the voltage of the generator, and a new scan begins.

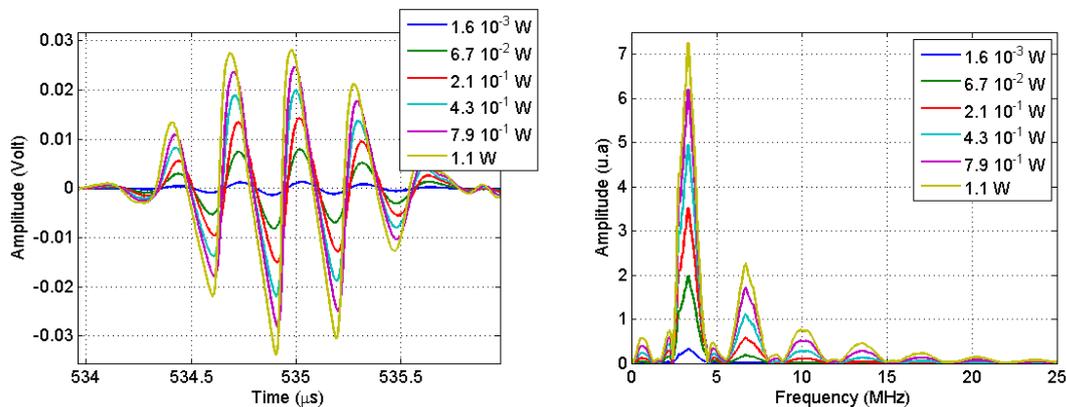
Transmitted signals in water (without the scattering slab) are shown in figure 2. In the temporal domain, the waveform stiffens when the emission power is increased which is typical of the classical nonlinearities [3, 4, 5]. At the lowest emission power (1.6 mW), the transmitted signal only shows frequency components around  $f_0$  which indicates that even if the travelled distance (i.e.,  $\sim 80$  cm) is large, the regime of propagation stays linear. In comparison, at the highest emission power (1.1 W), frequency components spread at least until the 7th harmonic (i.e., 24.5 MHz).

For the determination of the frequency-resolved elastic mean free path,  $l_e(f)$ , a procedure from [6] is followed. The transmission coefficient for the energy of the coherent wave,  $T_c$ , is calculated for different thicknesses  $L$  of the slab on narrow frequency bands (width  $\delta f = 0.2$  MHz). Then, the frequency-resolved elastic mean free path  $l_e(f)$  is deduced from a linear fit of  $\log(T_c(L, f))$  as a function of  $L$ , for each value of  $f$ .

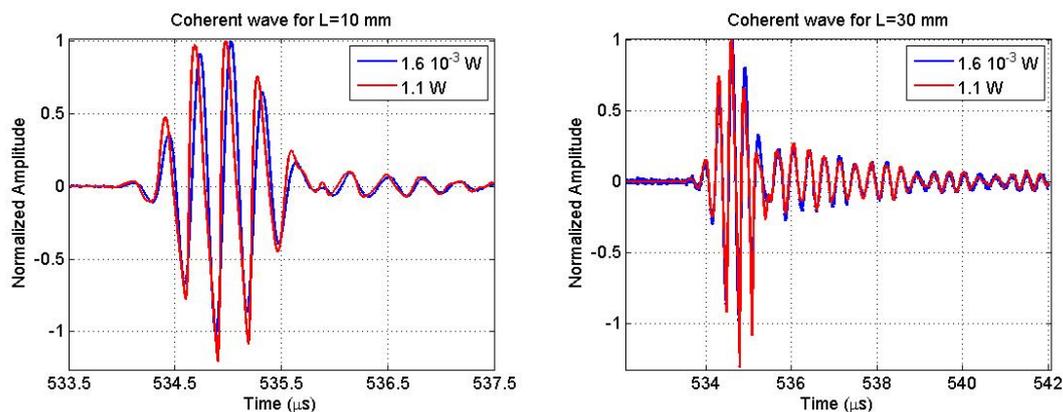
### 3. Results and discussion

Estimates of the coherent wave at the lowest and the highest emission amplitudes are presented in figure 3 for two different thicknesses  $L$ . For the finest sample ( $L = 10$  mm), the duration of the estimate of the coherent wave is really alike the duration of the measured signals in water, whereas for the sample with thickness  $L = 30$  mm, the duration of the estimate of the coherent wave is much greater than the duration of the measured signals in water. This is due to a resonance in the scattering cross section of the steel rod occurring at the 2.75 MHz frequency [6]. At a given thickness, the two estimates of the coherent wave corresponding to the lowest and to the highest emission powers look really alike, excepted that the curve corresponding to the highest power is stiffened compare to the one corresponding to the lowest emission power.

Figure 4 shows the harmonic amplitudes for the first -fundamental-, the second, the third and the fourth harmonics. For every thickness  $L$  of the sample, the harmonic amplitude is plotted as a function of the emission amplitude (here, the square root of the electrical power transmitted



**Figure 2.** Measured signals after propagation in water over 80 cm for different emission powers. At the lowest emission power (i.e., 1.6 mW), the spectrum has only components around 3.5 MHz, while, at higher power, other components successively appear around higher order harmonics: 7, 10.5, 14, 17.5 MHz...

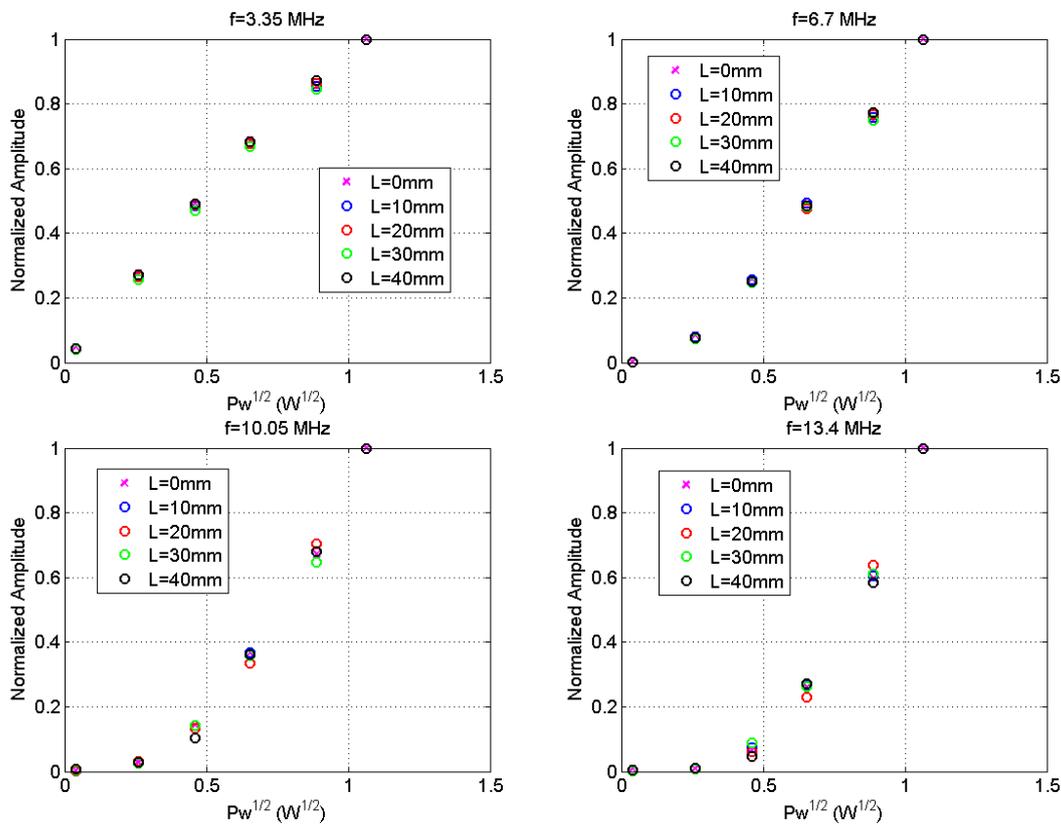


**Figure 3.** Estimates of the coherent wave ( $N = 401$  realisations) for the lowest and the highest emission powers. The thickness of the sample is equal to 10 mm (left) and 30 mm (right).

to the transducer). Each curve is normalized by its value at the highest emission amplitude so that it can be compared to the others. It is remarkable to see that, at a given frequency, whatever is the thickness of the multiple scattering slab, all the curves corresponding to the different thicknesses and to the transmission in water ( $L = 0 \text{ mm}$ ) overlap merely perfectly. This indicates that in our experimental configuration, the multiple scattering does not change the behavior of one given harmonic or does not act nonlinearly.

As shown in figure 2, the order of the highest harmonic that is measurable depends on the emission power. For example, at a 0.21 W emission power, the spectrum components of the transmitted wave in water are limited around the first and the second harmonics, while at a 0.79 W emission power, spectrum components extend to the third harmonic. Therefore, depending on the emission amplitude, the elastic mean free path  $l_e$  is determined in a different frequency range.

In figure 5, the estimates of the elastic mean free path versus frequency are represented for different emission powers. Over the frequency interval 2.4-4.4 MHz the elastic mean free path is estimated with four sets of data corresponding to the following emission powers : 1.6 mW, 0.21, 0.79 and 1.1 W. The four estimates are in very good agreement with each other. They



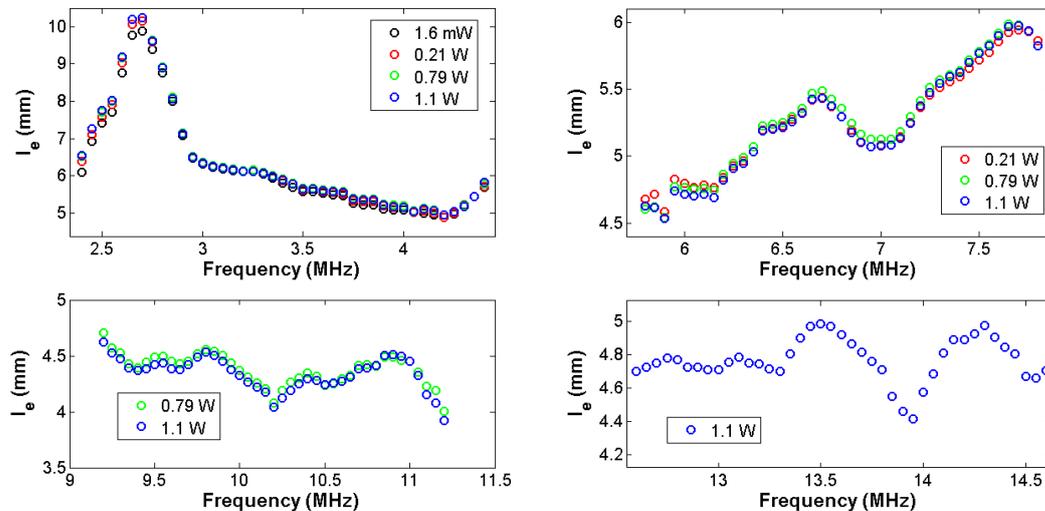
**Figure 4.** Harmonic amplitudes as a function of the square root of the electric power transmitted to the transducer, for different thicknesses  $L$ .

all reproduce the resonance at the frequency 2.8 MHz. Over the upper frequency interval (5.8–7.8 MHz) corresponding to the second harmonic, the estimate of the elastic mean free path is done with three sets of data corresponding to emission powers of 0.21, 0.79 and 1.1 W. Again, the agreement is good. Around the third harmonic (9.2–11.2 MHz), the elastic mean free path is determined with the two sets of data corresponding to emission powers of 0.79 and 1.1 W. The two estimates overlap. At last, with the set of data corresponding to the highest emission power (1.1 W), we could also estimate the elastic mean free path  $l_e$  over the frequency interval 12.6–14.6 MHz. The comparison with another set of data is not possible because this harmonic is not enough generated at lower emission amplitude.

#### 4. Conclusion

Experimental measurements of the coherent field resulting from the interaction of a shock wave with a multiple scattering medium is reported. It shows that the transmitted coherent wave remains a shock wave despite high-order multiple scattering. The stiffening of the transmitted coherent waveform generates, in the frequency domain, higher order harmonics. Surprisingly, their amplitudes behave the same as in water, whatever the thickness of the scattering slab. Besides, estimates of the elastic mean free path  $l_e$  do not change with the emission power. It seems that the only effect associated with the shock wave is that it allows to estimate the elastic mean free path  $l_e$  on a frequency band whose extension increases with the emission power. Therefore, we can conclude that in our experimental configuration, multiple scattering does not act nonlinearly.

Diffuse acoustic wave spectroscopy makes use of the high sensitiveness of long time scattered



**Figure 5.** Frequency resolved estimate of the elastic mean free path  $l_e$ , for different emission powers. At the lowest emission power (1.6 mW), estimate of the elastic mean free path is only possible around the fondamental -first harmonic- whereas at the highest emission power (1.1 W), estimate of the elastic mean free path is made over 4 frequency bands corresponding to harmonics one to four.

signals (i.e., coda waves) to monitor small changes in a scattering medium [7, 8, 9, 10]. Future work will focus on these signals for which the cumulative nonlinear effects should be stronger.

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### References

- [1] Press A (ed) 1997 *Nonlinear Acoustics* (Hamilton, M. F. and Blackstock, D. T.)
- [2] Markham J J, Beyer R T and Lindsay R B 1951 *Reviews of Modern Physics* **23** 353
- [3] Averkiou M A and Hamilton M F 1997 *The Journal of the Acoustical Society of America* **102** 2539–2548
- [4] Baker A C and Humphrey V F 1992 *The Journal of the Acoustical Society of America* **92** 1699–1705
- [5] Baker A C, Anastasiadis K and Humphrey V F 1988 *The Journal of the Acoustical Society of America* **84** 1483–1487
- [6] Derode A, Mamou V and Tourin A 2006 *Phys. Rev. E* **74** 036606
- [7] Leroy V and Derode A 2008 *Phys. Rev. E* **77** 036602
- [8] Cowan M L, Jones I P, Page J H and Weitz D A 2002 *Phys. Rev. E* **65** 066605
- [9] Page J H, Cowan M L, Sheng P and Weitz D A 2001 *Diffusing acoustic wave transport and spectroscopy* (Kluwer Academic)
- [10] Page J H, Cowan M L and Weitz D A 2000 *Physica B: Condensed Matter* **279** 130