

Fermionic condensation in ultracold atoms, nuclear matter and neutron stars

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Abstract. We investigate the Bose-Einstein condensation of fermionic pairs in three different superfluid systems: ultracold and dilute atomic gases, bulk neutron matter, and neutron stars. In the case of dilute gases made of fermionic atoms the average distance between atoms is much larger than the effective radius of the inter-atomic potential. Here the condensation of fermionic pairs is analyzed as a function of the s-wave scattering length, which can be tuned in experiments by using the technique of Feshbach resonances from a small and negative value (corresponding to the Bardeen-Cooper-Schrieffer (BCS) regime of Cooper Fermi pairs) to a small and positive value (corresponding to the regime of the Bose-Einstein condensate (BEC) of molecular dimers), crossing the unitarity regime where the scattering length diverges. In the case of bulk neutron matter the s-wave scattering length of neutron-neutron potential is negative but fixed, and the condensate fraction of neutron-neutron pairs is studied as a function of the total neutron density. Our results clearly show a BCS-quasiunitary-BCS crossover by increasing the neutron density. Finally, in the case of neutron stars, where again the neutron-neutron scattering length is negative and fixed, we determine the condensate fraction as a function of the distance from the center of the neutron star, finding that the maximum condensate fraction appears in the crust of the neutron star.

1. Introduction

In 1951 Penrose [1] and Onsager [2] introduced the idea of off-diagonal long-range order (ODLRO) of the one-body density matrix to determine the Bose-Einstein condensate fraction in a system of interacting bosons. In 1962 Yang [3] proved that for attractive fermions the Bose-Einstein condensation of fermionic pairs is instead related to the ODLRO of the two-body density matrix.

It is now established that at zero temperature the Bose-Einstein condensate fraction of bosonic liquid ^4He is below 10% [5], while for dilute and ultracold bosonic alkali-metal atoms it can reach 100% [4]. Some years ago, by using the ODLRO of the two-body density matrix, Salasnich, Manini and Parola [6] calculated the condensate fraction of fermionic pairs in the crossover from the Bardeen-Cooper-Schrieffer (BCS) state of Cooper Fermi pairs to the Bose-Einstein condensate (BEC) of molecular dimers at zero temperature (later in the same year there were other two papers [7, 8] on the same topic). In particular, it was found that the condensate fraction grows from zero to one in the BCS-BEC crossover [6, 7, 8]. These theoretical predictions are in quite good agreement with the data obtained in two experiments [9, 10] with Fermi vapors of ^6Li atoms. Recently, the condensate fraction of bulk neutron matter has been calculated by Wlazlowski and Magierski [11, 12] and also by Salasnich [13].



In this paper we review the zero-temperature mean-field theory we have used to extract the fermionic condensate fraction in ultracold atoms [6] and bulk neutron matter [13]. In addition we determine the condensate fraction of neutron-neutron pairs as a function of the distance from the center of a neutron star [14, 15]. In particular, we find the maximum condensate fraction at the distance $r/R \simeq 0.96$, where R is the star radius.

2. Bosonic and fermionic condensation

A quantum system of interacting identical bosons can be described by the bosonic field operator $\hat{\phi}(\mathbf{r})$, which satisfies the familiar commutation rules [16]

$$[\hat{\phi}(\mathbf{r}), \hat{\phi}^+(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}') , \quad [\hat{\phi}(\mathbf{r}), \hat{\phi}(\mathbf{r}')] = [\hat{\phi}^+(\mathbf{r}), \hat{\phi}^+(\mathbf{r}')] = 0 , \quad (1)$$

where $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ and $\delta(\mathbf{r})$ is the Dirac delta function. The bosonic one-body density matrix is given by

$$n(\mathbf{r}, \mathbf{r}') = \langle \hat{\phi}^+(\mathbf{r}) \hat{\phi}(\mathbf{r}') \rangle , \quad (2)$$

where the average $\langle \dots \rangle$ can be a thermal average or a zero-temperature average. Its diagonal part is the average local density of bosons, i.e. $n(\mathbf{r}) = n(\mathbf{r}, \mathbf{r}) = \langle \hat{\phi}^+(\mathbf{r}) \hat{\phi}(\mathbf{r}) \rangle$, while the average total number of bosons reads

$$N = \int \langle \hat{\phi}^+(\mathbf{r}) \hat{\phi}(\mathbf{r}) \rangle d^3\mathbf{r} . \quad (3)$$

As previously discussed, Penrose and Onsager [1, 2] used the ODLRO of the one-body density matrix of a uniform bosonic system to determine the condensate number N_0 of bosons. For a non-uniform bosonic system this condensate number N_0 is nothing else than the largest eigenvalue of the one-body density matrix [16]. As shown by Yukalov [17], in the thermodynamic limit this is equivalent to the spontaneous symmetry breaking of $U(1)$ gauge symmetry, which gives

$$N_0 = \int |\langle \hat{\phi}(\mathbf{r}) \rangle|^2 d^3\mathbf{r} \quad (4)$$

as the average number of condensed bosons.

A quantum system of interacting identical fermions with two spin components ($\sigma = \uparrow, \downarrow$) can be described by the fermionic field operator $\hat{\psi}_\sigma(\mathbf{r})$, which satisfies the familiar anti-commutation rules [16]

$$\{\hat{\psi}_\sigma(\mathbf{r}), \hat{\psi}_{\sigma'}^+(\mathbf{r}')\} = \delta(\mathbf{r} - \mathbf{r}') \delta_{\sigma, \sigma'} , \quad \{\hat{\psi}_\sigma(\mathbf{r}), \hat{\psi}_{\sigma'}(\mathbf{r}')\} = \{\hat{\psi}_\sigma^+(\mathbf{r}), \hat{\psi}_{\sigma'}^+(\mathbf{r}')\} = 0 , \quad (5)$$

where $\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$ and $\delta_{\sigma, \sigma'}$ is the Kronecher symbol. The fermionic one-body density matrix is given by

$$n_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}') = \langle \hat{\psi}_\sigma^+(\mathbf{r}) \hat{\psi}_{\sigma'}(\mathbf{r}') \rangle . \quad (6)$$

Its diagonal part is the average local density of fermions with spin σ , i.e. $n_\sigma(\mathbf{r}) = n_{\sigma, \sigma}(\mathbf{r}, \mathbf{r}) = \langle \hat{\psi}_\sigma^+(\mathbf{r}) \hat{\psi}_\sigma(\mathbf{r}) \rangle$, while the average total number of fermions reads

$$N = \sum_{\sigma=\uparrow, \downarrow} \int \langle \hat{\psi}_\sigma^+(\mathbf{r}) \hat{\psi}_\sigma(\mathbf{r}) \rangle d^3\mathbf{r} . \quad (7)$$

Yang [3] suggested that for a uniform strongly-interacting fermionic system the number N_0 of Bose-condensed fermions, that is twice the number of condensed fermionic pairs, is related by the ODLRO of the fermionic two-body density matrix, given by

$$n_{\sigma_1, \sigma_2, \sigma'_1, \sigma'_2}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}'_1, \mathbf{r}'_2) = \langle \hat{\psi}_{\sigma_1}^+(\mathbf{r}_1) \hat{\psi}_{\sigma_2}^+(\mathbf{r}_2) \hat{\psi}_{\sigma'_2}(\mathbf{r}'_2) \hat{\psi}_{\sigma'_1}(\mathbf{r}'_1) \rangle . \quad (8)$$

For a generic (non-uniform) strongly-interacting system of identical fermions this condensate number N_0 is nothing else than twice the largest eigenvalue of the fermionic two-body density matrix [16]. In the thermodynamic limit this is equivalent [18] to the spontaneous symmetry breaking of $SU(2)$ gauge symmetry, which gives

$$N_0 = 2 \sum_{\sigma, \sigma'=\uparrow, \downarrow} \int |\langle \hat{\psi}_\sigma(\mathbf{r}) \hat{\psi}_{\sigma'}(\mathbf{r}') \rangle|^2 d^3\mathbf{r} d^3\mathbf{r}' \quad (9)$$

as the average number of condensed fermions. It is important to stress that within the BCS theory of superconductors and fermionic superfluids the condensate number N_0 fermions can be much smaller than the average total number of fermions N . Moreover, in a uniform system N_0 is twice the average number of fermionic pairs in the (pseudo) Bose-Einstein condensate, i.e. the number of fermionic pairs which have their center of mass with zero linear momentum.

3. Ultracold and dilute atomic gases

The shifted Hamiltonian of the uniform two-spin-component Fermi superfluid made of ultracold atoms is given by

$$\hat{H}' = \int d^3\mathbf{r} \sum_{\sigma=\uparrow, \downarrow} \hat{\psi}_\sigma^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 - \mu \right) \hat{\psi}_\sigma(\mathbf{r}) + g \hat{\psi}_\uparrow^\dagger(\mathbf{r}) \hat{\psi}_\downarrow^\dagger(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) \hat{\psi}_\uparrow(\mathbf{r}), \quad (10)$$

where $\hat{\psi}_\sigma(\mathbf{r})$ is the field operator that annihilates a fermion of spin σ in the position \mathbf{r} , while $\hat{\psi}_\sigma^\dagger(\mathbf{r})$ creates a fermion of spin σ in \mathbf{r} . Here $g < 0$ is the strength of the attractive fermion-fermion interaction, which is approximated by a contact Fermi pseudo-potential [4] because for ultracold and dilute gases the average distance between atoms is much larger than the effective radius of the inter-atomic potential [4, 16]. The ground-state average of the number of fermions is given by Eq. (7). This total number N is fixed by the chemical potential μ which appears in Eq. (10).

Within the Bogoliubov approach the mean-field Hamiltonian derived from Eq. (10) can be diagonalized by using the Bogoliubov-Valatin representation of the field operator $\hat{\psi}_\sigma(\mathbf{r})$ in terms of the anti-commuting quasi-particle Bogoliubov operators $\hat{b}_{\mathbf{k}\sigma}$ with amplitudes $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ and the quasi-particle energy $E_{\mathbf{k}}$. In this way one finds familiar expressions for these quantities:

$$E_{\mathbf{k}} = [(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2]^{1/2} \quad (11)$$

and

$$u_{\mathbf{k}}^2 = (1 + (\epsilon_{\mathbf{k}} - \mu)/E_{\mathbf{k}})/2 \quad (12)$$

$$v_{\mathbf{k}}^2 = (1 - (\epsilon_{\mathbf{k}} - \mu)/E_{\mathbf{k}})/2, \quad (13)$$

where $\epsilon_{\mathbf{k}} = \hbar^2 k^2/(2m)$ is the single-particle energy. The parameter Δ is the pairing gap, which satisfies the gap equation

$$-\frac{1}{g} = \frac{1}{\Omega} \sum_{\mathbf{k}} \frac{1}{2E_{\mathbf{k}}}, \quad (14)$$

where Ω is the volume of the uniform system. Notice that this equation is ultraviolet divergent and it must be regularized. The equation for the total density $n = N/\Omega$ of fermions is obtained from Eq. (7) as

$$n = \frac{2}{\Omega} \sum_{\mathbf{k}} v_{\mathbf{k}}^2. \quad (15)$$

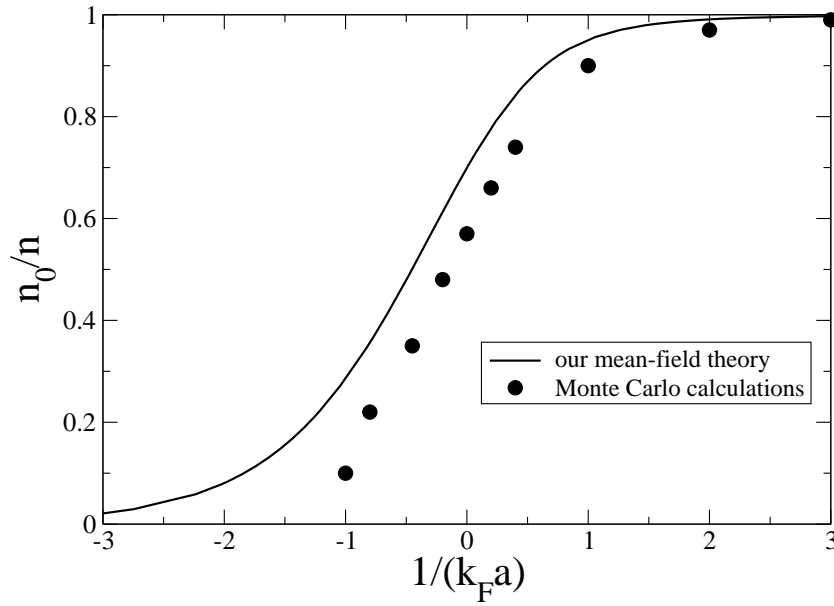


Figure 1. Condensate fraction of fermionic atoms as a function of the inverse interaction strength $1/(k_F a)$: our mean-field theory [6] (solid line); fixed-node diffusion Monte Carlo results [8] (filled circles). Here $k_F = (3\pi^2 n)^{1/3}$ is the Fermi wavenumber, with n the total number density of atoms, and a is the s-wave scattering length of the inter-atomic potential.

Finally, from Eq. (9) one finds that the condensate density $n_0 = N_0/\Omega$ of paired fermions is given by [6, 18]

$$n_0 = \frac{2}{\Omega} \sum_{\mathbf{k}} u_{\mathbf{k}}^2 v_{\mathbf{k}}^2. \quad (16)$$

In three dimensions, a suitable regularization [19] of the gap equation is obtained by introducing the s-wave scattering length a via the equation

$$-\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{1}{\Omega} \sum_{\mathbf{k}} \frac{m}{\hbar^2 k^2}, \quad (17)$$

and then subtracting this equation from the gap equation (14). In this way one obtains the three-dimensional regularized gap equation

$$-\frac{m}{4\pi\hbar^2 a} = \frac{1}{\Omega} \sum_{\mathbf{k}} \left(\frac{1}{2E_{\mathbf{k}}} - \frac{m}{\hbar^2 k^2} \right), \quad (18)$$

which can be used to study the full BCS-BEC crossover [6] by changing the amplitude and sign of the s-wave scattering length a .

Taking into account the functional dependence of the amplitudes $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ on μ and Δ , one finds [6] the very nice formula

$$n_0 = \frac{m^{3/2}}{8\pi\hbar^3} \Delta^{3/2} \sqrt{\frac{\mu}{\Delta} + \sqrt{1 + \frac{\mu^2}{\Delta^2}}}, \quad (19)$$

which shows the not trivial relationship between the energy gap Δ and the condensate density n_0 . By the same techniques, also the two BCS-BEC equations can be written in a more compact

form as

$$-\frac{1}{a} = \frac{2(2m)^{1/2}}{\pi\hbar^3} \Delta^{1/2} I_1\left(\frac{\mu}{\Delta}\right), \quad (20)$$

$$n = \frac{(2m)^{3/2}}{2\pi^2\hbar^3} \Delta^{3/2} I_2\left(\frac{\mu}{\Delta}\right), \quad (21)$$

where $I_1(x)$ and $I_2(x)$ are two monotonic functions [19] given by

$$I_1(x) = \int_0^{+\infty} y^2 \left(\frac{1}{\sqrt{(y^2 - x)^2 + 1}} - \frac{1}{y^2} \right) dy, \quad (22)$$

$$I_2(x) = \int_0^{+\infty} y^2 \left(1 - \frac{y^2 - x}{\sqrt{(y^2 - x)^2 + 1}} \right) dy. \quad (23)$$

In Fig. 1 we report the condensate fraction n_0/n of fermionic atoms in the BCS-BEC crossover as a function of the inverse interaction strength $1/(k_F a)$ obtained with this mean-field theory [6, 7] (solid line). In the figure we compare our calculations [6] with the fixed-node diffusion Monte Carlo results (filled circles) obtained by Astrakharchik, Boronat, Casulleras, and S. Giorgini [8] with $N = 66$ fermions and a tunable square-well potential. Remarkably, the agreement between the two theoretical approaches is better in the BEC side of the crossover. On the other hand, as discussed in Refs. [6, 20], our Eq. (19) is in full agreement with the experimental data of the MIT group [9] in the BCS side of the crossover.

4. Nuclear matter

Let us now consider the nuclear matter, and in particular the bulk neutron matter, which is a dense Fermi liquid made of two-component (spin up and down) neutrons. The shifted Hamiltonian of the uniform neutron matter can be written as

$$\begin{aligned} \hat{H}' &= \int d^3\mathbf{r} \sum_{\sigma=\uparrow,\downarrow} \hat{\psi}_\sigma^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 - \mu \right) \hat{\psi}_\sigma(\mathbf{r}) \\ &+ \int d^3\mathbf{r} d^3\mathbf{r}' \hat{\psi}_\uparrow^\dagger(\mathbf{r}) \hat{\psi}_\downarrow^\dagger(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \hat{\psi}_\downarrow(\mathbf{r}') \hat{\psi}_\uparrow(\mathbf{r}), \end{aligned} \quad (24)$$

where $\hat{\psi}_\sigma(\mathbf{r})$ is the field operator that annihilates a neutron of spin σ in the position \mathbf{r} , while $\hat{\psi}_\sigma^\dagger(\mathbf{r})$ creates a neutron of spin σ in \mathbf{r} . Here $V(\mathbf{r} - \mathbf{r}')$ is the neutron-neutron potential characterized by s-wave scattering length $a = -18.5$ fm and effective range $r_e = 2.7$ fm [21].

One can apply the familiar Bogoliubov approach to diagonalize the mean-field quadratic Hamiltonian derived from Eq. (24), but now the pairing gap $\Delta_{\mathbf{k}}$ depends explicitly on the wave number \mathbf{k} and satisfies the integral equation

$$\Delta_{\mathbf{q}} = \sum_{\mathbf{k}} V_{\mathbf{q}\mathbf{k}} \frac{\Delta_{\mathbf{k}}}{2E_{\mathbf{k}}}, \quad (25)$$

where $V_{\mathbf{q}\mathbf{k}} = \langle \mathbf{q}, -\mathbf{q} | V | \mathbf{k}, -\mathbf{k} \rangle$ is the wave-number representation of the neutron-neutron potential, and

$$E_{\mathbf{k}} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu \right)^2 + |\Delta_{\mathbf{k}}|^2}. \quad (26)$$

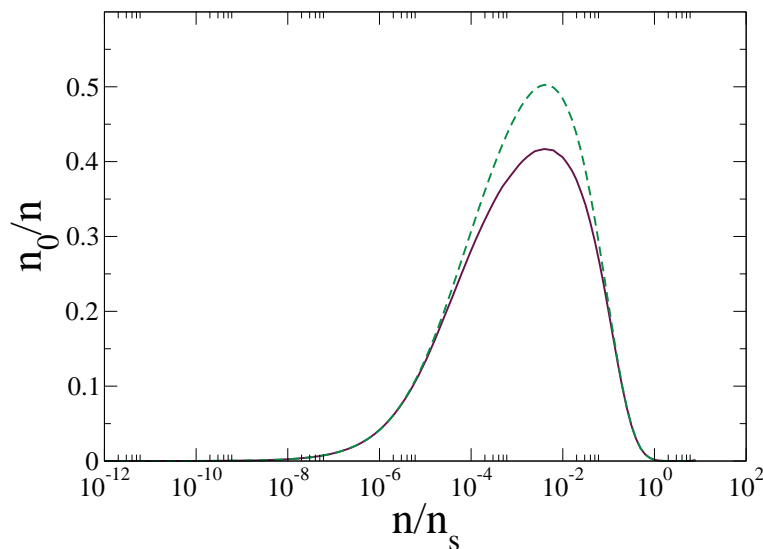


Figure 2. Condensate fraction n_0/n of neutron pairs in neutron matter as a function of the scaled neutron number density n/n_s , where $n_s = 0.16 \text{ fm}^{-3}$ is the nuclear saturation density (see also [13]). The solid line is obtained by using Eqs. (27) and (29). The dashed line is obtained by using Eqs. (28) and (29).

Under the simplifying assumptions $\mu \simeq \epsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$ and $\Delta_{\mathbf{k}} \simeq \Delta$, in the continuum limit we determine the condensate fraction as [13]

$$\frac{n_0}{n} = \frac{\pi}{2^{5/2}} \frac{\sqrt{\frac{\epsilon_F}{\Delta} + \sqrt{1 + \frac{\epsilon_F^2}{\Delta^2}}}}{I_2(\frac{\epsilon_F}{\Delta})} \quad (27)$$

Notice that in the deep BCS regime where $\Delta/\epsilon_F \ll 1$ one finds

$$\frac{n_0}{n} = \frac{3\pi}{8} \frac{\Delta}{\epsilon_F}. \quad (28)$$

Fitting the numerical data of Δ/ϵ_F vs k_F obtained by Matsuo [21] from realistic neutron-neutron potentials we get the formula

$$\frac{\Delta}{\epsilon_F} = \frac{\beta_0 k_F^{\beta_1}}{\exp(k_F^{\beta_2}/\beta_3) - \beta_3} \quad (29)$$

with the following fitting parameters: $\beta_0 = 2.851$, $\beta_1 = 1.942$, $\beta_2 = 1.672$, $\beta_3 = 0.276$, $\beta_4 = 0.975$. By using this fitting formula and Eq. (27) we finally get the condensate fraction of neutron matter as a function of the neutron density n [13].

The condensate fraction n_0/n of neutron pairs is shown in Fig. 2 as a function of the scaled density n/n_s , where $n_s = 0.16 \text{ fm}^{-3}$ is the nuclear saturation density. Notice that the horizontal axis is in logarithmic scale. At very low neutron density n the neutron matter behaves like a quasi-ideal Fermi gas with weakly correlated Cooper pairs and the condensate fraction n_0/n is exponentially small. By increasing the neutron density n the attractive tail of the neutron-neutron potential becomes relevant and the condensate fraction n_0/n grows significantly. The maximum of the condensate fraction is $(n_0/n)_{max} = 0.42$ at the neutron density $n = 5.3 \cdot 10^{-4}$

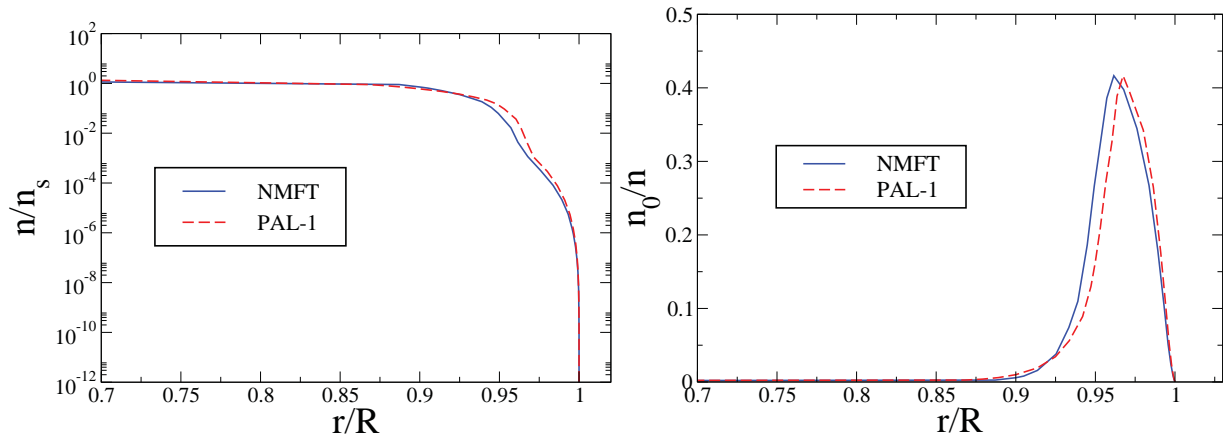


Figure 3. Properties of the 1.4 solar mass neutron star. Left panel: Scaled density profile n/n_s vs scaled distance r/R . Here $n_s = 0.16 \text{ fm}^{-3}$ is the nuclear saturation density and R is the radius of the star. Right panel: condensate fraction n_0/n of neutron pairs vs scaled distance r/R . Solid lines are obtained with the Walecka model of bulk neutron matter [23]. Dashed lines are obtained with the more sophisticated model 1 of Prakash, Ainsworth, and Lattimer [24].

fm^{-3} which corresponds to the Fermi wave number $k_F = (3\pi^2 n)^{1/3} = 0.25 \text{ fm}^{-1}$. By further increasing the density n the repulsive core of the neutron-neutron potential plays an important role in destroying the correlation of Cooper pairs and the condensate fraction n_0/n slowly goes to zero. Remarkably, the results of Fig. 2 are fully consistent with the Monte Carlo value $n_0/n \simeq 0.35$ at $n = 0.003 \text{ fm}^{-3}$ one extracts from the finite-temperature Path Integral Monte Carlo data of Wlazlowski and Magierski [11, 12].

5. Neutron stars

Neutron stars are astronomical compact objects which can result from the gravitational collapse of a massive star during a supernova event. Such stars are mainly composed of neutrons. Neutron stars are very hot and are supported against further collapse by Fermi pressure. A typical neutron star has a mass M between 1.35 and about 2.0 solar masses with a corresponding radius R of about 12 km. Notice that in the crust of neutron stars one estimates [15] a temperature $T \simeq 10^8 \text{ K}$, while the critical temperature of the normal-superfluid transition is $T_c \simeq 10^{10} \text{ K}$. Thus the crust of neutron stars is superfluid.

Some years ago, Datta, Thampan, and Bhattacharya [22] have calculated several mass density profiles $\rho(r)$ of spherical and non-rotating neutron stars by solving the Tolman- Oppenheimer- Volkoff (TOV) equation, which describes the interplay between the expulsive kinetic pressure of the star and its gravitational self-attraction [14, 15]. Datta, Thampan, and Bhattacharya [22] have solved the TOV equation by using various equations of state (EOS) of the nuclear matter. In the left panel of Fig. 3 we plot their results in the case of 1.4 solar mass neutron star. In particular, we report the scaled density profile $n(r)/n_s$ of the neutron star as a function of the scaled distance r/R from the center of the star, where n_s is the nuclear saturation density and R is the radius of the star. Notice that $n(r) = \rho(r)/m_N$ with m_N the neutron mass. The solid line is obtained [22] solving the TOV equation with the simple nuclear EOS of Walecka [23], while the dashed line is obtained [22] solving the TOV equation with a more sophisticated EOS, called model 1, of Prakash, Ainsworth and Lattimer [22, 24].

In the previous section we have found a fitting formula for the condensate fraction n_0/n of neutron matter as a function of the bulk neutron density n . Knowing the density profile $n(r)$ of a neutron star, i.e. the neutron density n as a function of the distance r from the center of

a neutron star, we can determine (local density approximation) the condensate fraction n_0/n of the neutron star as a function of the distance r .

The results are shown in the right panel of Fig. 3. The figure shows that the two EOS give very similar results and that a relevant condensate fraction appears only in the crust of the neutron star in the region between $r/R = 0.85$ and $r/R = 1$ with a maximum value $\simeq 0.4$ at $r/R \simeq 0.96$. We stress that this suggestive plot can be certainly improved because in neutron stars the hadronic matter is not made only of neutrons [14, 15]. On the other hand, it is exactly in the crust of neutron stars that it is expected to find the dilute neutron matter we have considered to derive Eq. (27).

6. Conclusions

We have seen that the condensate fraction of Cooper pairs can be calculated in various superfluid fermionic systems: dilute atomic gases, dense neutron matter and neutron stars. We observe that, while the condensate fraction in ultracold gases of fermionic atoms has been measured in two sophisticated experiments by measuring the momentum distribution of pairs [9, 10], it remains open the exciting problem of finding reliable observational signatures of the condensate fraction of neutron-neutron pairs in atomic nuclei and in neutron stars. In conclusion, we point out that recently we have studied the behavior of the condensate fraction in ultracold and dilute gases of fermionic atoms not only in the 3D uniform system but also in other configurations: 2D uniform system [25], 2D system on a square lattice [26], 3D and 2D uniform system with spin-orbit coupling [27, 28], 3D and 2D uniform system with three-spin components [29], and 3D uniform system with a narrow Feshbach resonance [30].

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