

Ionization and entanglement of two interacting Rydberg atoms in a strong laser field

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Abstract. Dynamics of ionization and entanglement of two Rydberg atoms interacting with each other and with external strong laser field is investigated. The phenomenon of interference stabilization of the bipartite atomic system is established in a strong field limit. The production of highly-entangled state of the studied atoms is shown to be realized. The possibility to control the evolution and time-dependent entanglement of the examined multilevel q-dit system is demonstrated.

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1. Introduction

Coherent control of multi-particle systems is one of the most interesting and important problems of the modern quantum optics. There are many types of the q-bit systems and many different ways to control them [1, 2, 3, 4]. Many papers are aimed on the creation of highly entangled atomic q-bit systems and on their applications in different problems of quantum information [5, 6, 7, 8, 9, 10]. Usually coupling between atomic q-bits is supposed to arise due to the interaction with a common reservoir (atom trap, microcavity, resonator, etc.) and quantum control is performed by applying an external field. Now it is possible to trap Rydberg atoms and ions together in pairs or chains during rather long time [11] with different constructions of traps being used for these purposes [12, 13]. According to recent investigations a pair of tightly trapped atoms excited into the Rydberg states appears to be very attractive as a simple q-bit system and can be controlled either by optical or microwave field [5, 6, 14]. Coupling of these highly-excited atoms results from their dipole interaction caused by rather broad electron density distribution of both Rydberg atoms. This interaction is known to influence strongly on the eigenstate energies of individual atoms and leads to



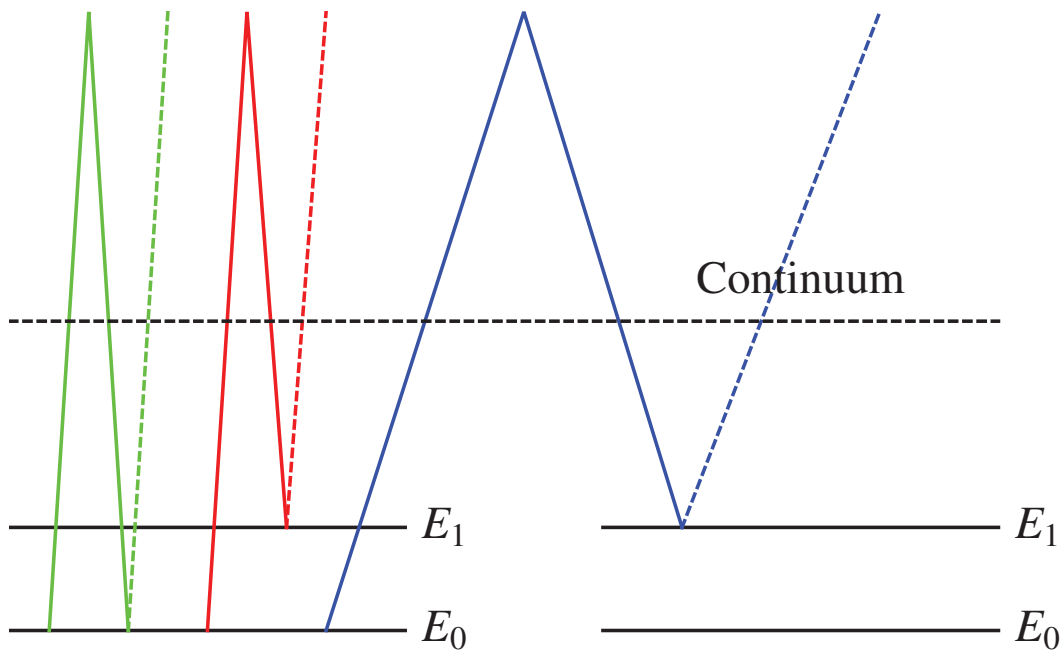


Figure 1. Different types of Raman Λ -type transitions taken into consideration for the system of two interacting Rydberg atoms.

so-called "dipole blockade" effect [6, 14, 15, 16]. Nevertheless tuning of the laser frequency to the shifted by interaction excitation energy results in "anti-blockade". In such situation it is very important to find and analyze the quasienergy states of the bipartite system in a laser field, including the interaction between subsystems and a laser field action. The wave functions and their features are strongly dependent on the interatomic separation R . This problem does not seem to be solved up to now. In addition to study the influence of each type of interaction on the atomic dynamics and the possibility to obtain and to control the entanglement in such bipartite system remains still an open question.

Another interesting aspect of the interaction of a Rydberg q-bit with a laser field can lead to the ionization of a system which is usually not taken into account in theoretical considerations. Often a large number of neighbouring Rydberg levels existing in a real atom can be simultaneously involved both in the excitation/ionization process. The ionization process can be partially suppressed in a strong laser field due to the well known interference stabilization phenomenon [17, 18]. In such case again the influence of the strong external field and the role of the interatomic interaction should be correctly taken into account to analyze the dynamics of the system and to perform the coherent control of atomic q-bits.

In this paper we investigate the behavior of two coupled Rydberg atoms interacting with each other in a presence of external non-resonant laser field and analyze the possibility to control the dynamics of the system including the suppression of the ionization process and the production highly entangled bipartite states. Each atom is considered in a model of "n bound Rydberg states + continuum" [17] and the non resonant laser field is supposed to provide multi-photon transitions to the continuum being accompanied by the repopulation of

Rydberg states via the Raman Λ -type transitions. The scheme of the considered field induced transitions is presented on Fig. 1 and different multi-photon processes of odd order are seen to be taken into account. Direct field-induced transitions between different Rydberg levels of each atom are supposed to be small. In approach used the quasi-energies of the system are found in a presence of both the external laser field and interatomic interaction and the dynamics of the system is analyzed. The strong laser field will be shown to result in ionization suppression and coherent control of the two atomic system, that can be used in many Rydberg q-bit applications. Production of the highly entangled bipartite states is demonstrated.

2. Analytical Model

The evolution of considered bipartite system is described by the time dependent Schroedinger equation (TDSE):

$$i\hbar\partial_t\psi(\vec{r}_1, \vec{r}_2, t) = [\hat{H}_1 + \hat{H}_2 + \hat{V}_1 + \hat{V}_2 + \hat{W}_{12}] \psi, \quad (1)$$

Here $\psi(\vec{r}_1, \vec{r}_2, t)$ is the wave function (WF) of electronic subsystem of two atoms depending on time, indexes "1" and "2" correspond to electrons of different atoms, the Hamiltonian $\hat{H}_i$ includes the kinetic energy and interaction of "i" electron with the own and other nucleus:

$$\hat{H}_i = \hat{T}_i - \frac{ze^2}{|\vec{r}_i - \vec{r}_A|} - \frac{ze^2}{|\vec{r}_i - \vec{r}_B|}, \quad (2)$$

where ze and $\vec{r}_{A,B}$ are the charge and the radius-vector of nucleus "A" and "B" correspondingly. The operator $\hat{W} = e^2/|\vec{r}_1 - \vec{r}_2|$ stands for the interaction between electrons and the interaction with the laser field is considered in the dipole approximation $\hat{V}_i = -(\vec{d}_i \cdot \vec{\epsilon})$ where $\vec{d}_i = e\vec{r}_i$ is the dipole moment corresponding to "i"-th electron and $\vec{\epsilon}$ is the electric field of the laser pulse with the polarization direction chosen along the direction from one nucleus to another. The laser field carrier envelope can be constant or depend slowly on time $\vec{\epsilon}(t) = \vec{\epsilon}_0(t) \cos \omega t$ and the laser field wavelength is assumed to exceed the interatomic separation R to keep us in the frame of the dipole approximation.

To describe the field induced evolution of the system correctly an appropriate basis of the field-free two electron WF should be used, which obey the proper symmetry conditions in relation to exchange of both electron and nucleus positions. Such states can be found as a product of the single electron eigenfunctions $\varphi_i(\vec{r}_1, R)$ and $\varphi_k(\vec{r}_2, R)$ (including the continuum states), depending on the relative position between nuclei $R = |\vec{r}_B - \vec{r}_A|$ and obeying the following stationary problems:

$$\begin{aligned} \hat{H}_1\varphi_i(\vec{r}_1, R) &= E_i\varphi_i(\vec{r}_1, R), \\ \hat{H}_2\varphi_k(\vec{r}_2, R) &= E_k\varphi_k(\vec{r}_2, R), \end{aligned} \quad (3)$$

The solution of (3) can be found using the elliptical coordinates [19] and obey the symmetry conditions in relation to the exchange between nuclear positions. The two-electron WF $|ik\rangle$ are the solutions of the eigenproblem:

$$\begin{aligned} (\hat{H}_1 + \hat{H}_2)|ik\rangle &= E_{ik}|ik\rangle \\ |ik\rangle &= \varphi_i(\vec{r}_1, R)\varphi_k(\vec{r}_2, R) \end{aligned} \quad (4)$$

and are characterized by the energies $E_{ik} = E_i + E_k$. To avoid the autoionization process we suppose internuclear separation R or the charge z of the ion in (2) are large enough provide electron states to be bound.

To provide the symmetry conditions in relation to the electron positions we use the following states:

$$\begin{aligned} & |ii\rangle \text{ for } i = k, \\ \Psi_{ik}^{\pm} &= \frac{|ik\rangle \pm |ki\rangle}{\sqrt{2}} \text{ for } i \neq k, \end{aligned} \quad (5)$$

where $\Psi_{ik}^{\pm}$ can be considered as generalized Bell functions [20].

It should be noticed that many of states (5) should be taken into account to describe correctly the behavior of the two-atomic system in a laser field. However we would like to demonstrate that some general features of the system dynamics - the ionization suppression and time dependent entanglement - can be found in the frame of a rather simple model when only two bound states (3) for each electron are taken into account, and these features can be also seen in the case of larger number of the basis functions (5).

Let us consider firstly the continuum and only two bound states (3) for each electron keeping in mind that they do not coincide with the eigenfunctions of the individual atom since each Hamiltonian $\hat{H}_i$ includes the interaction of the i -th electron with the own and the other nuclei (2). Under these condition using the procedure of adiabatic elimination of the continuum [17, 18] we obtain the differential equations for probability amplitudes of bound bipartite states separately for spatially symmetric (singlet) $\Psi_{S=0}^{bound}$ and anti-symmetric (triplet) $\Psi_{S=1}^{bound}$ states, that can be represented by following way:

$$\begin{aligned} \Psi_{S=0}^{bound} &= C_{00}|00\rangle + C_{11}|11\rangle + \\ &+ C_{01}^+ \Psi_{01}^+ \\ \Psi_{S=1}^{bound} &= C_{01}^- \Psi_{01}^- \end{aligned} \quad (6)$$

Then the system of equations for spatially symmetric and antisymmetric WF are independent in accordance with the intercombination prohibition principle and are given by:

$$\begin{cases} i\dot{C}_{00} = (2E_0 - i\Gamma + W)C_{00} + WC_{11} + \frac{2W-i\Gamma}{\sqrt{2}}C_{01}^+ \\ i\dot{C}_{11} = WC_{00} + (2E_1 - i\Gamma + W)C_{11} + \frac{2W-i\Gamma}{\sqrt{2}}C_{01}^+ \\ i\dot{C}_{01}^+ = \frac{2W-i\Gamma}{\sqrt{2}}(C_{00} + C_{11} + \sqrt{2}C_{01}^+) + (E_0 + E_1)C_{01}^+ \end{cases} \quad (7)$$

$$i\dot{C}_{01}^- = (E_0 + E_1 - i\Gamma)C_{01}^-. \quad (8)$$

Equations (7-8) are written with following significations:

$$\begin{aligned} \Gamma &= \Gamma_{i,k} = 2\pi \frac{V_{i,E}}{2} \frac{V_{E,i}}{2} \\ V_{\alpha,\beta} &= -\vec{e} \langle \alpha | \hat{d} | \beta \rangle \\ W &= \langle ik | \hat{W} | i'k' \rangle = W_{ik}^{i'k'}, \end{aligned} \quad (9)$$

where for simplicity we have supposed that all elements of tensor $\Gamma_{i,k}$ are the same and all matrix elements $\langle ik | \hat{W} | i'k' \rangle$ are equal to each other. It should be noticed that the value of W depends strongly on the interatomic separation R and the further will analyze different cases for wide range of values $W(R)$ in relation to the values of ionization width Γ and energy separation between atomic levels $\Delta = E_2 - E_1$.

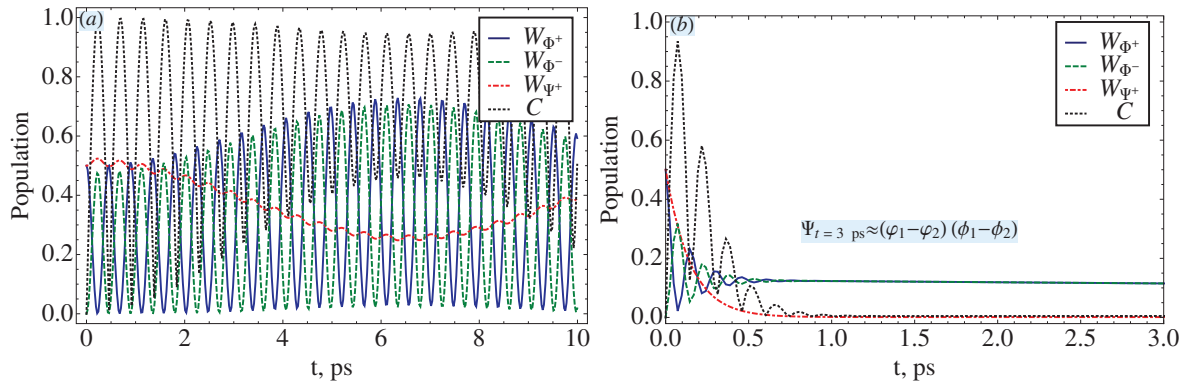


Figure 2. Population of different Bell states (W_{Φ^+} , W_{Ψ^+} , W_{Φ^-}) atomic states in (6) and concurrence C calculated in dependence on time for two interacting Rydberg atoms (with 2 bound Rydberg levels) in the absence of laser field (a) and in a strong field regime (b) for $W = 30\Delta$, $\Gamma = 10\Delta$ and $\Delta = \frac{2Ry}{50^3}$. The state $\varphi_0(\vec{r}_1)\varphi_0(\vec{r}_2)$ is populated initially.

3. Results and discussion

According to equation (8) in the case of only two bound levels (3) taken into account in each atom there is the only spatially antisymmetric bipartite state which decays fast in a strong laser field. However as we will show later in the case of larger number of bound levels (3) the dynamics is more rich and the stabilization can take place in the case of spatially antisymmetric states too.

Before we'll analyze the solution of equation (7) let us discuss a fundamental and very interesting feature of composite systems referred to as the entanglement. The physical meaning of entanglement consists in the fact that it is impossible to characterize each subsystem of a multi-particle system by its own wave function. In other words, the total WF of the system can not be factorized into the product of two WF separately describing each of the subsystems.

To characterize the entanglement of the system different quantitative degrees can be used: the Schmidt number [21, 22], concurrence [23, 24], negativity, etc. According to [21, 22] the Schmidt number defined as following:

$$K^{-1} = Tr\{\rho_r^2\}, \quad (10)$$

where ρ_r is the reduced density matrix of the system. And for our spatially symmetric state $\Psi_{S=0}^{bound}$ the expression (10) can be calculated as:

$$K_S^{-1} = 1 - 2|C_{00}C_{11} - (C_{01}^+)^2/2|^2. \quad (11)$$

This value appears to coincide exactly with that obtained for the biphoton q-trite state considered in [24]. This fact is a direct evidence that in our case the considered two atomic system is an example of two atomic q-bit, which can be analyzed in terms of different q-bit states without special attention on the continuous variable dependence of the single atom WF. In addition, using the well known connection between Schmidt number K and concurrence

C [23, 24] one can obtain for the case of the symmetric singlet state Ψ^{bound} :

$$S=0$$

$$C_S = 2 |C_{00}C_{11} - C_{01}^2|. \quad (12)$$

Taking into account the antisymmetric spin part of the total WF in this case we found the total Schmidt number to be twice larger:

$$K_{SA} = 2K_S. \quad (13)$$

Since the spin variable is not included in the Hamiltonian The factor "2" is not changed during the evolution and results in a renormalization of the concurrence from the range $[0, 1]$ to $[1, \sqrt{3/2}]$ in a case of (12).

The change of the range of possible values of the concurrence (or of the Schmidt parameter) means that now (including the spin variables) the system is always entangled. However in our work we will be interested firstly in the possible entanglement over spatial variables (excluding the independent spin degree of freedom). So only K_S (11) and C_S (12) are calculated and presented in this paper.

Let us consider firstly the field-free dynamics of our system. Fig. 2a represents the time-dependent population of Bell states:

$$\begin{aligned} \Phi^\pm &= \frac{\varphi_0(\vec{r}_1)\varphi_0(\vec{r}_2) \pm \varphi_1(\vec{r}_1)\varphi_1(\vec{r}_2)}{\sqrt{2}}, \\ \Psi^\pm &= \frac{\varphi_0(\vec{r}_1)\varphi_1(\vec{r}_2) \pm \varphi_1(\vec{r}_1)\varphi_0(\vec{r}_2)}{\sqrt{2}} \end{aligned} \quad (14)$$

in order to demonstrate that maximally entangled states are populated periodically during the evolution of the system, even though the dis-entangled ground state of two atomic system was populated initially. Fast oscillations of the populations are caused by the electron-electron (interatomic) interaction. Period of the oscillations is determined by the interaction energy W and slow modulation is proportional to the energy difference Δ between bound atomic levels of each atom.

In a presence of a rather strong field these fast oscillations of atomic population are damped out at times of about $\sim \hbar/\Gamma$ (see figure 2b) due to the dominant role of the laser field in the dynamics of the system. In a strong field regime ($\Gamma \gg \Delta$) the ionization process is found to be partly suppressed and there is a non-zero probability to find the system bound even at large time, so the stabilization phenomenon is observed. The mechanism of stabilization is similar to that predicted for a single Ry atom in a strong field [17, 18] and is referred to as "the interference stabilization". The state which describes the bound fraction of population of two atomic q-bits is found to be given by:

$$\Psi^{--} = \frac{(\varphi_0(\vec{r}_1) - \varphi_1(\vec{r}_1))(\varphi_0(\vec{r}_2) - \varphi_1(\vec{r}_2))}{2}. \quad (15)$$

and appears to be factorized so as the two atomic subsystems are totally dis-entangled. The state (15) is found to be the quasienergy state (QES) of the system in a strong field limit and is seen to be a product of stable in a strong field QES of each independent atom discussed in [17, 18].

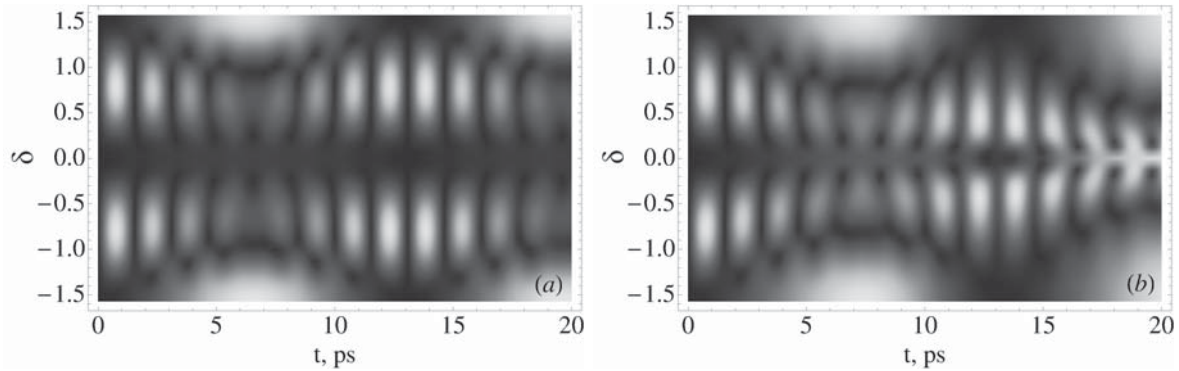


Figure 3. Concurrence calculated for two interacting atoms ($W = 3\Delta$) in dependence on time and relative phase of initial state (16) for the field free regime (a) and in a presence of a weak field with $\Gamma = \Delta/4$ (b).

In the case of the intermediate field strength ($\Gamma \sim \Delta$) it is possible to provide both ionization suppression and entanglement in the system. Fig. 3 represents entanglement map calculated without laser field and in a presence of a rather week field at different phases of initial state chosen for the two atomic q-bit system:

$$\Psi|_{t=0} = \frac{(\varphi_0(\vec{r}_1) + \varphi_1(\vec{r}_1)e^{2i\delta})(\varphi_0(\vec{r}_2) + \varphi_1(\vec{r}_2)e^{2i\delta})}{2}. \quad (16)$$

What is interesting here is that the presence of a classical field leads to higher degree of entanglement at some initial conditions (see the case of $\delta = 0$ of fig. 3b) and such entanglement can not be achieved in a field free dynamics of even strongly interacting atoms.

It should be emphasized that the factorized state Ψ^{--} which describes two atomic q-bits remaining bound in a strong laser field can be used to produce a maximally entangled Bell state. If we prepare the system in a stable Ψ^{--} state, then turn off the laser field and look at the field-free dynamics of the system, the population of this state will be changed in time due to the coupling between atoms caused by electron (interatomic) interaction. It is found that two of four QES characterizing the field-free dynamics of two coupled atoms are the superpositions of the state Ψ^{--} and the Bell state Φ^- (14):

$$\begin{aligned} \Psi_-^{QE} e^{-\frac{i}{\hbar}\gamma_- t} &= \frac{\Psi^{--} - \Phi^-}{\sqrt{2}} e^{-\frac{i}{\hbar}\gamma_- t}, \\ \Psi_+^{QE} e^{-\frac{i}{\hbar}\gamma_+ t} &= \frac{\Psi^{--} + \Phi^-}{\sqrt{2}} e^{-\frac{i}{\hbar}\gamma_+ t}, \end{aligned} \quad (17)$$

with the corresponding approximate quasienergies:

$$\begin{aligned} \gamma_+ &\approx E_0 + E_1 + \Delta/\sqrt{2} + o(\Delta^2), \\ \gamma_- &\approx E_0 + E_1 - \Delta/\sqrt{2} + o(\Delta^2). \end{aligned} \quad (18)$$

Thus the population of state Ψ^{--} corresponds to the population (with equal probability amplitudes) of both QES (17) and the time dependent probability to find system in Ψ^{--} state is given by:

$$\begin{aligned} W_{\Psi^{--}} &= \frac{1}{2} \left| \langle \Psi^{--} | \Psi_-^{QE} e^{-\frac{i}{\hbar}\gamma_- t} + \Psi_+^{QE} e^{-\frac{i}{\hbar}\gamma_+ t} \rangle \right|^2 = \\ &= \cos^2 \left(\frac{t\Delta}{\hbar\sqrt{2}} \right). \end{aligned} \quad (19)$$

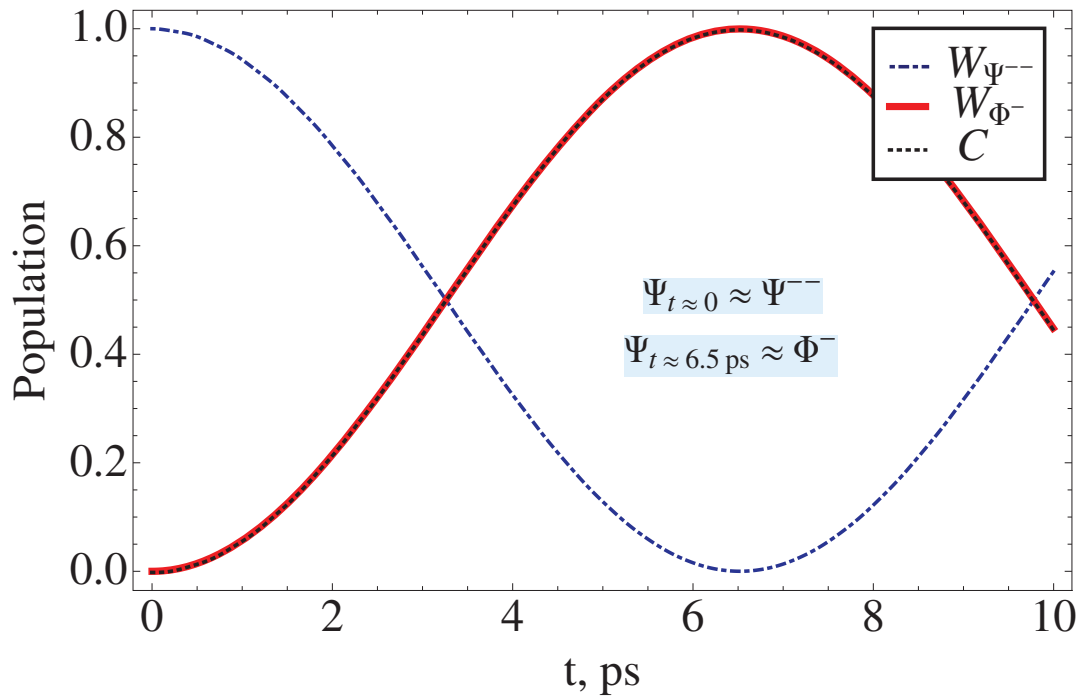


Figure 4. The time dependent population of the state Ψ^{--} ($W_{\Psi^{--}}$), the Bell state Φ^{-} ($W_{\Phi^{-}}$) and the concurrence C of the two-atomic q-bit in the after pulse regime.

The population (19) oscillates in time with the frequency $\Delta\sqrt{2}/\hbar$ and there is a population exchange between only two states: Ψ^{--} and the Bell state Φ^{-} (see fig. 4), so as $W_{\Psi^{--}} + W_{\Phi^{-}} = 1$. In other words it is possible to create maximally entangled electron spatial Bell state of the two atomic system existing for relatively long time. Since the system reveals periodically maximal entanglement or the highest resistance to the the ionization by a second strong laser pulse, it seems to be rather easy to measure experimentally the population of the stable state Ψ^{--} or the degree of entanglement (concurrence) which coincides exactly with population $W_{\Phi^{-}} = 1 - W_{\Psi^{--}}$. For this purpose the second ("probe") strong laser pulse should be applied to the system with the delay τ in relation to the first pulse, which prepared Ψ^{--} state. Since Ψ^{--} is almost stable against ionization in a strong laser field the ionization signal W_i obtained in the second ("probe") pulse in dependence on the time delay τ gives the population $W_{\Phi^{-}}$ or concurrence of the system in dependence on time ($W_{\Phi^{-}} = W_i = 1 - W_{\Psi^{--}} = C$). As a result the dynamics of the population of the Bell state Φ^{-} and the entanglement of the system can be measured experimentally and can be used to extract information about the QES (17) in their dynamics as well as about the average number of the populated Rydberg states because the oscillation period is proportional to Δ and in the case of large number of highly excited states (3) corresponds to the modified Kepler period of the system.

Now let us consider the case of large number n of bound states (3) for each electron. Now we obtain again the system of independent equation for spatially symmetric and antisymmetric states. However for $n > 2$ we do observe the interference stabilization regime for states of both symmetries. The stabilization assumes here that several of the QES are

characterized by much more slower decay in a strong field in comparison to the ionization rate calculated according to the Fermi Golden Rule, similarly to the case of individual Rydberg atom with large number of Rydberg bound states discussed in [17, 18]. The initially fast and further very slow decay to the continuum is observed for two coupled Rydberg atoms interacting with a strong laser field (see fig. 5) in the case of $n = 4$.

During this interaction the system reveals high degree of stability due to significant population of several QES that are very resistant to the ionization process. Since these QES are characterized by different quasienergy values (taking into account their real parts), the population of different spatially symmetric states is found to be changed significantly during the laser field action, as it can be seen on fig. 5. Because of the simple initial condition chosen, the stable QES appear to be populated in such a way that the system remains almost totally dis-entangled during all the time of interaction with a laser field. This fact is confirmed by the time dependent behavior of the Schmidt parameter K presented on fig. 5 and the value of K in the case of large number of bound levels is calculated using the expression:

$$K = \frac{\sum_{i,k} |C_{ik}|^2}{\sum_{i,k,l,n} C_{il} C_{in}^* C_{kn} C_{kl}^*} \quad (20)$$

However in the after pulse regime the interatomic interaction $\hat{W}$ plays significant role and appears to provide efficient entanglement between atoms and the increase of the Schmidt parameter possibly to its maximum value $K_{max} = n$. Thus for spatially symmetric states the regime of interference stabilization is found to be accompanied by almost independent dynamics of two atoms in a strong field while the the significant entanglement can be achieved due to the interaction between the atoms $\hat{W}$ in the after pulse regime only.

Another situation is observed for spatially antisymmetric initial state. In this case similarly to the case of two bound level ($n = 2$) for each electron the interatomic interaction term doesn't influence the dynamics of the system (under the assumption of equal matrix elements $W_{ik}^{i'k'}$) and the antisymmetric Bell states ψ_{ik}^- appear to be the eigenstates of the field-free two atomic system. In a strong laser field the QES of the system can be found as a antisymmetrized product of the QES of two non-interacting atoms in a strong field. All the antisymmetric QES can be represented then as a superposition of different Bell states ψ_{ik}^- (5):

$$\psi_l^{QES} = \sum_{i,k} \alpha_{ik}^l \Psi_{ik}^- \quad (21)$$

and some of ψ_l^{QES} appear to be almost stable in a strong field limit if the number of considered Rydberg levels n for each atom is greater than 2. Thus for $n > 2$ the interference stabilization is observed for antisymmetric states too.

At fig. 6 the dynamics of the total bound probability and the populations of different Bell states are presented during the strong laser pulse action and in the after pulse regime for the case when one of the stable antisymmetric QES is initially populated. During the laser pulse the populations of different antisymmetric Bell states as well as the total bound probability are seen to be characterized by the very slow exponential decay with the same small ionization

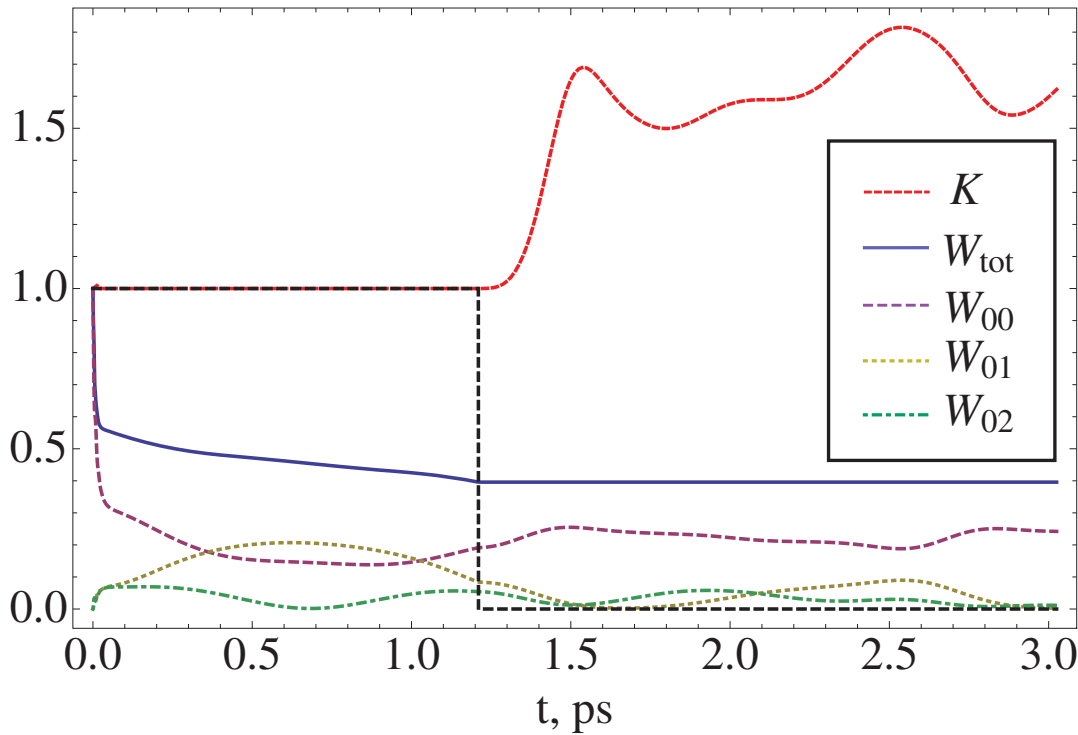


Figure 5. The population of several bipartite states (W_{00} , W_{01} , W_{02}) and Schmidt parameter K calculated in dependence on time for two interacting Rydberg atoms (with $n = 4$ electron bound excited levels) during the laser pulse (dashed line) and in the after pulse regime, for $W = 10\Delta$ and $\Gamma = 10\Delta$. The state $\varphi_0(\vec{r}_1)\varphi_0(\vec{r}_2)$ is populated initially.

rate Γ_{stable} for all states since the dynamics of initially populated QES with quasienergy γ is given by:

$$\psi^{QES}(t) = \left(\sum_{i,k} \alpha_{ik} \Psi_{ik}^- \right) e^{-i\gamma t/\hbar} \quad (22)$$

where α_{ik} are the probability amplitudes of the Bell states ψ_{ik}^- at $t = 0$. Thus the total bound probability is not changed much during many periods of the laser field and the system reveals high stability against ionization in a strong laser field. In the after pulse regime the presented probabilities are conserved. As for entanglement, it is shown that for any superposition of antisymmetric Bell states the value of Schmidt parameter (20) equals 2 ($K = 2$), so as the system is found to be always entangled. As a result, if the system is initially prepared in any arbitrary spatially antisymmetric state, its dynamics is characterized by surviving part of bound population in a strong field due to trapping in the stable QES and by conservation of the degree of entanglement with $K = 2$ both during the laser pulse and in the after pulse regime. Due to different energies of the Bell states ψ_{ik}^- the projection of the total WF on the QES (22) existing in the laser field is changed significantly in the after pulse regime (fig. 6). However full reconstruction of the wave packet is seen to take place periodically in time so as the system periodically reveals a very high resistance against ionization if the second strong laser pulse will be applied.

It should be emphasized that the stability of the two atomic system in a strong laser field

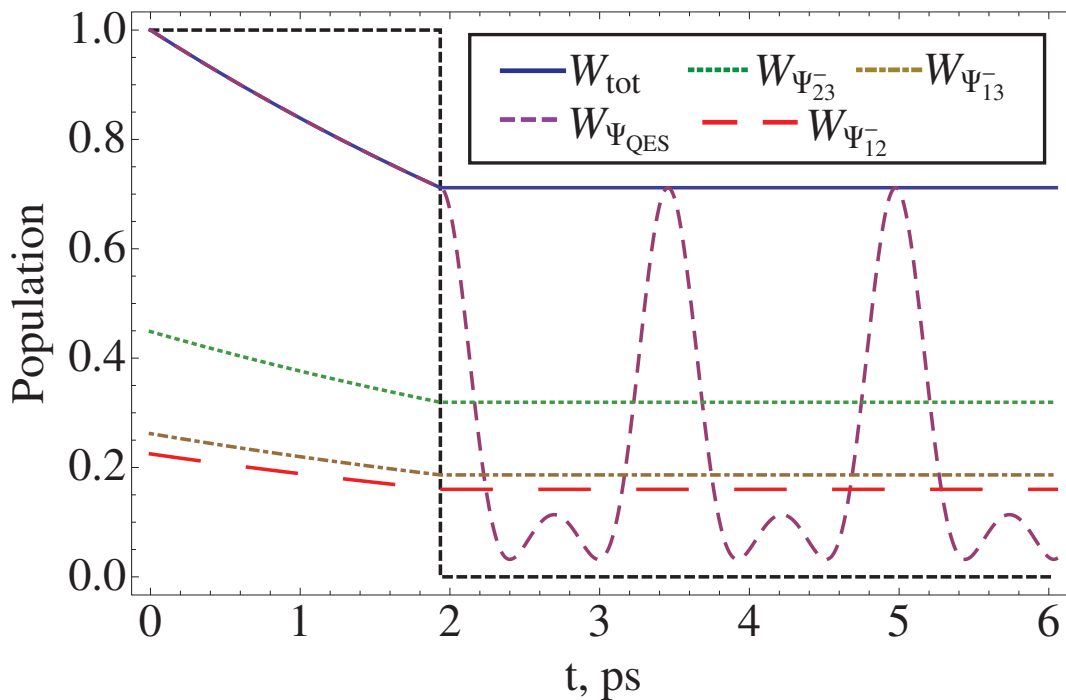


Figure 6. The total bound probability W_{tot} , the population of antisymmetric initially populated QES $W_{\Psi_{QES}}$ and population of several antisymmetric Bell states during the laser pulse and in the after pulse regime for two atomic system (with $n = 4$ bound Rydberg levels for each atom) for $\Gamma = 10\Delta$

increases with the number of bound levels (3) taken into account for each electron. According to [17, 18] the individual Rydberg atom (with n Rydberg levels) in a strong field regime has only one "unstable" QES, which decays fast in a presence of the laser field. Since in a presence of a strong laser field the influence of the electron-electron interaction on the system dynamics appears to be relatively small the field induced QES of the system can be approximated simply as a product of single electron QES with a performed symmetrization if needed. As a result for considered two atomic system it can be shown that there are n "unstable" and $n(n-1)/2$ "stable" spatially symmetric states and $(n-1)$ "unstable" and $(n-2)(n-1)/2$ "stable" antisymmetric states. Thus the number of "stable" states grows much faster than the number of states which decays rapidly in a strong laser field. So the interference stabilization expected to be well pronounced for the system of two real Rydberg atoms.

4. Conclusions

The dynamics of two interacting Rydberg atoms in external laser field is studied and interference stabilization against ionization is found in a strong field regime. The mechanism of stabilization is similar to that proposed in [18] for isolated Rydberg atom in a strong field. In the case of more than 2 bound levels taken into account for each electron the interference stabilization is found to occur for both spatially symmetric and antisymmetric states and appears to be more pronounced for larger number of electron bound levels. The

surviving of the stable QES under the spatially symmetric initial condition leads to the full dis-entanglement between two electrons in the system, while in the case of antisymmetric initial state the entanglement is found to be conserved both for stable QES in the strong laser field and in the after pulse regime. The methods to obtain strongly entangled bipartite states and to measure the degree of entanglement experimentally are suggested. Thus we show the possibility to control the studied two atom q-bit system by external strong non-resonant laser field and create highly entangled and dis-entangled bipartite states providing significant stabilization against the field induced ionization process. The main approximation made in the worked out theoretical approach consists in the assumption of equal matrix elements (9). Since the wave functions for each electron (3) depend on the interatomic separation and can be a superposition of the large number of Rydberg states of individual atom, the matrix elements $\langle ik|\hat{W}|i'k'\rangle$ can be different for different states and depend dramatically on R . For this reason we analyzed the dynamics of the system in a wide range of the this matrix element values. The predominant role of the strong laser field was found in relation to the electron-electron interaction even under condition of large value of W . In a real system the difference of matrix elements $W_{ik}^{i'k'}$ can lead to the influence of the electron-electron interaction on the dynamics of the antisymmetric electron states. This can reduce the degree of entanglement quantitatively. However the main features of observed dynamics of the studied two atomic system - the strong field stabilization and the possibility of entanglement - seem to be general. Thus the general analytical approach to describe the dynamics of two interacting Rydberg atoms in a strong laser field is developed, providing simple way to calculate for wide range of quantum system parameters, based on the scaling parameter Δ - separation between atomic levels (inverse Kepler period).

4.1. Acknowledgments

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The calculations were performed on the supercomputer SKIF-MSU “Chebyshev”.

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