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# Quantum Mechanics and physical calculations

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**Abstract.** We suggest to realize the computer simulation and calculation by the algebraic structure built on the basis of the logic inherent to processes in physical systems (called physical computing). We suggest a principle for the construction of quantum algorithms of neuroinformatics of quantum neural networks. The role of academician Sahakyan is emphasized in the development of quantum physics in Armenia.

## 1. Academician Sahakyan and the development of quantum physics in Armenia

Gurgen Sahakyan, being a well-known physicist in the fields of gravity, astrophysics and microphysics, showed a big interest to quantum physics. In the middle of last century, he founded the Chair of Theoretical Physics at the Physics Department of Yerevan State University (YSU). At that time, the quantum concept entered into all areas of science and technology. Already the first quantum-mechanical calculations revealed a band character of the energy spectrum of multi-particle systems which became the basis for solid-state physics, semiconductors and semiconductor electronics, and then for quantum electronics. That time in Armenia well-known experts in microphysics were developing their directions. Being a highly erudite and multi-sided developed scientist, G.S. Sahakyan felt unacceptable the indirect role of quantum mechanics and bravely took to put it on a fundamental course. Times were hard, there were no textbooks in Armenian language. On the basis of his first lectures he and his student E.V. Chubaryan have written the first, unfortunately so far the only textbook on quantum physics in the Armenian language (it is republished, as well as translated into Russian language [1]). Academician Sahakyan could induce students' great interest in quantum physics and give them his enthusiasm. I still remember with gratitude his pride and pathos in the introductory lecture on quantum mechanics in our group when he presented the essence of the concept of quantum physics: "At present, the quantum theory is the most common, complete, beautiful and experimentally justified among all scientific theories... But more radical properties are inherent to the concept of quantum physics: a new way of thinking, a new level of civilization and a new logic – the logic of Nature."

## 2. Quantum physics in informatics

The first machines carried out calculations in the classical Boolean logic system by using given algorithms and programs. Algebra of Boolean logic (ABL) is universal, so a mathematical machine realizing it is universal as well. This is reflected in the famous Church-Turing thesis [2]. In the middle of the last century the mathematical theory of information and cybernetics were built. In fact, it was believed that any problem of informatics is principally solvable on


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such machines and the physics was considered simply and only as a means of realization of mathematical ideas and algorithms. The complete set of these tools composed the element base of informatics, which is responsible for the efficient operation of information systems. For this purpose, high expectations were associated with quantum mechanics. The successes are impressive to the extent that mathematicians-informatics were convinced in its "omnipotence." The classics of physics did not shared this opinion and predicted more "omnipotence" but principally new information systems (Sahakyan was also a supporter of the last statement).

The behavior of a micro-object is described by the amplitude of probability, the set of which forms a Hilbert space $\mathcal{H}$. It was discovered that the quantum probabilities are not described by the classical probability theory [2]-[4]. Further, the nature of the physical and mathematical information is different [5]. L. Brillouin developed the foundations of the theory of physical informatics with the use of the quantum concept of physics and showed that in the nature we meet the physical rather than abstract information and he received a number of restrictions on the mathematical information [5]. Further, infinite amount of information is inherent to a quantum system, a part of which is redundant and can not be put out from the system, but that is involved in all information processes. This property can not be modelled by finite resources on the basis of ABL.

A. Einstein, E. Schrodinger, and others have indicated entangled states of combined quantum systems which have no counterparts in macro-world and contain more information than non-entangled states. This is a manifestation of the properties of logic, in accordance of which processes occur in Nature. G. Birkhoff and J. von Neumann have constructed the algebra of quantum logic, which contains as a subalgebra ABL [2, 6]. It is more precise to say that "the classical theory is an essential part of quantum theory, which also contains all the limiting cases." Moreover, in macro-world ABL is only valid, so Feynman concluded that any physical system (machine) realizing only the ABL, can not model quantum mechanics, but it can only imitate. He also noted that the information performance of ABL-machines depends of their size polynomially, but for quantum machines, this dependence is exponential. R. Feynman formulated these ideas as quantum informatics [2]-[4].

## 3. Algebras of physical logic and physical mathematics

Mathematical logic (and its algebras) is an axiomatic science, which is not always possible to realize in nature. Being abstract, it may contain an infinitely large, infinitely small or the other kind of elements which do not exist in nature. Only the completeness and consistency are required from its axioms, so it can be physically unrealizable. If, in addition, physical realizability is required, not for all axiomatics in our universe the "Maxwell's Demon", realizing them, exists [2, 3, 5]. But judging from the condition of the realizability and universality of logic, we come to the inverse problem: using the logic of the processes occurring in nature to form the axiomatic of the logic algebra with the properties of completeness, consistency and universality. Then its existence and realizability are automatically provided. We refer to such logical systems as physical and the computation in them physical [7]. A physical logical system contains only physical quantities as carriers of information and processes of their evolution. Mathematical physics is directed to construct mathematical images of physical objects. The representation of mathematical objects by physical phenomena is the essence of the so-called physical mathematics. Both of them have a structure on the logic, abstract mathematical and physical, respectively. We present two examples of algebraic structure of multi-valued physical logic.

Let one has a set $A_p = \{0, 1, 2, ..., (p-1) = E\}$ of equidistant dimensionless $p$ values for some parameter of the state of a physical system. Logical $p$-level statements can be presented by the numbers $0, 1, \ldots, E$. Moreover, in order to construct an appropriate algebra for $p$-valued physical logic, we can identify the logical operations, algebraic actions and physical transformations of

these parameters. As transformers one can take a multilevel controllable physical system, the input (control) and output parameters of which coincide with $A_p$ (or with $A_{q\leq p}$, in particular, $A_2$). Such systems exist, for example, polistors [7], and it is proved that the logic of processes in the group $\mathcal{L}_p$ of polistors is sufficient to form a closed, complete, consistent and universal $p$-valued deterministic physical logical system. $\mathcal{L}_p$ consists of 5 polistors that produce the following transformations:

$$I_2x \equiv 1-x,\ A_2,\ I_px \equiv E-x,\ S(x_1,x_2,\ldots,x_n) = \begin{cases} 1, \text{ if } \sum_{i=1}^n x_i \leq E \\ 0, \text{ if } \sum_{i=1}^n x_i > E \end{cases},$$

$$h(x_1,\ldots,x_n) = \begin{cases} E, \text{ if } \sum_{i=1}^n x_i \leq E \\ 0, \text{ if } \sum_{i=1}^n x_i > E \end{cases},\ K(x_1,\ldots,x_n) = \begin{cases} I_p(\sum_i x_i), \text{ if } \sum_{i=1}^n x_i \leq E \\ 0, \text{ if } \sum_{i=1}^n x_i > E \end{cases}.$$

It is not difficult to find in $\mathcal{L}_p$ physical images (that is, to make physical modelling) of the action of ABL and of mathematical models.

The presentation of indeterministic processes of nature is not possible in any deterministic system of finite resources, whether mathematical or physical. G. Birkhoff and J. von Neumann noted the identity (up to isomorphism) of lattices for a number of constructions. Lattice (similar to the atomic one) is the set $\mathcal{M}$, ordered by the relation of order $a \leq$ and satisfying the conditions (in standard notations): $1.\forall x,y \in \mathcal{M}$, $\max(x,y) \equiv x \vee y$; $x \leq x \vee y$; $y \leq x \vee y$; $z \leq x$ and $z \leq y \Rightarrow x \vee y \leq zz$; $2.\forall x,y \in \mathcal{M}, \min(x,y) \equiv x \wedge y$; $x \wedge y \leq x$; $x \wedge y \leq y$; $z \leq x$ and $z \leq y \Rightarrow z \leq x \wedge y$. The element $a$ in the lattice is called an atom if $x \leq a \Rightarrow x = 0$ or $x = a$. In particular, the set of statements with the implication closed with respect to the operations of conjunction and disjunction (that is $Q \leq P \iff P$ implies $Q$) and the set of subspaces of the space of states for a quantum system are isomorphic lattices. Both these lattices are orthocomplementary (that is $\forall x$ is associated with the orthogonal complement $\overline{x}$ of such that the conditions: $\overline{\overline{x}} = x$; $x \vee \overline{x}$; $x \leq y \iff \overline{y} \leq \overline{x}$ are satisfied) if $\overline{x}$ belongs to the orthogonal complement of the subspace spanned on the state $x$. In the algebra of logic the measure of the truth 0 and 1 correspond to the "lie" and "truth", and in the Hilbert space it corresponds to eigenvalues of the orthogonal projection operator. The indeterminism of microworld is reflected in projectors, so Gleason's theorem states that the metastates $\mu(M) = Tr(P_M\rho)$, where $\rho$ is the density matrix and $P_M$ is a projector. If an observable $A$ takes a value $a$ in the state $\psi$, then it takes the same value in all the states of the form $(\psi + \phi)$, where $\phi$ is an arbitrary state orthogonal to $\psi$. Because of this the lattice of quantum states becomes non-distributive and is called orthomodular: $x \leq y \Rightarrow (x \vee \overline{y}) \wedge y = x$. In general, its elements are not compatible, but in the lattice there are also many compatible elements for which the distributive law takes place: $(x \wedge y) \vee (x \wedge \overline{y}) = x$. The ortho-complementary distributive lattice is called classical logic, and the ortho-complementary orthomodular lattice is called quantum logic. That the classical logic is completely incorporated into the quantum logic is the law of Nature. The quantum logic is isomorphic to the lattice of projective space, which contains all his hyperplanes as points and which generates the geometric Clifford algebra. It became clear that quantum logic, being the logic of nature and able to implement the algebras $S(U_n)$ and $S(O_n)$ of matrices, can be used to build quantum informatics. In this case, the carriers of the information are multi-level quantum systems, in particular, qubits.

## 4. Quantum computers and quantum computing

For modelling and exponentially fast computing only machines are suitable based on quantum-correlated systems, because the dependence of their productivity on the number of qubits is exponential. In this way, two classes of problems arise: (i) how to build a quantum algorithm for computing, that is to present it by quantum processes, and (ii) what kind of machine is the best for the computing?

Feynman has constructed an example of a quantum computer working by the algebra of quantum logic in the "language of second quantization" [4]. To this end, he chose a redundant set of operators, represented by $n \times n$ matrices and realized by the application of the phase change for a state by the angle $\varphi$. The main operations are: annihilation and creation operators, $a$ and $a^+$, the operator of the particle number $n$, the negation operator $N$, Walsh-Hadamard operator $h$, the operator of the phase shift $s_\varphi$, the operator of the "element $\pi/8$" and Pauli matrices $\sigma_i, i = 0, 1, 2, 3$, rotation operators $R_{e_i}(\varphi) = \exp(i\varphi e_i/2)$ around the axis $OX$ (similarly $OY$) and $OZ$:

$$R_x(\varphi) = \begin{pmatrix} \cos\varphi/2 & -i\sin\varphi/2 \\ -i\sin\varphi/2 & \cos\varphi/2 \end{pmatrix}, \; R_z(\varphi) = \begin{pmatrix} e^{-i\varphi/2} & 0 \\ 0 & e^{i\varphi/2} \end{pmatrix}.$$

Multiplace operations are realized by the conditional dynamics of a closed quantum combined system. We give just a matrix representations of the basic double-place operations: controllable phase element $S$, controllable element $C_{\sigma_z}$, element $C_N$ (controllable $N$) and the element of exchange $Sw$ (see [2, 4] and references cited therein):

$$S \equiv \begin{pmatrix} \sigma_0 & 0 \\ 0 & s_\varphi \end{pmatrix}, \; C_{\sigma_z} \equiv \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}, \; C_N \equiv \begin{pmatrix} \sigma_0 & 0 \\ 0 & N \end{pmatrix}, \; Sw \equiv \begin{pmatrix} n & a^+ \\ a & 1-n \end{pmatrix}.$$

In essence, a quantum algorithm of this problem is reduced to the construction of such a time-dependent Hamiltonian, which contains these operators and their compositions, with the exception of the input and output processes (measurements), which require projectors as well. Currently, a number of quantum algorithms are found for specific tasks, such as Deutsch - Jos, Grover, Shor and others algorithms [2].

Here we should distinguish two classes of problems. The first one is reduced to the organization of the mathematical calculation process on a quantum computer in order to speed up the calculations in comparison with a classical computer. This direction is most likely a quantum cybernetics. The second class of problems does not contain mathematical calculations, as such, an example of which is Shor's algorithm on the factorization of an integer $N$. The expansion of the number $N$ in prime factors is realized on the base of quantum Fourier transform and by measuring the period of some $N$-dependent function by the interference pattern. Shor's algorithm is a physical computing and the first example of computing exponentially faster than classical one.

## 5. Artifical Intelligence and Quantum Neuroinformatics

Classically correlated combined physical systems are widely used in the associative and parallel processing of information, in performing calculations and in physical modelling. In particular, currently the only way to create artificial intelligence is based so far on the use of these systems, also known as neural networks [8]. However, classically correlated combined physical systems are valid to perform polynomial algorithms only. At the same time for the construction of strong artificial intelligence, exponentially high efficiency of information processing is required. Now it becomes actual another new and more complicated problem: informal simulation of the mind and thinking. But in order the physical system could think of (that is, it is able to make an algorithm of its action, not only to realize the algorithms imposed in the system from outside), it is necessary the realization by it of the non-distributive algebra of quantum logic.

At least one of the ways to solve the above problems is associated with the unification of capabilities of classical neuroinformatics and quantum informatics, that is it is associated with the creation of quantum neuroinformatics. However, this unification is not trivial because of the incompatibility of the methods classical neuroinformatis and quantum dynamics of a closed system.

Consider a quantum implementation of a cyclic process of conditionally controlled iteration for the formation of knowledge. For the formation of conditions of repetition of an each cycle of

iteration, it is necessary to compare the state of a physical system to its state in the previous cycle or to some given state (expert state), the formation, by the results of comparison, of the conditions of the system dynamics variation with the help of feedback, the formation of the operator of conditionally controllable iteration and composition of the projection operator in the subspace of "codes of knowledge" of the state vector.

Since the state of a classical physical system can be known at any time, the realization of these operations is reduced merely to the realization of program operators induced by the algorithm [8]. The iteration is continued as long as the specified state is not found or some condition of closeness of the two output states is not satisfied. The output state of a macroscopic system is described by the values of characteristic quantities, the numerical values of which determine their equality or closeness. The physical system is only responsible for the efficiency of realizing the mathematical operations by the program. Such a method of knowledge formation and getting artificial intelligence does not apply when using quantum-correlated systems. First of all, the difficulties are associated with the fact that in principle we can not know the state of a quantum system without destroying the state itself or its coherence. This fact prevents the use of classical methods of comparison between different states by proximity and consequently, the formation of conditions for the variation of dynamics [9]. Moreover, the comparison must be made only by unitary operations to ensure the physical reversibility. Moreover, the comparison must be made only by unitary operations to ensure the physical reversibility. Finally, in the quantum case the proximity of states is not a trivial notion by itself. The quantum state is presented by a function of a complex unitary space and it is impossible to establish the equality or closeness of two states by one or a finite number of numerical values without a significant loss of the information.

The basis in the space of states of a coherently correlated combined quantum system of several qubits $A_i$ is formed by the tensor product of the bases of all the qubits. In the space $\mathcal{H}_k$ of states of combined system, constructed from this basis, if the coefficients of two superpositions of the states coincide in one basis, then these superpositions, up to a global phase, present the same state of the system. This known fact allows to realize the comparison of quantum states in a given basis unitary by components, i.e. by the numerical values of the components of amplitude. $\mathcal{H}_k$ also includes the space $\mathcal{H}_e$ of all entangled states, which are absent in $\mathcal{H}_{\otimes A_i}$, they have no classical analog and contain information on the states of two or more qubits, that is: $\mathcal{H}_k = \otimes_i \mathcal{H}_i = \mathcal{H}_{\otimes A_i} \oplus \mathcal{H}_e$. The latter property is the basis for the unitary implementation of logical scheme "if ... then ..., if ... then ...", to which the comparison operation of quantum states is reduced with the subsequent formation of the condition of the iteration cycle of the shift operator $\widehat{T}_n(U)$ for the cycle number:

$$\widehat{T}_n(U) \begin{pmatrix} \psi_n(0) \\ \psi_{n-1}(\tau) > \end{pmatrix} = \begin{pmatrix} \psi_{n+1}(0) \\ \psi_n(\tau) > \end{pmatrix}.$$

Let $\Delta_{ab} = |\,|a\rangle - |b\rangle\,|$ be the symmetric difference of the basis states $|a\rangle$ and $|b\rangle$. Then, based on the identity

$$\widehat{C}_N \begin{pmatrix} |a\rangle \\ |b\rangle \end{pmatrix} \equiv \begin{pmatrix} |a\rangle \\ \Delta_{ab} \end{pmatrix}. \tag{1}$$

, the condition of the repeating of iteration cycle can be formed by using the difference $\Delta_n \equiv |\,|\psi_n(\tau)\rangle - |\psi_{n-1}(\tau)\rangle\,|$ (or $\Delta_n \equiv |\,|\psi_n(\tau)\rangle - |\psi_e\rangle\,|$, in case of the presence of expert state). Indeed, if the qubit with the state $\Delta_n$ is used as controlling, then the similarity transformation $\widehat{C}_N \Delta_n \widehat{C}_N$ does not change the state of two involved qubits (because the operator $\widehat{C}_N$ is self-reversing ) . But the qubit with $\Delta_n$ can act as an intermediate supervisory by combining with the third qubit to perform conditional operations. In particular, with the help of $\Delta_n$ we form the operator $\widehat{U}_\varphi$ that shifts the phase of controlled qubit by a specific angle $\varphi$, if the qubit is in the state $|1\rangle$, otherwise, it produces no changes in the state of the third qubit.

If for some number $n_{np}$ the condition $\Delta_n = 0$ is obeyed, then it is obeyed for all $n \geq n_{np}$ and the corresponding state is a limiting state $|\psi_{np}\rangle$. It is obvious that in this case $\widehat{T}_n$ becomes and remains as identity operator. Then, for $n \geq n_{np}$ the whole iteration process is presented by the operator:

$$\widehat{T}_{it} = \widehat{1}\widehat{1} \ldots \widehat{T}_{n_{np}} \widehat{T}_{n_{np}-1} \ldots \widehat{T}_2 \widehat{T}_1 \equiv (\widehat{T})^{n \geq n_{np}}, \tag{2}$$

because the structure of the operators $\widehat{T}_n$ is the same for all cycles, it is denoted by $\widehat{T}$:

$$\widehat{T} \equiv \widehat{I}(2)\widehat{S}_w(2,3)\widehat{C}_N(3,2)\widehat{U}_\varphi(2,1)\widehat{C}_N(3,2)\widehat{U}_\tau(2)\widehat{C}_N(1,2). \tag{3}$$

Here, in (2) and (3), in order to avoid cumbersome matrix form of the presentation, near each operator the number of the qubit is written in the parentheses on which it acts and for binary operators the first number corresponds to the controlling, and the second one to the controlled. However, the algorithm presented above does not contain the indication to establish the existence of the limiting state, and also to find the value of $n_{np}$. This narrows the range of the algorithm application and reduces the efficiency of the search of limiting state. The probability of finding the limiting state can be increased by random variation of the phase in the operator $\widehat{U}_\varphi$. But this does not solve the problem of finding the values of $n_{np}$.

It seems that the extension to invariance of the search algorithm for a limiting state is the most consistent. In other words, it is necessary to find an algorithm that for a given $N$ steps would lead to the state closest to limiting one regardless of the existence of the limiting state and the nature of the convergence to it. For this purpose, in turn, it is necessary to "extend" the measure of closeness of quantum states, based on the identity (1). In this relation $\widehat{C}_N$ and $\Delta_{ab}$ can be generalized. For example, we replace the operator $\widehat{C}_N$ by the operator $\widehat{K}_N$ which reverses the phase of the state $|b\rangle$ in the case when the eigenvalues of the states $|a\rangle$ and $|1\rangle$ differ by some nonzero quantity $\delta$. Physically, this means that instead of a "naked" entangled states of vectors $|a\rangle$ and $|b\rangle$ in the subspace $\mathcal{H}_e$, we use a kind of "dressed" entanglement of "$\delta$-dressed" state $|a\rangle$ with the state $|b\rangle$. It is mathematically equivalent to replace $\mathcal{H}_e$ by some of its non-orthogonal "$\delta$-dressed" extension.

## 6. Conclusion

Academician Sahakyan managed to put on a good track the birth and development of a powerful school of physicists of the microworld. And this school has participated in the implementation of predictions by Sahakyan. Fundamental researches were conducted on quantum field theory, on the creation of quantum neuroinformatics. He and his followers, E. Chubaryan, D. Sedrakian, Yu. Vardanyan, R. Avakian, G. Harutyunyan, A. Saharian and many others transferred and will transfer the ideas and methods of quantum physics to astrophysics and gravitation.

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